On δg*-Closed Sets In Bitopological Spaces

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Abstract

The aim of this paper is to introduce the concept of (i, j) - δg-closed sets in bitopological spaces and study their properties. We prove that this class lies between the class of (i, j) - δ-closed sets and the class of (i, j) - δg-closed sets. Also we discuss some basic properties and applications of (i, j)- δg*-closed sets, which defines a new class of spaces namely (i, j)-* $_{\delta g^*}T_{1/2}$ *-spaces, (i, j)-* $_{\delta g^*}T^*_{1/2}$ *-spaces, (i, j)-* $_{\delta g^*}T_{\delta_{1/2}}$ *-spaces and (i, j)-* $_{\delta g}T_{1/2}$ *-spaces.*

Keywords: (1, 2)- δg-closed set, (1, 2)- δ-closed set, (1, 2)- δg*-closed set.

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1. Introduction

A triple (X, τ_1, τ_2) where X is a non-empty set and τ_1 and τ_2 are topologies on X is called a bitopological space and Kelly [8] initiated the study of such spaces. Njastad[12], Velicko [20] introduced the concept of α-open sets and δ-closed sets respectively. Dontchev and Ganster [4] studied δ-generalized closed set in topological spaces. Levine [10] introduced generalization of closed sets and discussed their properties. In 1985, Fukutake [5] introduced the concepts of g-closed sets in bitopological spaces and after that several authors turned their attention towards generalizations of various concepts of topology by considering bitopological spaces. Also M. E. Abd El-Monsef [1] et al investigated α-closed sets in topological spaces. Sheik John et al [14] introduced g*-closed sets in bitopological spaces. Sudha et al. [16] introduced the concept of δg*-closed sets in topological spaces and investigated its relationship with the other types of closed sets. The purpose of the present paper is to define a new class of closed sets called (i, j) - δg*-closed sets and we discuss some basic properties of (i, j)-δg*-closed sets in bitopological spaces. Applying these sets, we obtain the new spaces called (i, j)- $_{\delta g^*}T_{1/2}$ - space, (i, j)- $_{\delta g^*}T^*_{1/2}$ -space, (i, j)- $_{\delta g^*}T_{\delta_{1/2}}$ - space and (i, j)- $_{\delta g}T_{1/2}$ - space.

2. Preliminaries

If A is a subset of X with the topology τ, then the closure of A is denoted by τ-cl(A) or cl(A), the interior of A is denoted by τ-int(A) or int(A) and the complement of A in X is denoted by A^c.

2.1. Definition

A subset A of a topological space (X, τ) is called a

(i) semi-open set [9] if A ⊆ cl(int(A)).
(ii) α-open set [12] if A ⊆ int(cl(int(A))).
(iii) regular open set [16] if A = int (cl(A)).
(iv) Pre-open set [11] if A ⊆ int(cl(A)).

The complement of a semi open (resp. α-open, regular open, pre-open) set is called semi-closed (resp. α-closed, regular closed, pre-closed).

The semi-closure [3] (resp. α-closure [12], pre-closure [11]) of a subset A of (X, τ), denoted by scl(A) (resp. $cl_\alpha(A)$, pcl(A)) is defined to be the intersection of all semi-closed (resp.α-closed, pre-closed) sets containing A. It is known that scl(A) (resp. $cl_\alpha(A)$, pcl(A)) is a semi-closed (resp.α-closed, pre-closed) set.

2.2. Definition

The δ-interior [20] of a subset A of X is the union of all regular open sets of X contained in A and is denoted by $int_\delta(A)$ .The subset A is called δ-open [20] if $A = int_{\delta}(A)$. i.e., a set is δ-open if it is the union of regular open sets, the complement of a δ-open is called δ-closed. Alternatively, a set $A \subseteq X$ is called δ-closed [20] if $A = cl_{\alpha}(A)$, where $cl_{\delta}(A) = \{x \in X; \text{int}(cl(U)) \cap A \neq \phi, U \in \tau \text{ and } x \in U\}$. Every δ-closed set is closed [20].

2.3. Definition

A subset A of (X,τ) is called

1) δ-generalized closed (briefly δg-closed) [4] if $cl_{\delta}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
2) generalized closed (briefly g-closed) [10] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
3) g^*- closed [19] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (X, τ).

Throughout this paper by the spaces X and Y represent non-empty bitopological spaces on which no separation axioms are assumed, unless otherwise mentioned and the integers $i, j \in \{1, 2\}$.

For a subset A of X, $\tau_i - cl(A)$ (resp. $\tau_i - int(A)$, $\tau_i - pcl(A)$) denote the closure (resp. interior, pre closure) of A with respect to the topology τ_i. We denote the family of all g-open subsets of X with respect to the topology τ_i by $GO(X, \tau_i)$ and the family of all τ_j-closed sets is denoted by the symbol F_j. By (i, j) we mean the pair of topologies (τ_i, τ_j).

2.4. Definition

A subset A of a bitopological space (X, τ_1, τ_2) is called

1) (i, j) g-closed [5] if τ_j-cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is open in τ_i.
2) (i, j) g*-closed [14] if τ_j-cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is g-open in τ_i.
3) (i, j) rg-closed [2] if τ_j-cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is regular open in τ_i.
4) (i, j) wg-closed [6] if τ_j-cl(τ_i-int(A)) $\subseteq$ U whenever $A \subseteq U$ and U is open in τ_i.
5) (i, j) gpr-closed [6] if τ_j-pcl(A) $\subseteq$ U whenever $A \subseteq U$ and U is regular open in τ_i.
6) (i, j) αg*-closed [18] if τ_j-cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is α-open in τ_i.
7) (i, j) g*p-closed [17] if τ_j-pcl(A) $\subseteq$ U whenever $A \subseteq U$ and U is g-open in τ_i.
8) (i, j) w-closed [7] if τ_j-cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is semi-open in τ_i.
9) (i, j) sαg*-closed [13] if τ_j-αcl(A) $\subseteq$ U whenever $A \subseteq U$ and U is g*-open in τ_i.

2.5. Definition

A bitopological space (X, τ_1, τ_2) is called

1) (i, j)-$T_{1/2}$-space [5] if every (i, j)-g-closed set is τ_j-closed.
2) (i, j)-$T^*_{1/2}$-space [14] if every (i, j)-g*-closed set is τ_j-closed.
3) (i, j)-${}^*T_{1/2}$-space [14] if every (i, j)-g-closed set is (i, j)-g*-closed.

3. (i, j) - δg*-closed sets in bitopological spaces

In this section we introduce the concept of (i, j) - δg*-closed sets in bitopological spaces and discuss the related properties.

3.1. Definition

A subset A of a bitopological space (X, τ_1, τ_2) is said to be an (i, j) - δg*-closed set if $\tau_j - cl_{\delta}(A) \subseteq U$, whenever $A \subseteq U$ and $U \in GO(X, \tau_i)$

We denote the family of all (i, j) - δg*-closed sets in (X, τ_1, τ_2) by D^*_{δ}(i, j).

3.2. Remark

By setting $\tau_1 = \tau_2$ in Definition 3.1., a (i, j) - δg*-closed set is δg*-closed.

3.3. Proposition

If A is τ_j-δ-closed subset of (X, τ_1, τ_2), then A is (i, j)-δg*-closed.

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Proof: Let A be a τ_j-δ-closed subset of (X, τ_1, τ_2). Then $\tau_j - cl_\delta(A) = A$. Let $U \in GO(X, \tau_i)$ such that $A \subseteq U$, then $\tau_j - cl_\delta(A) = A \subseteq U$ which implies A is (i, j) - δg*-closed.

The converse of the above proposition is not true as seen from the following example.

3.4. Example

Let X = {a, b, c}, τ_1 = {X, ϕ, {a}}, τ_2 = {X, ϕ ,{b}, {c},{a, b},{b, c}}. Then the subset {b, c} is (1, 2) - δg*-closed but not τ_2 - δ-closed set.

3.5. Proposition

If A is both τ_i-g-open and (i, j) - δg*-closed, then A is τ_j-δ-closed.

Proof: Let A be both τ_i-g-open and (i, j)-δg*-closed. Since A is (i, j)-δg*-closed, we have $A \subseteq U$ and $U \in GO(X, \tau_i)$ which implies $\tau_j - cl_\delta(A) \subseteq U$ and since A is τ_i-g-open. Put A = U, then we have $\tau_j - cl_\delta(A) \subseteq A$, implies A is a τ_j-δ-closed set.

3.6. Proposition

If A is both τ_i-g-open and (i, j)- δg*-closed, then A is τ_j-closed.

Proof: Since δ – closedness ⇒ closedness, the result follows the above Proposition 3.5.

3.7. Proposition

If $A, B \in D^*_\delta(i, j)$, then $A \cup B \in D^*_\delta(i, j)$.

Proof: Let A and B be (i,j)-δg*-closed. Let $A \cup B \subseteq U$ where $U \in GO(X, \tau_i)$. Now $A \cup B \subseteq U$ implies $A \subseteq U$ and $B \subseteq U$. Since $A, B \in D^*_\delta(i, j)$, implies $\tau_j - cl_\delta(A) \subseteq U$ and $\tau_j - cl_\delta(B) \subseteq U$. Then $(\tau_j - cl_\delta(A) \cup \tau_j - cl_\delta(B)) \subseteq U$. That is $\tau_j - cl_\delta(A \cup B) \subseteq U$. Hence $A \cup B \in D^*_\delta(i, j)$,

3.8. Remark

The intersection of two (i, j)-δg*-closed need not be (i, j)-δg*-closed as seen from the following example.

3.9. Example

Let X = {a, b, c}, τ_1 = {X, ϕ, {a}}, τ_2 = {X, ϕ, { b}, {c},{a, b},{b, c}}. Then {a, b} & {b, c} are (1, 2) - δg*-closed sets but {a, b} ∩ {b, c} = {b} is not (1, 2) - δg*-closed.

3.10. Proposition

For each element x of (X, τ_1, τ_2), $\{x\}$ is τ_i-g-closed or $\{x\}^c$ is (i, j)- δg*-closed.

Proof: If $\{x\}$ is τ_i-g-closed, then the proof is over. Assume $\{x\}$ is not τ_i-g-closed. Then $\{x\}^c$ is not τ_i-g-open. So the only τ_i-g-open containing $\{x\}^c$ in X. Hence $\{x\}^c$ is (i, j) - δg*-closed.

3.11. Proposition

If A is (i, j) - δg*-closed, then $\tau_j - cl_\delta(A)/A$ contains no non-empty τ_i-g-closed set.

Proof: Let A be (i, j)-δg*-closed and F be a non empty τ_i-g-closed subset of $\tau_j - cl_\delta(A)/A$. Now $F \subseteq \tau_j - cl_\delta(A)/A = \tau_j - cl_\delta(A) \cap A^c$ which implies $F \subseteq \tau_j - cl_\delta(A)$ and $F \subseteq A^c$. Therefore $A \subseteq F^c$. Since F^c is τ_i-g-open and A is (i, j)-δg*-closed in X, we have $\tau_j - cl_\delta(A) \subseteq F^c$ which implies that $F \subseteq (\tau_j - cl_\delta(A)) \cap (\tau_j - cl_\delta(A))^c$. Therefore F = ϕ. Hence $\tau_j - cl_\delta(A)/A$ contains no non-empty τ_i-g-closed set.

The following example shows that the reverse implication of the above theorem is not true.

3.12. Example

Let X = {a, b, c}, τ_1 = {X, ϕ, {a}, {a, c}}, τ_2 = {X, ϕ, {a, b}}. If A = {a}, then $\tau_j - cl_\delta(A)/A$ = {b, c} does not contain any non-

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empty τ_1-g-closed set. But A is not (1, 2) - δg*-closed.

3.13. Corollary

If A is (i, j)-δg*-closed in (X, τ_1, τ_2), then A is τ_j-δ-closed if and only if $\tau_j - cl_\delta(A)/A$ is τ_i-g-closed.

Proof: (Necessity) Let A $\subseteq D^*_\delta(i, j)$ and let A be τ_j-δ-closed.

Then $\tau_j - cl_\delta(A) = A$. i.e., $\tau_j - cl_\delta(A)/A = \phi$ and hence $\tau_j - cl_\delta(A)/A$ is τ_i-g-closed.

(Sufficiency) If $\tau_j - cl_\delta(A)/A$ is τ_i-g-closed, then by Proposition 3.11, $\tau_j - cl_\delta(A)/A = \phi$, since A is (i, j)-δg*-closed. Hence $\tau_j - cl_\delta(A) = A$. Therefore A is τ_j-δ-closed.

3.14. Proposition

If A is an (i, j)-δg*-closed set, then $\tau_i - cl_\delta(x) \cap A \neq \phi$ holds for each $x \in \tau_j - cl_\delta(A)$

Proof: Let A be (i, j)- δg*-closed and we know $\tau_i \subseteq GO(X, \tau_i)$. Suppose $\tau_i - cl_\delta(x) \cap A \neq \phi$ for some $x \in \tau_j - cl_\delta(A)$, then $A \subseteq X - \tau_j - cl_\delta(x) = B$, say. Then B is a τ_i-δ-open set. Since a δ-open set is an open set and a open set is g-open, B is g-open in τ_i. Since A is (i, j)- δg*-closed, we get $\tau_j - cl_\delta(A) \subseteq B = X - \tau_j - cl_\delta(x)$. Then $\tau_j - cl_\delta(A) \cap \tau_j - cl_\delta(x) = \phi$ which implies that $\tau_j - cl_\delta(A) \cap (x) = \phi$. Hence $x \notin \tau_j - cl_\delta(A)$, which is a contradiction.

The converse of the above proposition is not true as seen in the following example.

3.15. Example

Let X = {a, b, c}, τ_1 = {X, ϕ, {a}}, τ_2 = {X, ϕ, {a}, {b, c}}. The subset A = {b} in (X, τ_1, τ_2) is not (1, 2) - δg*-closed. However $\tau_1 - cl_\delta(x) \cap A \neq \phi$ holds for each $x \in \tau_2 - cl_\delta(A)$.

IJERT

3.16. Proposition

If A is an (i, j)-δg*-closed set of (X, τ_i, τ_j) such that $A \subseteq B \subseteq \tau_j - cl_\delta(A)$, then B is also an (i, j)- δg*-closed set of (X, τ_i, τ_j).

Proof: Let U be a τ_i -g-open set in (X, τ_i, τ_j) such that B ⊆ U and hence A ⊆ U. Since A is (i, j)-δg*-closed, $\tau_j - cl_\delta(A) \subseteq U$. Since $A \subseteq B \subseteq \tau_j - cl_\delta(A)$, $\tau_j - cl_\delta(B) \subseteq \tau_j - cl_\delta(\tau_j - cl_\delta(A)) = \tau_j - cl_\delta(A) \subseteq U$. Hence $\tau_j - cl_\delta(B) \subseteq U$ which implies that B is a (i, j)- δg*-closed set of (X, τ_i, τ_j).

3.17. Proposition

Let A ⊆ Y ⊆ X and suppose that A is (i, j)-δg*-closed in X. Then A is (i, j)- δg*-closed relative to Y.

Proof: Let $A \in D^*_\delta(i, j)$ & A ⊆ Y ∩ U, U is g-open in X. A ⊆ Y ∩ U implies A ⊆ U and since A $\in D^*_\delta(i, j)$, $\tau_j - cl_\delta(A) \subseteq U$. That is $\tau_j - cl_\delta(A) \cap Y \subseteq U \cap Y$. Hence $\tau_j - cl_{\delta_Y}(A) \subseteq U \cap Y$. Therefore A is (i, j) - δg*-closed relative to Y.

3.18. Theorem

In a bitopological space (X, τ_1, τ_2), $GO(X, \tau_i) \subseteq F_{\delta_j}$ if and only if every subset of X is an (i, j) - δg*-closed set, where F_{δ_j} is the collection of δ-closed sets with respect to τ_j.

Proof: Suppose that $GO(X, \tau_i) \subseteq F_{\delta_j}$. Let A be a subset of (X, τ_1, τ_2) such that A ⊆ U where $U \in GO(X, \tau_i)$. Then $\tau_j - cl_\delta(A) \subseteq \tau_j - cl_\delta(U) = U$. Therefore A is (i, j)-δg*-closed set.

Conversely, suppose that every subset of X is (i, j)- δg*-closed. Let $U \in GO(X, \tau_i)$. Since U is (i, j) - δg*-closed, we have $\tau_j - cl_\delta(U) \subseteq U$. Therefore $U \in F_{\delta_j}$ and hence $GO(X, \tau_i) \subseteq F_{\delta_j}$.

3.19. Proposition

Every (i, j)- δg*-closed set is (i, j)-g-closed.

Proof: Let A be (i, j) - δg*-closed. Let A ⊆ U and U be a open set in τ_i. Since every open set is g-open, U is a g-open set. Then $\tau_j - cl_\delta(A) \subseteq U$, we know that $\tau_j - cl(U) \subseteq \tau_j - cl_\delta(U) \subseteq U$. Hence A is (i, j)- g-closed.

3.20. Remark

A (i, j)-g-closed need not be (i, j)- δg*-closed as shown in the following example.

3.21. Example

Let X = {a, b, c}, τ_1 = {X, ϕ, {a}}, τ_2 = {X, ϕ, {a, b}}. Then the set {b} is (1, 2)-g-closed but not (1, 2) - δg*-closed.

3.22. Proposition

Every (i, j) - δg*-closed set is (i, j) - g*-closed.

Proof: Let A be (i, j) - δg*-closed. Let A ⊆ U and U be a g-open set in τ_i Then $\tau_j - cl_\delta(A) \subseteq U$, we know that $\tau_j - cl(U) \subseteq \tau_j - cl_\delta(U) \subseteq U$. Hence A is (i, j)- g*-closed.

3.23. Remark

A (i, j)-g*-closed need not be (i, j) - δg*-closed as shown in the following example.

3.24. Example

Let X = {a, b, c}, τ_1={X, ϕ, {a, b}}, τ_2={X, ϕ, {b, c}}. Then the set {a} is (1, 2)-g*-closed but not (1, 2) - δg*-closed.

3.25. Proposition

Every (i, j)- δg*-closed set is (i, j)-rg-closed.

Proof: The proof follows from every regular open set is g-open.

3.26. Remark

A (i, j)-rg-closed need not be (i, j)- δg*-closed as shown in the following example.

3.27. Example

Let X = {a, b, c}, τ_1 = {X, ϕ, {a}, {a, b}}, τ_2 = {X, ϕ, {a, b}}. Then the set {a, b} is (1, 2)-rg-closed but not (1, 2)- δg*-closed.

3.28. Proposition

Every (i, j)- δg*-closed set is (i, j)-wg-closed.

Proof: Let A be (i, j) - δg*-closed. Let A ⊆ U and U be a open set in τ_i Since every open set is g-open, U is g-open in τ_i Now $\tau_i - int(A) \subseteq A$, implies $\tau_j - cl(\tau_i - int(A)) \subseteq \tau_j - cl(A) \subseteq \tau_j - cl_\delta(A)$. Since A is (i, j) - δg*-closed, $\tau_j - cl_\delta(A) \subseteq U$. Therefore $\tau_j - cl(\tau_i - int(A)) \subseteq U$. Hence A is (i, j)-wg-closed.

3.29. Remark

A (i, j)-wg-closed need not be (i, j) - δg*-closed as shown in the following example.

3.30. Example

Let X = {a, b, c}, τ_1= {X, ϕ, {a}}, τ_2= {X, ϕ, {b, c}}. Then the set {b} is (1, 2)-wg-closed but not (1, 2)- δg*-closed.

3.31. Proposition

Every (i,j)- δg*-closed set is (i, j)-αg*-closed.

Proof: Let A be (i, j)- δg*-closed. Let A ⊆ U ∈ GO(X, τ_i), since τ_i ∈ GO(X, τ_i).Then $\tau_j - cl_\delta(A)$ ⊆ U. We know τ_j-αcl(A) ⊆ $\tau_j - cl_\delta(A)$ which implies $\tau_j - \alpha cl(A) \subseteq U$ Therefore A is (i, j)-αg*-closed.

3.32. Remark

A (i, j)-αg*-closed need not be (i, j) - δg*-closed as shown in the following example.

3.33. Example

Let X = {a, b, c}, τ_1= {X, ϕ, {a, b}}, τ_2= {X, ϕ, {a},{b},{a, b}}. Then the set {a} is (1, 2)-αg*-closed but not (1, 2)- δg*-closed.

3.34. Proposition

Every (i,j)-δg*-closed set is(i,j)-gpr-closed.

Proof: Let A be (i, j)- δg*-closed. Let A ⊆ U and U be regular open. Since every regular open set is g-open, U is g-open. Since A is (i, j)- δg*-closed, $\tau_j - cl_\delta(A)$ ⊆ U, We

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know that $\tau_j - pcl(A) \subseteq \tau_j - cl_\delta(A)$. That is, $\tau_j - pcl(A) \subseteq \tau_j - cl_\delta(A) \subseteq U$ Therefore A is (i, j)-gpr-closed.

3.35. Remark

A (i, j)-gpr-closed need not be (i, j)- δg*-closed as shown in the following example.

3.36. Example

Let X = {a, b, c}, τ_1 = {X, ϕ, {a, b}}, τ_2 = {X, ϕ, {a},{b, c}}. Then the set {b} is (1, 2)-gpr-closed but not (1, 2)- δg*-closed.

3.37. Proposition

Every (i, j)- δg*-closed set is (i, j)-g*p-closed.

Proof: Let A be (i, j)- δg*-closed. Let $A \subseteq U$ and U is g-open in τ_i. Then $\tau_j - cl_\delta(A) \subseteq U$. We know $\tau_j - pcl(A) \subseteq \tau_j - cl_\delta(A)$. Therefore $\tau_j - pcl(A) \subseteq U$. Hence A is (i, j)- g*p-closed.

3.38. Remark

A (i, j)-g*p-closed need not be (i, j) - δg*-closed as shown in the following example.

3.39. Example

Let X = {a, b, c}, τ_1 = {X, ϕ, {a, b}}, τ_2 = {X, ϕ, {a},{b, c}}. Then the set {b} is (1, 2)-g*p-closed but not (1, 2)- δg*-closed.

3.40. Proposition

Every (i, j)- δg*-closed set is (i, j)-sαg*-closed.

Proof: Let A be (i, j) - δg*-closed. Let $A \subseteq U$ and U is g*-open set in τ_i. Since every g*-open set is g-open, U is g-open. Then $\tau_j - cl_\delta(A) \subseteq U$. We know $\tau_j - \alpha cl(A) \subseteq \tau_j - cl_\delta(A)$, which implies $\tau_j - \alpha cl(A) \subseteq U$. Therefore A is (i, j)-sαg*-closed.

3.41. Remark

A (i, j)-sαg*-closed need not be (i, j)- δg*-closed as shown in the following example.

3.42. Example

Let X = {a, b, c}, τ_1 = {X, ϕ, {a, b}}, τ_2 = {X, ϕ, {a},{b},{a, b}}. Then the set {b} is (1, 2)-sαg*-closed but not (1, 2)- δg*-closed.

3.43. Proposition

Every (i, j)- δg*-closed set is (i, j)-δg-closed.

Proof: The proof follows from the fact that every open set is g-open.

3.44. Remark

A (i, j)- δg -closed need not be (i, j)- δg*-closed as shown in the following example.

3.45. Example

Let X = {a, b, c}, τ_1 = {X, ϕ, {a}}, τ_2 = {X, ϕ, {a, b}}. Then the set {b} is (1, 2)- δg -closed but not (1,2)- δg*-closed.

3.46. Remark

The following examples show that (i, j)- w-closed and (i, j)- δg*-closed are independent to each other.

3.47. Example

Let X = {a, b, c}, τ_1 = {X, ϕ, {a}}, τ_2 = {X, ϕ, {a},{a, b}}. Then the set {a} is (1, 2)-w-closed but not (1,2)- δg*-closed.

3.48. Example

Let X = {a, b, c}, τ_1 = {X, ϕ, {a}, {a, b}}, τ_2 = {X, ϕ, {a, b}}. Then the set {a} is (1, 2) - δg*-closed but not (1,2)- w-closed.

3.49. Remark

The following diagram has shown the relationship of (i, j)- δg*-closed sets with other known existing sets. A ⟶ B represents A implies B but not conversely and A ⟷ B represents A and B are independent to each other.

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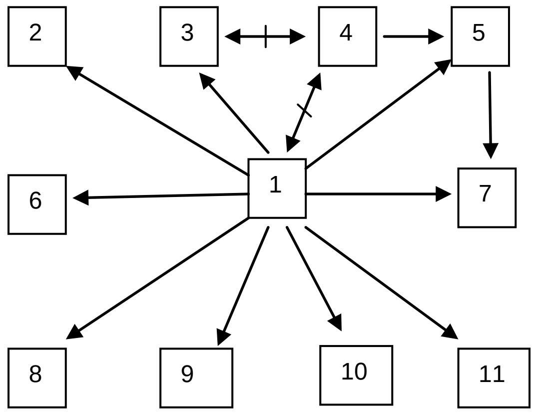

Figure 1

1. (i, j)- δg*-closed set, 2. (i, j)- wg-closed set, 3. (i, j)- g*-closed set, 4. (i, j)- w-closed set, 5. (i, j)- g-closed set, 6. (i, j)- sαg*-closed set, 7. (i, j)- rg-closed set, 8. (i, j)- g*p-closed set, 9. (i, j)- gpr-closed set, 10. (i, j)- δg-closed set, 11. (i, j)- αg*-closed set,

4. Applications

In this section we introduce the new closed spaces namely (i, j)- $_{\delta g^*}T_{\frac{1}{2}}$-space, (i, j)- $_{\delta g^*}T^*_{\frac{1}{2}}$-space, (i, j)- $_{\delta g^*}T_{\delta_{\frac{1}{2}}}$-space and (i, j)- $_{\delta g}T_{\frac{1}{2}}$-space in bitopological spaces.

4.1. Definition

A bitopological space (X, τ_1, τ_2) is said to be a

1) (i, j)- $_{\delta g}T_{\frac{1}{2}}$- space if every (i, j)- g-closed set is (i, j)-δg-closed.
2) (i, j)- $_{\delta g^*}T_{\frac{1}{2}}$-space if every (i, j)-g-closed set is (i, j)- δg*-closed.
3) (i, j)- $_{\delta g^*}T^*_{\frac{1}{2}}$-space if every (i, j)-g*-closed set is (i, j)- δg*-closed.
4) (i, j)- $_{\delta g^*}T_{\delta_{\frac{1}{2}}}$-space if every (i, j)- δg-closed set is (i, j)- δg*-closed.

4.2. Proposition

Every (i, j)- $_{\delta g^*}T_{\frac{1}{2}}$-space is a (i, j) - $_{\delta g^*}T^*_{\frac{1}{2}}$-space.

Proof: Let X be a (i, j) - $_{\delta g^*}T_{\frac{1}{2}}$-space and A be (i, j)- g*-closed. Since every (i, j)- g*-closed set is (i, j)- g-closed. Then A is (i, j)- g-closed. By assumption, we get A is (i, j)- δg*-closed. Hence X is a (i, j) - $_{\delta g^*}T^*_{\frac{1}{2}}$-space.

The converse of the above proposition is not true as seen by the following example.

4.3. Example

Let X = {a, b, c}, τ_1 = {X, ϕ, {a, b}}, τ_2 = {X, ϕ, {b}, {a, b}}. Then (X, τ_1, τ_2) is (i, j)- $_{\delta g^*}T^*_{\frac{1}{2}}$-space. But {a, b} is (i, j)- g-closed but not (i, j)-δg*-closed. Hence (X, τ_1, τ_2) is not $_{\delta g^*}T_{\frac{1}{2}}$-space.

4.4. Proposition

Every (i, j) - $_{\delta g^*}T_{\frac{1}{2}}$-space is a (i, j) - $_{\delta g^*}T_{\delta_{\frac{1}{2}}}$-space.

Proof: Let X be a (i, j) - $_{\delta g^*}T_{\frac{1}{2}}$-space and A be (i, j)- δg-closed. Since every (i, j)- δg -closed set is (i, j)- g-closed. Then A is (i, j)- g-closed. By assumption, we get A is (i, j)- δg*-closed. Hence X is a (i, j)- $_{\delta g^*}T_{\delta_{\frac{1}{2}}}$-space.

The converse of the above proposition is not true as seen by the following example.

4.5. Example

Let X = {a, b, c}, τ_1 = {X, ϕ, {b}, {c}, {b, c}, {a, b}}, τ_2 = {X, ϕ, {a}}. Then (X, τ_1, τ_2) is (i, j)- $_{\delta g^*}T_{\delta_{\frac{1}{2}}}$- space not (i, j)- $_{\delta g^*}T_{\frac{1}{2}}$-space. Since {b, c} is (i, j)- g-closed but not (i, j)-δg*-closed. Hence (X, τ_1, τ_2) is not $_{\delta g^*}T_{\frac{1}{2}}$-space.

4.6. Proposition

Every (i, j)- $_{\delta g^*}T_{\frac{1}{2}}$- space is (i, j)- $_{\delta g}T_{\frac{1}{2}}$-space.

Proof: Let X be a (i, j)- $_{\delta g^*}T_{\frac{1}{2}}$- space and A be (i, j)- g-closed. Then A is (i, j)- δg*-closed. Since every (i, j)-δg*-closed set is (i, j)- δg-closed. We get A is (i, j)- δg-closed. Hence X is a (i, j)- $_{\delta g}T_{\frac{1}{2}}$-space.

The converse of the above proposition is not true as seen by the following example.

IJERT

4.7. Example

Let X = {a, b, c}, τ_1 = {X, ϕ, {a}}, τ_2 = {X, ϕ, {a, b}}. Then (X, τ_1, τ_2) is (i, j)- ${}_{\delta g}T_{1/2}$-space not (i, j)- ${}_{\delta g^*}T_{1/2}$- space. Since {b} is (i, j)- g-closed but not (i, j)–δg*–closed.

4.8. Proposition

(X, τ_1, τ_2) is both (i, j)- ${}_{\delta g}T_{1/2}$-space and (i, j)- ${}_{\delta g^*}T_{\delta_{1/2}}$-space if and only if it is a (i, j)- ${}_{\delta g^*}T_{1/2}$- space.

Proof : (Necessity): Let (X, τ_1, τ_2) be (i, j)- ${}_{\delta g}T_{1/2}$-space and (i, j)- ${}_{\delta g^*}T_{\delta_{1/2}}$-space. Consider A is (i, j)-g*-closed. Then A is (i, j)-g-closed. Since (X, τ_1, τ_2) be (i, j)- ${}_{\delta g}T_{1/2}$-space, A is (i, j)- δg-closed. Since (X, τ_1, τ_2) be a (i, j)- ${}_{\delta g^*}T_{\delta_{1/2}}$-space, A is (i, j)- δg*-closed. Therefore (X, τ_1, τ_2) is a (i, j)- ${}_{\delta g^*}T_{1/2}$- space.

(**Sufficiency**): It satisfies by Proposition 4.4 and Proposition 4.6.

4.9. Remark

The following examples show that (i, j)- ${}_{\delta g^*}T_{\delta_{1/2}}$ and (i, j)- ${}_{\delta g^*}T^*_{1/2}$ are independent to each other.

4.10. Example

Let X = {a, b, c}, τ_1 = {X, ϕ, {b}, {c}, {b, c}, {a, b}}, τ_2 = {X, ϕ, {a}}. Then (X, τ_1, τ_2) is (i, j)- ${}_{\delta g^*}T_{\delta_{1/2}}$-space. But {b, c} is (i, j)- g*-closed but not (i, j)–δg*–closed.

4.11. Example

Let X = {a, b, c}, τ_1 = {X, ϕ, {a}, {b, c}}, τ_2 = {X, ϕ}. Then (X, τ_1, τ_2) is (i, j)- ${}_{\delta g^*}T^*_{1/2}$-space. But {b} is (i, j)- δg-closed but not (i, j)–δg*–closed.

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4.12. Remark

The following examples show that (i, j)- ${}_{\delta g^*}T_{\delta_{1/2}}$ and (i, j)- ${}_{\delta g}T_{1/2}$ are independent to each other.

4.13. Example

Let X = {a, b, c}, τ_1 = {X, ϕ, {a}}, τ_2 = {X, ϕ, {a, b}}. Then (X, τ_1, τ_2) is (i, j)- ${}_{\delta g}T_{1/2}$-space. But {b} is (i, j)- δg-closed but not (i, j)–δg*–closed.

4.14. Example

Let X = {a, b, c}, τ_1 = {X, ϕ, {a, b}}, τ_2 = {X, ϕ, {a}, {b}, {a, b}}. Then (X, τ_1, τ_2) is (i, j)- ${}_{\delta g^*}T_{\delta_{1/2}}$-space. But {a, b} is (i, j)- g-closed but not (i, j)–δg–closed.

4.15. Remark

The following examples shows that (i, j)- ${}_{\delta g^*}T^*_{1/2}$ and (i, j)- ${}_{\delta g}T_{1/2}$ are independent to each other.

4.16. Example

Let X = {a, b, c}, τ_1 = {X, ϕ, {a}}, τ_2 = {X, ϕ, {a, b}}. Then (X, τ_1, τ_2) is (i, j)- ${}_{\delta g}T_{1/2}$-space. But {c} is (i, j)- g*-closed but not (i, j)-δg*-closed.

4.17. Example

Let X = {a, b, c}, τ_1 = {X, ϕ, {a, b}}, τ_2 = {X, ϕ, {a}, {b}, {a, b}}. Then (X, τ_1, τ_2) is (i, j)- ${}_{\delta g^*}T^*_{1/2}$-space. But {a, b} is (i, j)- g-closed but not (i, j)–δg–closed.

4.18. Remark

The following diagram has shown the relationship of (i, j)–δg*–closed spaces with other known existing space. A ⟶ B represents A implies B but not conversely and A⟷B represents A and B are independent to each other.

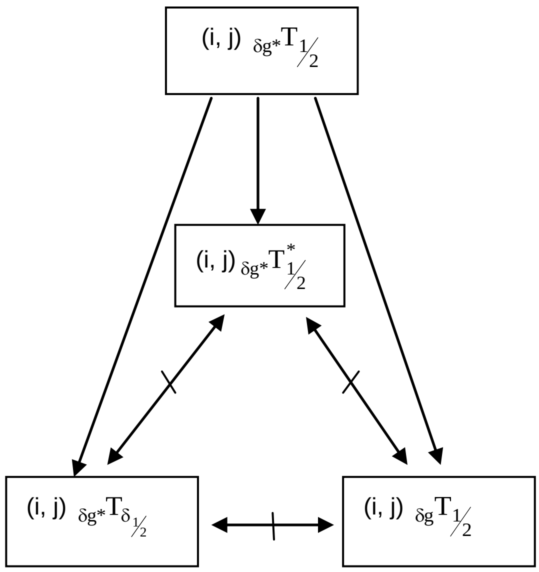

Figure 2

5. References

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