ISSN: 2278-0181
Vol. 1 Issue 6, August - 2012

ON MHD FREE CONVECTION SLIP FLOW OF AN ELASTICO-VISCOUS FLUID WITH HEAT SOURCE AND MASS TRANSFER THROUGH A POROUS MEDIUM

***M. M. Hamza, *B.Y Isah, *N. S Dauran and **I. D Yale**

*Department of Mathematics, Usmanu Danfodiyo University, Sokoto, Nigeria

**Department of Mathematics, Kebbi State University of Science and Technology, Aleiro, Nigeria

ABSTRACT

MHD free convection elastic-viscous slip fluid flow with Heat source and chemical reaction through a porous medium is analyzed, by taking into account the effect of mass transfer and Slip parameter. A uniform magnetic field acts perpendicular to the porous surface, which absorbs the fluid with a constant suction velocity. The dimensionless governing equations for this investigation are solved analytically using two-term harmonic and non-harmonic functions. The effects of various controlling parameters associated with this flow problem are discussed with the help of graphs. The results show that the velocity increases with increasing Slip and permeability parameters, while decreases with increasing Prandtl number, Magnetic parameter, Schmidt number, Chemical reaction parameter, and Heat source parameter. The study also revealed that the visco-elastic parameter has no significant effect on velocity profile.

Key words: Mass Transfer, Elastico-Viscous Fluid, Slip Flow, Free Convection, Porous Medium, MHD

1. INTRODUCTION

In many practical engineering applications the free convective heat and mass transfer flow of an electrically conducting fluid is of immense importance in the design of control systems for modern free convection heat exchange devices, magnetohydrodynamic (MHD) generator, oil exploration, plasma studies and geothermal energy extractions. Many common liquids such as oils, certain paints, polymer solution, some organic liquids and many new material of industrial importance exhibit both viscous and elastic properties. Therefore, the above fluid called elastico-viscous fluids is being studied extensively. Many researchers have shown their interest in the fluctuating and non fluctuating flow of a viscous incompressible fluid past an infinite or semi-infinite flat plate. For class of problems related to geothermal energy system there is a need for including constant heat sources or at times heat generating in porous media. Despite the importance of the unsteady heat and mass transfer of an elastico-viscous fluid with heat source in many engineering systems and applications, the attention it has received to date is not enough.

Mention may be made of the research studies of Raptis and Tzivanidis (1981), Chowdhury and Islam (2000). Basant Kumar Jha and Ravindra Prasad (1991) have studies the effects of heat source on MHD free convection and mass transfer flow through a porous medium. Rahman and Sattar (1999) analyzed MHD free convection and mass transfer flow with oscillating plate velocity and constant heat source in a rotating frame of reference. Recently Rajesh (2011) investigated the effects of heat source and mass transfer on MHD flow of an elasto-viscous fluid through a porous medium.

The phenomenon of slip flow regime has attracted the attention of a large numbers of researchers due to its wide ranging applications. The fluid slippage at the solid boundaries appeared in many applications such as micro channels or Nano channels etc. Several excellent studies have also considered slip condition and chemical reaction effects on unsteady heat and mass transfer flow of an electrically conducting fluid. Excellent articles include those by Watanebe *et al* (1998), Khalid and Vafai (2004), Makinde and Osalusi (2006), Mehmood and Ali (2007). More recently Seth *et al* (2010) investigated the unsteady MHD convective flow within a parallel plate rotating channel with thermal source/ Sink in a porous medium under slip conditions. Rakesh (2011) studied the effect of slip conditions and hall current on unsteady MHD flow of a visco-elastic fluid past an infinite vertical porous plate through porous medium.

This paper investigate the combined effects of mass transfer, heat source, chemical reaction and slip conditions on MHD free convection flow of a visco-elastic fluid past an infinite vertical oscillating plate through a channel filled with porous medium.

2. MATHEMATICAL FORMULATION

We consider unsteady free convection and mass transfer flow of a viscous incompressible and electrically conducting visco-elastic fluid past an infinite vertical plate through a porous medium in the presence of first order chemical reaction and heat source. A magnetic field of uniform strength B_0 is applied transversely to the plate. The flow is assumed to be in x'-direction which is taken along the vertical plate in the upward direction. The y'-axis is taken to be normal to the plate. The plate is subjected to a constant suction velocity. The effect of viscous dissipation is assumed to be negligible. Then by usual Boussinesq approximation, the flow governing equations are as follows:

$$\frac{\partial u'}{\partial t'} - v_o \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) - k_o \frac{\partial^3 u'}{\partial y' \partial t'} - \frac{\sigma B_o^2 u'}{\rho} - \frac{\nu u'}{K'} \tag{1}$$

$$\frac{\partial T'}{\partial t'} - v_o \frac{\partial T'}{\partial y'} = \frac{k}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{Q^*(T' - T'_\infty)}{\rho C_p} \tag{2}$$

$$\frac{\partial C'}{\partial t'} - v_o \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} - K_r(C' - C'_\infty) \tag{3}$$

The boundary conditions are

$$\left.\begin{aligned} &u' - \gamma^* \frac{\partial u'}{\partial y'} = 0,\ T' = T'_\omega + (T'_\omega - T'_\infty)e^{i\omega t}, C' = C'_\omega + (C'_\omega - C'_\infty)e^{i\omega t},\ at\ y' = 0 \\ &u' \to 0, \qquad T' \to T'_\infty, \qquad C' \to C'_\infty, \quad as\ y' \to \infty \end{aligned}\right\} \tag{4}$$

On introducing the following dimensionless quantities and parameters:

$$\left.\begin{aligned} &u = \frac{u'}{v_0}, t = \frac{v_o^2 t'}{v}, t = \frac{v_o y'}{v}, \omega = \frac{v\omega'}{v_o^2}, \Pr = \frac{v\rho C_p}{k}, t = \frac{v_o^2 t'}{v}, F = \frac{vQ*}{v_o^2 \rho C_p} \\ &\theta = \frac{T' - T'_\infty}{T'_\omega - T'_\infty}, C = \frac{C' - C'_\infty}{C'_\omega - C'_\infty}, Gr = \frac{g\beta(T'_\omega - T'_\infty)v}{v_o^3}, Gc = \frac{g\beta^*(C'_\omega - C'_\infty)v}{v_o^3} \\ &M = \frac{\sigma B_o^2 v}{\rho v_o^2},\ K = \frac{K' v_o^2}{v^2},\ S = \frac{k_o v_o^2}{\rho v^2},\ K_r = \frac{K'_r v}{v_o^2},\ Sc = \frac{v}{D}, \gamma = \frac{\gamma^* v_o}{v} \end{aligned}\right\} \tag{5}$$

Where u' is the axial velocity, t' is the time, T'_ω denotes the temperature of the fluid at the plate, T'_∞ denotes the temperature of the fluid far away from the plate, g the gravitational force, C'_ω denotes the concentration of the fluid at the plate, C'_0 ,denotes the concentration of the fluid far away from the plate, D is mass diffusivity, K is the mean permeability of the porous medium , ω is the frequency of the oscillation, β is the coefficient of thermal expansion, β^* is the coefficient of concentration expansion, B_o the electromagnetic induction, σ_e the conductivity of the fluid, ρ the density of the fluid, v is the kinematics viscosity coefficient, v_o is mean suction velocity, Gr is the Grashoff number, Gc is the mass Grashoff number, M is the Hartmann number, Pr is the Prandtl number, γ is the Slip parameter, Sc is the Schmidt number, θ is the dimensionless temperature, C is the dimensionless concentration, K_r is the chemical reaction parameter, S is the visco-elastic parameter and F is the heat source parameter.

Equations (1) to (3) with the help of (5) reduce to

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + GcC - Mu - \frac{u}{K} - S\frac{\partial^3 u}{\partial y^2 \partial t} \tag{6}$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} - F\theta \tag{7}$$

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - K_r C \tag{8}$$

The corresponding boundary conditions become

$$\left.\begin{array}{l} u-\gamma\dfrac{\partial u}{\partial y}=0,\ \theta=1+\varepsilon e^{i\omega t},\ C=1+\varepsilon e^{i\omega t}\ at\ y=0 \\ u\to 0,\qquad \theta\to 0,\qquad C\to 0\qquad as\ y\to\infty \end{array}\right\} \tag{9}$$

3. METHOD OF SOLUTION

In order to solve equations (6) to (8) under the boundary conditions (9), we assumed the fluid velocity, temperature and concentration of the fluid as follows:

$$\left.\begin{array}{l} U(y,t)=u_o(y)+\varepsilon u_1(y)e^{i\omega t} \\ \theta(y,t)=\theta_o(y)+\varepsilon\theta_1(y)e^{i\omega t} \\ C(y,t)=C_o(y)+\varepsilon C_1(y)e^{i\omega t} \end{array}\right\} \tag{10}$$

Putting equation (10) into equations (6) to (8), we obtain

$$u_0''+u_0'-m_1u_o=-Gr\theta_o-GcC_o \tag{11}$$

$$(1-iA)u_1''+u_1'-m_2u_1=-Gr\theta_1-GcC_1 \tag{12}$$

$$\theta_0''+\Pr\theta_0'-F\Pr\theta_o=0 \tag{13}$$

$$\theta_1''+\Pr\theta_1'-\Pr(i\omega+F)\theta_1=0 \tag{14}$$

$$C_0''+ScC_0'-ScK_rC_o=0 \tag{15}$$

$$C_1''+ScC_1'-Sc(i\omega+K_r)C_1=0 \tag{16}$$

The corresponding boundary conditions become

$$\left.\begin{array}{l} u_o-\gamma u_o'=0, u_1-\gamma u_1'=0,\quad \theta_o=\theta_1=1,\quad C_o=C_1=1,\ \ at\ y=0 \\ u_o=u_1=0,\quad \theta_o=\theta_1=0,\quad C_o=C_1=0\quad as\ y\to\infty \end{array}\right\} \tag{17}$$

Where the prime's denoted differentiation.

Solving equations (11) to (16) with the help of (17), and using (10), we have

$$\theta(y,t)=e^{-h_1y}+\varepsilon e^{i\omega t-h_2y} \tag{18}$$

$$C(y,t)=e^{-r_1y}+\varepsilon e^{i\omega t-r_2y} \tag{19}$$

$$U(y,t)=B_1e^{-\alpha_2 y}+De^{-h_1 y}+D_1e^{-r_1 y}+\varepsilon(B_2e^{-\alpha_4 y}+D_2e^{-h_2 y}+D_3e^{-r_2 y})e^{i\omega t} \tag{20}$$

The shear stress at the lower wall of the channel is given by

$$\left.\frac{\partial U}{\partial y}\right|_{y=0}=-\alpha_2B_1-h_1D-r_1D_1+\varepsilon(-\alpha_4B_2-h_2D_2-r_2D_3)e^{i\omega t} \tag{21}$$

Where

$$m_1=M+\frac{1}{K},\, m_2=M+\frac{1}{K}+i\omega,\, h_1=\frac{\Pr+\sqrt{\Pr^2+4F\Pr}}{2},\, h_2=\frac{\Pr+\sqrt{\Pr^2+4\Pr(i\omega+F)}}{2},$$

$$r_1=\frac{Sc+\sqrt{Sc^2+4ScK_r}}{2},\, r_2=\frac{Sc+\sqrt{Sc^2+4Sc(i\omega+K_r)}}{2},\, \alpha_2=\frac{1+\sqrt{1+4m_1}}{2},$$

$$\alpha_2=\frac{1+\sqrt{1+4(1-iA)m_2}}{2},\, D=\frac{-Gr}{h_1^2-h_1-m_1},\, D_1=\frac{-Gc}{r_1^2-r_1-m_1},\, D_2=\frac{-Gr}{(1-iA)h_2^2-h_2-m_2},$$

$$D_3=\frac{-Gc}{(1-iA)r_2^2-r_2-m_2},\, B_1=\frac{-[1+\gamma h_1]D-[1+\gamma r_1]D_1}{1+\gamma\alpha_2},\, B_2=\frac{-[1+\gamma h_2]D_2-[1+\gamma r_2]D_3}{1+\gamma\alpha_4}$$

4. RESULTS AND DISCUSSION

In order to investigate the physical situation of the problem, we plotted velocity profiles for different parameters such as Slip parameter, Magnetic parameter, chemical reaction parameter, permeability parameter, Heat source parameter, visco-elastic parameter, Prandtl number and Schmidt number. Figure (1) displays the influences of Slip parameter (γ = 0, 0.5, 1.0, 1.5) on the velocity field. From this figure we observed that, as the values of Slip parameter increases the velocity increase. Figure (2) represent the velocity profiles for different values of the Heat source parameter (F = 0, 0.5, 1.0, 1.5), it is found that the velocity decreases with increasing Heat source parameter. Figure (3) illustrate the velocity profiles for different values of the Prandtl number (Pr = 0.71, 0.85, 1.0, 2.0) from this figure it is observed that the velocity decreases with increasing the Prandtl number. Figure (4) represent the velocity profiles for different values of the Magnetic parameter (M = 1, 2, 3, 4), it is found that the velocity decreases with increasing magnetic parameter. It is because that the application of transverse magnetic field will result a resistivity type force (Lorentz force) similar to drag force, which tends to resist the fluid flow and thus reducing its velocity. Figure (5) display the velocity profiles for different values of the Schmidt number (Sc = 0.22, 0.60, 0.78, 2.00). From this figure the velocity is found to decrease with increasing Schmidt number. Figure (6) illustrate the velocity profile for different values of the visco-elastic parameter (S = 0, 5, 10, 100). It is observed that the velocity does not change with increasing visco-elastic parameter. Figure (7) display the velocity profiles for different values of the permeability parameter (K = 0.1, 0.3, 0.5, 0.7). It is observed that the velocity increases with increasing permeability parameter. This is due to the fact that the presence of a porous medium increases the resistance to flow. The influences of chemical reaction parameter (K_r = 1, 2, 3, 4) on the velocity profiles are presented in figure (8). It is observed that the velocity decreases with increasing chemical reaction parameter.

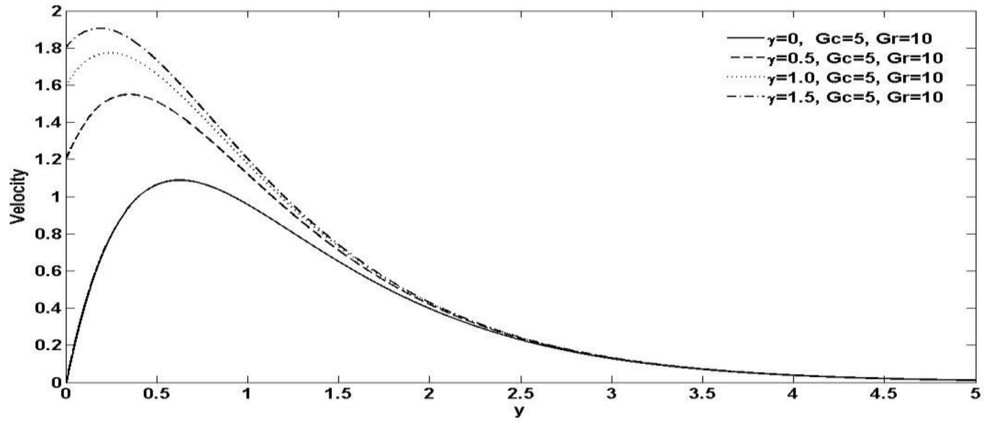

Figure1: Velocity profile for different values of Slip parameter.

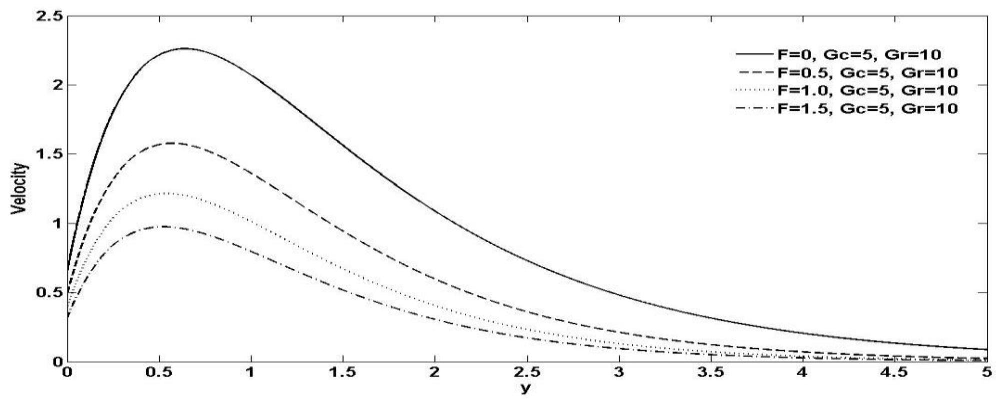

Figure2: Velocity profiles for different values of the Heat source parameter.

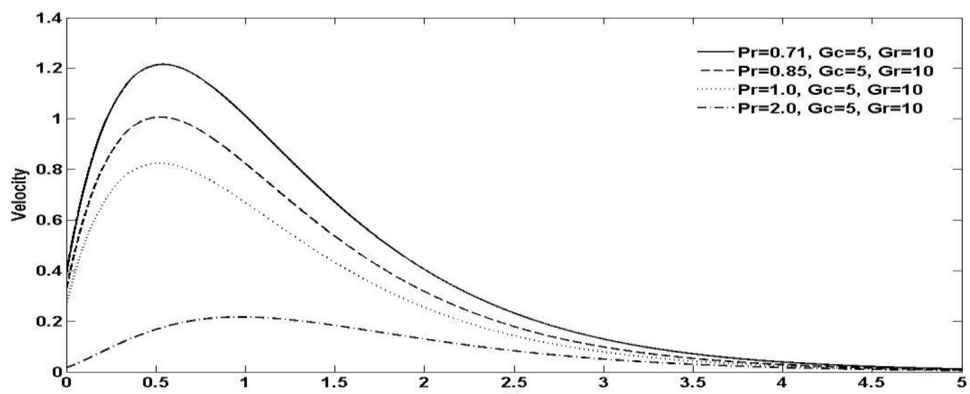

Figure3: Velocity profiles for different values of the Prandtl number.

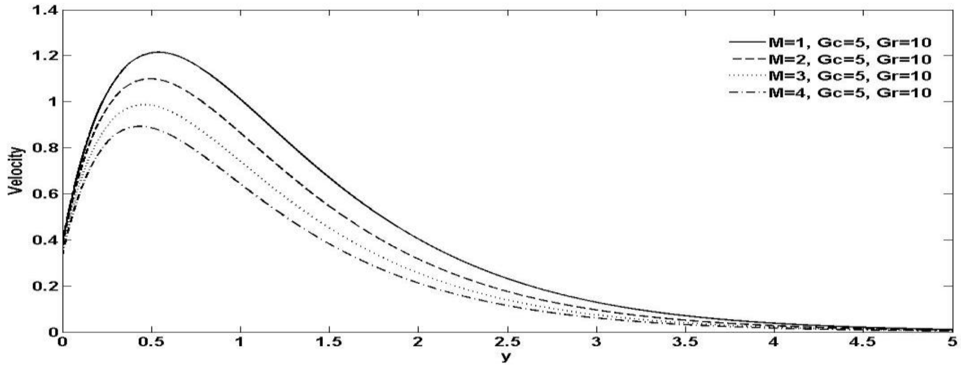

Figure4: Velocity profiles for different values of the Magnetic parameter.

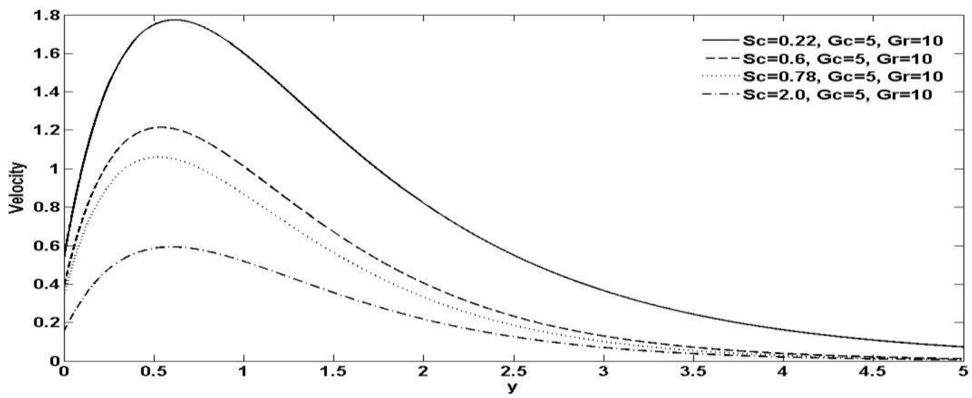

Figure5: Velocity profiles for different values of the Schmidt number.

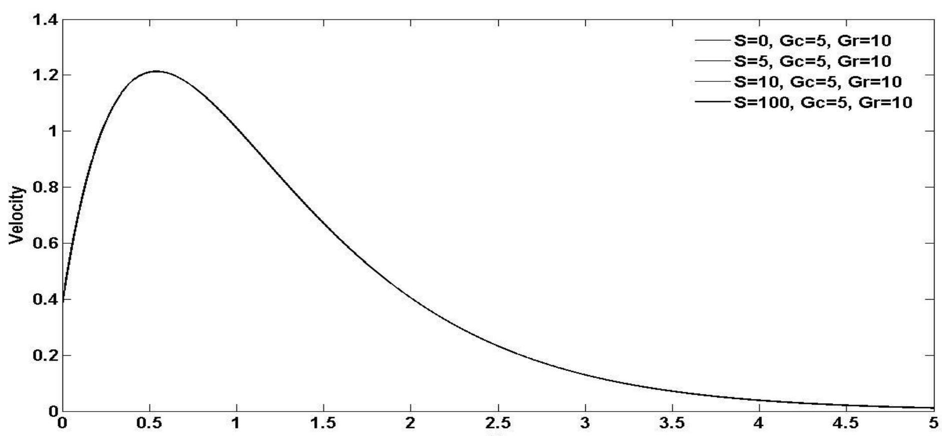

Figure6: Velocity profiles for different values of the Visco-elastic parameter.

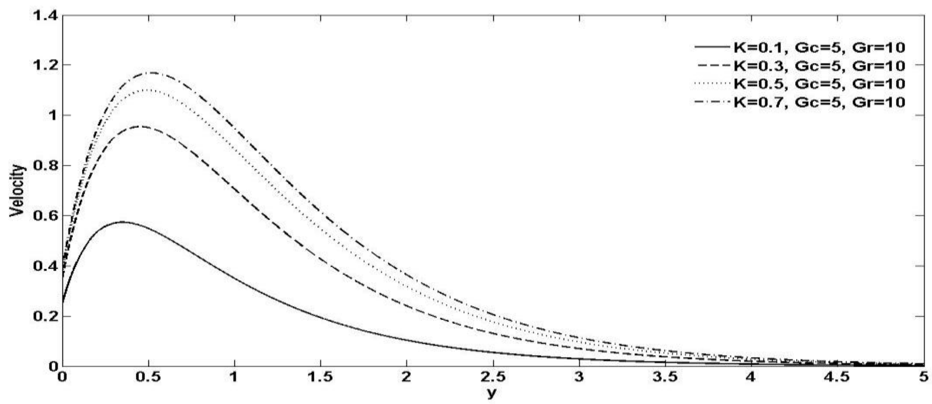

Figure7: Velocity profiles for different values of the Permeability parameter.

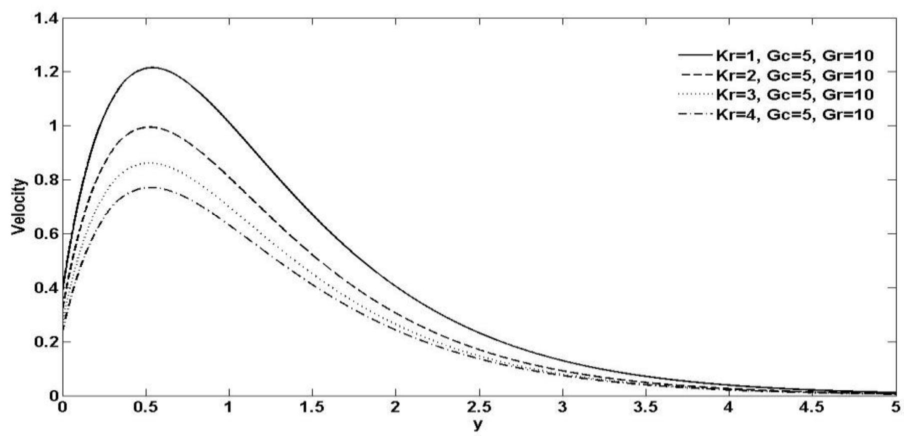

Figure8: Velocity profiles for different values of the Chemical reaction parameter.

5. CONCLUSION

We investigate the combined effects of mass transfer, heat source, chemical reaction and slip conditions on MHD free convection flow of a visco-elastic fluid past an infinite vertical oscillating plate through a channel filled with porous medium. The governing equations are solved using perturbation method. The result shows that the velocity rises with increasing the slip parameter and permeability parameter, and decreases with increasing the values of the Prandtl number, heat source, magnetic parameter, Schmidt number and chemical reaction parameter. While there is a negligible effect of visco-elastic parameter on velocity profiles.

REFERENCES

Chowdhury, M. K and Islam, M. N (2000): MHD free convection flow of visco-elastic fluid past an infinite vertical porous plate. *Heat and Mass Transfer*, 36, Pp. 439.

Jha, B. K, Prasad, R, and Rai, S (1991): Mass Transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux. *Astrophysics and Space Science*, 181, Pp. 125-134.

Khalid, A. R. A and Vafai, K (2004): The effect of the slip condition on Stokes and Couette flows due to an oscillating wall. *International Journal of Nonlinear Mechanics*, 39, Pp. 795.

Makinde, O. D and Osalusi, E (2006): MHD steady flow in a channel with slip at the permeable boundaries. *Romanian Journal of Physics*, 51, Pp. 319.

Mehmood, A and Ali, A (2007): The effect of slip condition on unsteady MHD oscillatory flow of a viscous fluid in a planner channel. *Rom. J. of Physics*, 52(1-2), PP. 85-91.

Rahman, M. M and Sattar, M. A (1999): MHD free convection and mass transfer flow with oscillating plate velocity and constant heat source in a rotating frame of reference. *The Dhaka University Journal of Science*, 47, Pp. 63.

Rajesh, V (2011): Heat source and mass transfer effects on MHD flow of an elasto-viscous fluid through a porous medium. *International Journal of Engineering*, 2, Pp. 205-212.

Rakesh, K and Khem, C (2011): Effect of slip conditions and Hall current on unsteady MHD flow of a visco-elastic fluid past an infinite vertical porous plate through porous medium. *International Journal of Engineering Science and Technology*, 3, Pp. 3124-3133.

Raptis, A. A and Tzivanidis, G. J (1981): Effects of mass transfer free convection currents and heat sources on the Stokes problem for an infinite vertical plate. *Astrophysics and Space Science*, 78(2), Pp. 351.

Watanebe, K; Yanuar; and Mizunuma, H (1998): Slip of Newtonian fluids at solid boundary. *Japan Society Mechanical Engineering, International Journal Series*, B41, pp. 525.