Optimal Inventory Model With Weibull Deterioration With Trapezoidal Demand And Shortages

Rekha Rani Chaudhary[1] & Vikas Sharma[2]

[1]H.O.D Department of Mathematics, Government Engineering College Bharatpur, Rajasthan, India

[2]Ph.D Scholar, Banasthali Vidyapith ,Rajasthan India

Abstract

In this paper, we discuss about an inventory model which is developed for deteriorating items. We have considered Weibull distribution with two parameters as a deterioration rate. The demand rate is trapezoidal type. Shortages are also considered in the model. Analytical solution of the model is obtained and the analytical condition is also given to illustrate the model developed. The model is solved analytically by minimizing the total cost.

Key words: - Inventory, EOQ Model, deteriorating items, Weibull distribution with two parameters, shortages, trapezoidal type demand.

1. Introduction

Deterioration is defined as decay or change in the quantity of the inventory. Maximum physical goods endure decay or deterioration over time. Fruits, vegetables and food items undergo from depletion by direct spoilage while stored. Highly unstable liquids such as gasoline, alcohol and turpentine suffer physical depletion over time through the process of evaporation. The deterioration rate is given in this paper is Weibull deterioration with two parameters. Items of physical goods are one of the important factors in any inventory and production system. The deteriorating items with shortages have received much attention of several researches in the recent years because most of the physical goods undergo decay or deterioration over the period of time. Commodities such as fruits, vegetables and food stuffs get depleted by direct spoilage while kept in store. Ghare and Schrader [5] developed a model for an exponentially decaying inventory. An order level inventory model for items deteriorating at a constant rate was proposed by Aggarwal [6], Dave and Patel [1].Inventory models with a time dependent rate of deterioration were considered by Covert and Philip [3]. The inventory model with ramp type demand rate was proposed the first time by Hill (4). The ramp type demand is very commonly seen when some fresh fruits come to the market. In case of ramp type demand rate, the demand increases linearly at the beginning and then the market grows into a stable stage such that the demand becomes a constant until the end of the inventory cycle. Mingbao Cheng, Guoqing Wang [2] developed "A note on the inventory model for deteriorating items with trapezoidal type demand rate." Hill first considered the inventory models for increasing demand followed by a constant demand. He derived the exact solution to compare with the Silver-Meal heuristic, and then resolved the similar problem by offering a rigorous and efficient method to derive the optimal solution. Mandal and Pal [10] extended the inventory model with ramp type demand for deterioration items and allowing shortage. Rearchers can consult for more work Aggarwal S.P and Jaggi C.K. [3] Benkherouf [16], Goswami and Chaudhuri [14], Hariga [18], Panda, Senapati, and Basu [12]. Some of the significant recent work in this field have been done by Chung and Ting [15], Giri [7] , Jalan [19] and Chaudhuri [7] extended the ramp type demand inventory model with a more generalized Weibull deterioration distribution. Burwell [9] developed economic lot size model for price-dependent demand under quantity and freight discounts. Inventory model for ameliorating items for price dependent demand rate was proposed by Mandal et.al [10] and inventory model with price and time dependent demand was developed by You [20]. In general holding cost is assumed to be known and constant.

In the following, we extend Mingbao Cheng, Guoqing Wang's "A inventory model for deteriorating items with trapezoidal demand rate", where deterioration rate follows two-parameters Weibull distribution. Assumption is that the inventory system consists of several replenishments. In this rate, demand is considered as dependent on time. The demand rate for such items increase with the time up to certain time and is ultimately stabilized and becomes constant, and finally demand rate approximately decreases to a constant or zero.All the ordering cycles have a fixed length, and consider only one of the ordering cycle in the paper. We think that such type of demand rate is quite realistic and a useful inventory replenishment policy for such type of inventory model is also proposed. In this model shortages are allowed. We consider that our work will provide a firm foundation for the further study of various inventory models for Weibull distribution with trapezoidal type demand rate.

2. Assumptions and Notations

The fundamental assumptions are used to developed the model.

1. The demand rate R(t) is dependent on time t, as follow

$$R(t) = \begin{cases} b_1 t & t \leq \gamma_1 \\ Z_0 & \gamma_1 \leq t \leq \gamma_2 \\ -b_2 t & \gamma_2 \leq t \leq T \end{cases}$$

γ_1 is time point where increasing linearly demand change to constant demand, and γ_2 is time point where constant demand change to decreasing linearly demand (see Fig. 1).

2. I(t) is the inventory level at time t. $0 \leq t \leq T$ . .
3. T is the length of the ordering cycle.
4. The ordering cost A is constant.
5. t_1 is the time when the inventory leavel reaches zero.
6. C_1 is the inventory holding cost per unit item per unit time.
7. The deterioration cost per unit item, per unit time is C_2. & the deterioration rate is proportional to time.
8. C_3 is the shortage cost per unit item per unit time.
9. S is the maximum inventory level for type ordering cycle such that S = I(0).
10. The ordering quantity per cycle is q_0.
11. The deterioration of time as follows by Weibull parameter (two) distribution θ (t) = $\alpha\beta t^{(\beta-1)}$ where $0<\alpha< 1$ is The scale parameter and $\beta >0$ is the shape parameter.

12. γ_1 time point changing from the increasing linearly demand to constant demand
13. γ_2 time point changing from the constant demand to the decreasing linearly demand
14. $P_1(t_1)$ is the total cost ,cost per unit time under the condition $t_1 \leq \gamma_1$.
15. $P_2(t_1)$ is the total cost ,cost per unit time under the condition $\gamma_1 \leq t_1 \leq \gamma_2$.
16. $P_3(t_1)$ is the total cost, cost per unit time under the condition $\gamma_2 \leq t_1 < T$.

3. Formulation and solution

The length of the cycle is T. we take the Weibull deterioration rate with trapezoidal type demand rate .

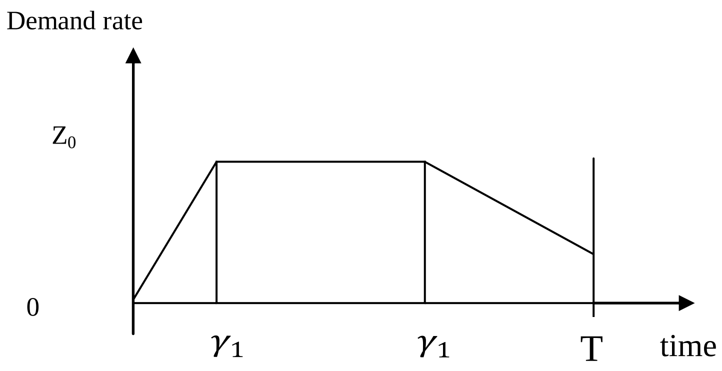


Fig. 1 A trapezoidal type function of the demand

The inventory level reduces due to demand and deterioration. At the time t_1 the inventory level achieves zero. Then shortages is occurs during the time interval (t_1,T). The shortages is completely backlogged .The total numbers of items is replaced by the next replenishment. By the above inventory system at any time can be described by the following differential equations:

$$\frac{dI(t)}{dt} + \propto\beta(t)^{\beta-1} I(t) = -R(t) \qquad 0 < t < t_1 \qquad (1)$$

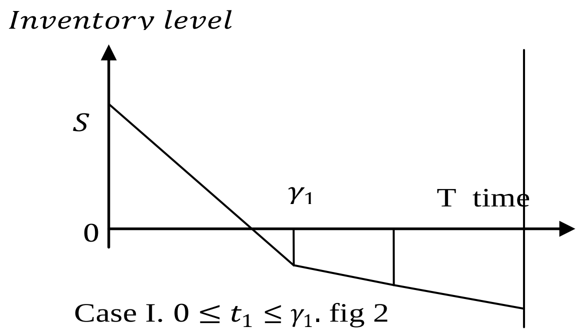
and

$$\frac{dI(t)}{dt} = -R(t) \qquad t_1<t<T \qquad (2)$$

With boundary condition $I(t_1) = 0$

In follow, let us consider three possible cases based on the values of t_1, γ_1 and γ_2.The all cases are given as follows (see fig 2, fig 3, fig 4).

Case I: $0 \le t_1 \le \gamma_1$



Case I. $0 \le t_1 \le \gamma_1$. fig 2

Due to the trapezoidal type demand rate and Weibull deterioration rate The inventory level decrease in the period $[0, t_1]$ and at last reduces to zero at t_1.

Then from the equation (1), we have

$$\frac{dI(t)}{dt} + \propto\beta(t)^{\beta-1} I(t) = -b_1(t) \qquad 0 < t < t_1 \qquad (3)$$

$$\frac{dI(t)}{dt} = -b_1(t) \qquad t_1 < t < \gamma_1 \qquad (4)$$

$$\frac{dI(t)}{dt} = -Z_0 \qquad \gamma_1 < t < \gamma_2 \qquad (5)$$

and

$$\frac{dI(t)}{dt}) = b_2(t) \qquad \gamma_1 < t < T \qquad (6)$$

Solving the differential equation (3), (4), (5) and (6) with $I(t_1) = 0$ we have

$$I(t) = b_1(1-\propto t^{\beta})\left[\frac{1}{2}(t_1^2 - t^2) + \frac{\propto}{\beta+2}\left(t_1^{(\beta+2)} - t^{(\beta+2)}\right)\right] \qquad 0 \le t \le t_1 \qquad (7)$$

$$I(t) = \frac{b_1}{2}(t_1^2 - t^2) \qquad t_1 \le t \le \gamma_1 \qquad (8)$$

$$I(t) = Z_0(t_1 - t) \qquad \gamma_1 \le t \le \gamma_2 \qquad (9)$$

$$I(t) = \frac{b_2}{2}(t^2 - t_1^2) \qquad \gamma_2 \le t \le T \qquad (10)$$

The initial Inventory level will be

$S = I(0)$ from Eq.(7), we have,

$$S = b_1\left[\frac{t_1^2}{2} + \frac{\alpha}{(\beta+2)} t_1^{(\beta+2)}\right] \qquad (11)$$

The total deterioration cost per unit time in the interval $[0, t_1]$, let D.c.

$$\text{D.c.}= C_2\left[S - \int_0^{t_1} R(t)dt\right]$$

$$\text{D.c.}= C_2\left\{b_1\left[\frac{t_1^2}{2} + \frac{\alpha}{(\beta+2)} t_1^{(\beta+2)}\right] - \int_0^{t_1} b_1 t\, dt\right\}$$

$$\text{D.c.}= \quad C_2 b_1\left\{\frac{\alpha}{(\beta+2)} t_1^{(\beta+2)}\right\} \tag{12}$$

The holding cost per unit time of the inventory in the interval $[0, t_1]$, let H.c.

$$\text{H.c.} = C_1 \int_0^{t_1} I(t)dt$$

$$\text{H.c.} = C_1 \int_0^{t_1} \mathrm{b1}\,(1-\propto t\beta)\left[\frac{1}{2}(t_1^2 - t^2) + \frac{\propto}{\beta+2}\left(t_1{}^{(\beta+2)} - t^{(\beta+2)}\right)\right] dt$$

$$H.c. = C_1\left\{\frac{b_1}{2} t_1^2\left(t_1 - \frac{1}{2}\right) + b_1\alpha t_1\left[\frac{1}{2(\beta+2)} + \frac{1}{(\beta+2)} - \frac{1}{(\beta+2)(\beta+3)}\right] - \frac{b_1}{2}\alpha t_1^{(\beta+2)} + \frac{b_1\alpha^2 t_1^{(2\beta+2)}}{(\beta+2)}\left[\frac{1}{(\beta+1)} - \frac{1}{(2\beta+2)}\right]\right\} \tag{13}$$

The Total shortage cost per unit time during the interval $[t_1, T]$ let S.c.

$$\text{S.c.} = -C_3 \int_{t_1}^{T} I(t)dt$$

$$\text{S.c.} = -C_3\left\{\int_{t_1}^{\gamma_1} \frac{b_1}{2}(t_1^2 - t^2)dt + \int_{\gamma_1}^{\gamma_2} Z_0(t_1 - t)\, dt + \int_{\gamma_2}^{T} \frac{b_2}{2}(t^2 - t_1^2)dt\right\}$$

$$.\ \text{S.c} = -C_3\left\{\frac{b_1}{2}\left[t_1^2\gamma_1 - \frac{\gamma_1^3}{3} - \frac{2t_1^3}{3}\right] + Z_0\left[t_1(\gamma_2-\gamma_1) - \frac{1}{2}(\gamma_2^2 - \gamma_1^2)\right] + \frac{b_2}{2}\left[\frac{1}{3}(T^3 - \gamma_2^3) + t_1^2(\gamma_2 - T)\right]\right\} \tag{14}$$

Then, the total cost per unit time under the condition $t_1 \le \gamma_1$ can be define

$$P_1(t_1) = \frac{1}{T}\{A_0 + D.c + H.c. + S.c.\} \tag{15}$$

Our main aim to minimize the total cost per unit time $P_1(t_1)$ .The necessary condition for minimize the total cost is $\frac{dP1(t_1)}{dt_1} = 0$

$$\frac{dP1(t_1)}{dt_1} = \frac{1}{T}\left\{C_1\left[\frac{3}{2}b_1 t_1^2 - \frac{b_1}{2}t_1 - \frac{b_1}{2}\alpha t_1^{(\beta+1)} + \frac{b_1}{2}\alpha t_1^{(\beta+2)}\left(\frac{3\beta^2+11\beta+10}{(\beta+1)(\beta+2)}\right) + \frac{b_1}{(\beta+1)}\alpha^2 t_1^{2(\beta+1)}\right] + C_2\left[b_1\alpha t_1^{(\beta+1)}\right] - C_3\left[\frac{b_1}{2}\left(2t_1\gamma_1 - 2t_1^2 + Z_0(\gamma_2-\gamma_1) + b_2 t_1(\gamma_2 - T)\right)\right]\right\} \tag{16}$$

$$\frac{1}{T}\left\{C_1\left[\frac{3}{2}b_1 t_1^2 - \frac{b_1}{2}t_1 - \frac{b_1}{2}\alpha t_1^{(\beta+1)} + \frac{b_1}{2}\alpha t_1^{(\beta+2)}\left(\frac{3\beta^2+11\beta+10}{(\beta+1)(\beta+2)}\right) + \frac{b_1}{(\beta+1)}\alpha^2 t_1^{2(\beta+1)}\right] + C_2\left[b_1\alpha t_1^{(\beta+1)}\right] - C_3\left[\frac{b_1}{2}\left(2t_1\gamma_1 - 2t_1^2 + Z_0(\gamma_2-\gamma_1) + b_2 t_1(\gamma_2 - T)\right)\right]\right\} = 0 \tag{17}$$

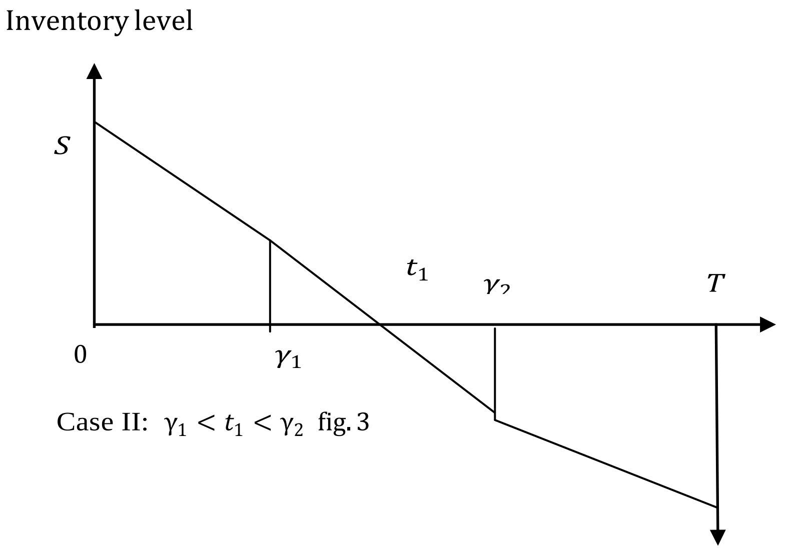
The minimum total average cost per unit time is obtain for those value of t_1 for which

$\frac{d^2P1(t_1)}{dt_1^2} > 0.$

By Eq.(17) the optimal value of t_1 satisfies $\frac{d^2P1(t_1)}{dt_1^2} > 0,$

It means the total cost $P_1(t_1)$ per unit time will be minimum.

Case II: $\gamma_1 < t_1 < \gamma_2$



Case II: $\gamma_1 < t_1 < \gamma_2$ fig. 3

In this case $t_1[\gamma_1, \gamma_2]$,The differential equation can be derived as below.

$$\frac{dI(t)}{dt} + \propto\beta(t)^{\beta-1} I(t) = -b_1(t) \qquad 0 < t < \gamma_1 \qquad (18)$$

$$\frac{dI(t)}{dt} + \propto\beta(t)^{\beta-1} I(t) = -Z_0 \qquad \gamma_1 < t < t_1 \qquad (19)$$

$$\frac{dI(t)}{dt} = -Z_0 \qquad t_1 < t < \gamma_2 \qquad (20)$$

and

$$\frac{dI(t)}{dt}) = b_2(t) \qquad \gamma_2 < t < T \qquad (21)$$

Solving the differential equation (18), (19), (20) and (21) with $I(t_1) = 0$ we have

$$I(t) = b_1(1-\propto t^\beta)\left[\frac{1}{2}(t_1^2 - t) + \frac{\propto}{\beta+2}\left(t_1{}^{(\beta+2)} - t^{(\beta+2)}\right)\right] \qquad 0 \le t \le \gamma_1 \qquad (22)$$

$$I(t) = Z_0(1-\propto t^\beta)\left[(t_1 - t) + \frac{\propto}{\beta+1}\left(t_1{}^{(\beta+1)} - t^{(\beta+1)}\right)\right] \qquad \gamma_1 \le t \le t_1 \qquad (23)$$

$$I(t) = Z_0(t_1 - t) \qquad t_1 \le t \le \gamma_2 \qquad (24)$$

$$I(t) = \frac{b_2}{2}(t^2 - t_1^2) \qquad \gamma_2 \le t \le T \qquad (25)$$

The beginning inventory level can be computed as

$S = I(0)$ from Eq.(22), we have,

$$S = b_1\left[\frac{t_1^2}{2} + \frac{\alpha}{(\beta+2)} t_1^{(\beta+2)}\right] \qquad (26)$$

The total deterioration cost per unit time in the interval $[0, t_1]$, let D.c.

$$D.c. = C_2\left[S - \int_0^{t_1} R(t)dt\right]$$

$$D.c. = C_2\left\{b_1\left[\frac{t_1^2}{2} + \frac{\alpha}{(\beta+2)} t_1^{(\beta+2)}\right] - \int_0^{\gamma_1} b_1 t\, dt - \int_{\gamma_1}^{t_1} Z_0\, dt\right\}$$

$$\text{D.c.}= \quad C_2\left\{ \frac{b_1}{2}(t_1^2-\gamma_1^2)+\frac{\alpha b_1}{(\beta+2)}t_1^{(\beta+2)}-Z_0(t_1-\gamma_1)\right\} \qquad (27)$$

The holding cost per unit time of the inventory in the interval $[0, t_1]$, let H.c.

$$\text{H.c.} = C_1\int_0^{t_1} I(t)dt$$

$$\text{H.c.} = C_1\left\{\int_0^{\gamma_1} b_1\left(1-\propto t^{\beta}\right)\left[\frac{1}{2}(t_1^2-t)+\frac{\alpha}{\beta+2}\left(t_1^{(\beta+2)}-t^{(\beta+2)}\right)\right]dt\right\}+ C_1\left\{\int_{\gamma_1}^{t_1} Z0\left(1-\propto t^{\beta}\right)\left[(t_1-t)+\frac{\alpha}{\beta+1}\left(t_1^{(\beta+1)}-t^{(\beta+1)}\right)\right]dt\right\}$$

$$\text{H.c} = P_1 + P_2 \qquad (28)$$

Where

$$P_1 = C_1\left\{\frac{b_1t_1^2\gamma_1}{2}+\frac{b_1\alpha\gamma_1^{(\beta+1)}}{(\beta+1)}\left[\frac{t_1^2}{2}-\frac{\alpha t_1^{(\beta+2)}}{(\beta+2)}\right]+\frac{b_1\gamma_1^2}{4}+\frac{b_1\alpha}{(\beta+2)}\left[t_1^{(\beta+2)}\gamma_1-\frac{1}{2}\gamma_1^{(\beta+2)}-\frac{\gamma_1^{(\beta+3)}}{(\beta+3)}-\frac{\alpha\gamma_1^{(2\beta+3)}}{(2\beta+3)}\right]\right\}$$

$$P_2 = C_1\left\{\frac{Z_0}{2}(t_1^2+\gamma_1^2)+\frac{Z_0\alpha}{(\beta+2)}\left(t_1^{(\beta+2)}-\gamma_1^{(\beta+2)}\right)+\frac{Z_0\alpha^2}{(\beta+1)^2}t_1^{(\beta+1)}\left(\gamma_1^{(\beta+1)}-t_1^{(\beta+1)}\right)+\frac{Z_0\alpha}{(\beta+1)(\beta+2)}\left(t_1^{(\beta+2)}+\gamma_1^{(\beta+2)}\right)-Z_0t_1\gamma_1+\frac{Z_0\alpha}{(\beta+1)}\gamma_1t_1\left(\gamma_1^{\beta}-t_1^{\beta}\right)+\frac{Z_0\alpha^2\gamma_1^{2(\beta+1)}}{2(\beta+1)^2}\right\}$$

The Total shortage cost per unit time during the interval $[t_1, T]$ let S.c.

$$\text{S.c.} = -C_3\left[\int_{t_1}^{\gamma_2} I(t)dt+\int_{\gamma_2}^{T} I(t)dt\right]$$

$$\text{S.c.} = -C_3\left[\int_{t_1}^{\gamma_2} Z_0(t_1-t)dt+\int_{\gamma_2}^{T}\frac{b_2}{2}(t^2-t_1^2)dt\right]$$

$$\text{S.c.} = C_3\left\{\frac{Z_0}{2}(\gamma_2^2+t_1^2)-Z_0t_1\gamma_2+\frac{b_2}{6}(\gamma_2^3-T^3)+\frac{b_2}{2}t_1^2(T-\gamma_2)\right\} \qquad (29)$$

Then, the total cost per unit time under the condition $\gamma_1 \le t_1 \le \gamma_2$ can be define

$$P_2(t_1) = \frac{1}{T}\{A_0 + D.c + H.c.+S.c.\} \qquad (30)$$

Our main aim to minimize the total cost per unit time $P_2(t_1)$ .The necessary condition for minimize the total cost is $\frac{dP_2(t_1)}{dt_1} = 0$

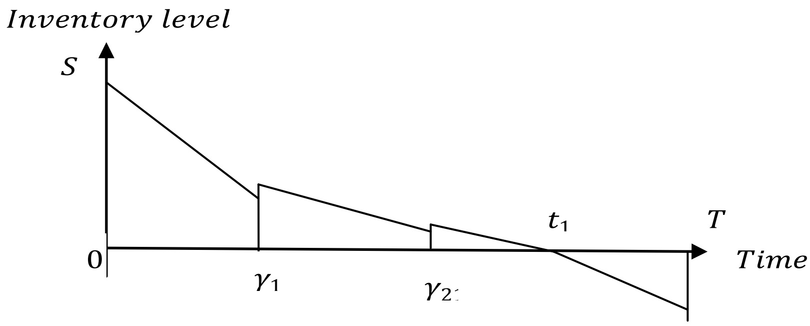
$$\frac{dP_2(t_1)}{dt_1} = C_1\left\{b_1t_1\gamma_1+\frac{b_1\alpha\gamma_1^{(\beta+1)}}{(\beta+1)}\left[t_1-\alpha t_1^{(\beta+1)}\right]+b_1\alpha\gamma_1t_1^{(\beta+1)}+Z_0t_1+Z_0\alpha t_1^{(\beta+1)}+\frac{Z_0\alpha^2}{(\beta+1)}\left[t_1^{\beta}\gamma_1^{(\beta+1)}-2t_1^{(2\beta+1)}\right]-\frac{\alpha Z_0}{(\beta+1)}t_1^{(\beta+1)}-Z_0\gamma_1+\frac{Z_0\alpha\gamma_1}{(\beta+1)}\left[\gamma_1^{\beta}+(\beta+1)t_1^{\beta}\right]\right\}+C_2\left\{b_1t_1+\alpha t_1^{(\beta+1)}-Z_0\right\}+C_3\{Z_0[t_1-\gamma_2]+b_2t_1[T-\gamma_1]\}$$

$$C_1\left\{b_1t_1\gamma_1+\frac{b_1\alpha\gamma_1^{(\beta+1)}}{(\beta+1)}\left[t_1-\alpha t_1^{(\beta+1)}\right]+b_1\alpha\gamma_1t_1^{(\beta+1)}+Z_0t_1+Z_0\alpha t_1^{(\beta+1)}+\frac{Z_0\alpha^2}{(\beta+1)}\left[t_1^{\beta}\gamma_1^{(\beta+1)}-2t_1^{(2\beta+1)}\right]-\frac{\alpha Z_0}{(\beta+1)}t_1^{(\beta+1)}-Z_0\gamma_1+\frac{Z_0\alpha\gamma_1}{(\beta+1)}\left[\gamma_1^{\beta}+(\beta+1)t_1^{\beta}\right]\right\}+C_2\left\{b_1t_1+\alpha t_1^{(\beta+1)}-Z_0\right\}+C_3\{Z_0[t_1-\gamma_2]+b_2t_1[T-\gamma_1]\}=0 \qquad (31)$$

The minimum total average cost per unit time is obtained for optimal value of t_1 for which $\frac{d^2P_2(t_1)}{dt_1^2} > 0$. As similar case first, By Eq.(17) the optimal value of t_1 satisfies $\frac{d^2 P1(t_1)}{dt_1^2} > 0$,

It means the total cost $P_2(t_1)$ per unit time will be minimum.

Case III: $\gamma_2 < t_1 < T$



Case III: $\gamma_2 < t_1 < T$, fig. 4

In this case $t_1[\gamma_2, T]$,The differential equation can be derived as below.

$$\frac{dI(t)}{dt} + \propto\beta(t)^{\beta-1} I(t) = -b_1(t) \qquad 0 < t < \gamma_1 \qquad (32)$$

$$\frac{dI(t)}{dt} + \propto\beta(t)^{\beta-1} I(t) = -Z_0 \qquad \gamma_1 < t < \gamma_2 \qquad (33)$$

$$\frac{dI(t)}{dt} + \propto\beta(t)^{\beta-1} I(t) = b_2(t) \qquad \gamma_2 < t < t_1 \qquad (34)$$

and

$$\frac{dI(t)}{dt}) = b_2(t) \qquad t_1 < t < T \qquad (35)$$

Solving the differential equation (32), (33), (34) and (35) with $I(t_1) = 0$ we have

$$I(t) = b_1 (1-\propto t^{\beta}) \left[\frac{1}{2}(t_1^2 - t) + \frac{\propto}{\beta+2}\left(t_1^{(\beta+2)} - t^{(\beta+2)}\right)\right] \qquad 0 \le t \le \gamma_1 \qquad (36)$$

$$I(t) = Z_0 (1-\propto t^{\beta}) \left[(t_1 - t) + \frac{\propto}{\beta+1}\left(t_1^{(\beta+1)} - t^{(\beta+1)}\right)\right] \qquad \gamma_1 \le t \le \gamma_2 \qquad (37)$$

$$I(t) = b_2 (1-\propto t^{\beta}) \left[\frac{1}{2}(t - t_1^2) + \frac{\propto}{\beta+2}\left(t^{(\beta+2)} - t_1^{(\beta+2)}\right)\right] \qquad \gamma_2 \le t \le t_1 \qquad (38)$$

$$I(t) = \frac{b_2}{2}(t^2 - t_1^2) \qquad \gamma_2 \le t \le T \qquad (39)$$

The beginning inventory level can be computed as

$S = I(0)$ from Eq.(36), we have,

$$S = b_1\left[\frac{t_1^2}{2} + \frac{\alpha}{(\beta+2)} t_1^{(\beta+2)}\right] \qquad (40)$$

The total deterioration cost per unit time in the interval $[0, t_1]$, let D.c.

$$\text{D.c.} = C_2\left[S - \int_0^{t_1} R(t)dt\right]$$

$$\text{D.c.}= C_2\left\{b_1\left[\frac{t_1^2}{2}+\frac{\alpha}{(\beta+2)}t_1^{(\beta+2)}\right]-\int_0^{\gamma_1} b_1 t\,dt-\int_{\gamma_1}^{\gamma_2} Z_0\,dt+\int_{\gamma_2}^{t_1} b_2 t\,dt\right\}$$

$$\text{D.c} = C_2\left\{\frac{b_1}{2}(t_1^2-\gamma_1^2)+\frac{\alpha b_1}{(\beta+2)}t_1^{(\beta+2)}+\frac{b_2}{2}(t_1^2-\gamma_2^2)-Z_0(\gamma_2-\gamma_1)\right\} \qquad (41)$$

The holding cost per unit time of the inventory in the interval $[0, t_1]$, let H.c.

$$\text{H.c.} = C_1\int_0^{t_1} I(t)dt$$

$$\text{H.c.} = C_1\left\{\int_0^{\gamma_1} b_1\,(1-\alpha\,t\beta)\left[\frac{1}{2}(t_1^2-t)+\frac{\alpha}{\beta+2}\left(t_1^{(\beta+2)}-t^{(\beta+2)}\right)\right]dt\right\}+ C_1\left\{\int_{\gamma_1}^{\gamma_2} Z_0\left(1-\alpha t^{\beta}\right)\left[(t_1-t)+\frac{\alpha}{\beta+1}\left(t_1^{(\beta+1)}-t^{(\beta+1)}\right)\right]dt\right\}+ C_1\left\{\int_{\gamma_2}^{t_1} b_2\left(1-\alpha\,t^{\beta}\right)\left[\frac{1}{2}(t_1^2-t)+\frac{\alpha}{\beta+2}\left(t_1^{(\beta+2)}-t^{(\beta+2)}\right)\right]dt\right\}$$

Let $\quad \text{H.c} = O_1 + O_2 + O_3 \qquad (42)$

Where

$$O_1 = C_1\left\{\frac{b_1 t_1^2\gamma_1}{2}+\frac{b_1\alpha\gamma_1^{(\beta+1)}}{(\beta+1)}\left[\frac{t_1^2}{2}-\frac{\alpha t_1^{(\beta+2)}}{(\beta+2)}\right]+\frac{b_1\gamma_1^2}{4}+\frac{b_1\alpha}{(\beta+2)}\left[t_1^{(\beta+2)}\gamma_1-\frac{1}{2}\gamma_1^{(\beta+2)}-\frac{\gamma_1^{(\beta+3)}}{(\beta+3)}-\frac{\alpha\gamma_1^{(2\beta+3)}}{(2\beta+3)}\right]\right\}$$

$$O_2 = C_1\left\{Z_0(\gamma_2-\gamma_1)\left[t_1-\frac{1}{2}(\gamma_2+\gamma_1)+\frac{\alpha}{(\beta+1)}t_1^{(\beta+1)}\right]+\frac{Z_0\alpha}{(\beta+1)}\left(\gamma_2^{(\beta+1)}-\gamma_1^{(\beta+1)}\right)\left[\frac{1}{2}\left(\gamma_2^{(\beta+1)}+\gamma_1^{(\beta+1)}\right)-t_1\left(1+t_1^{\beta}\right)\right]+\frac{Z_0}{(\beta+2)}\left(\gamma_2^{(\beta+1)}-\gamma_1^{(\beta+1)}\right)(\alpha-1)\right\}$$

$$O_3 = C_1\left\{\frac{b_2}{4}(t_1^2-\gamma_2^2)-\frac{b_2}{2}t_1^2(t_1-\gamma_2)-\frac{b_2\alpha}{2}t_1^2\left(t_1^{(\beta+1)}-\gamma_2^{(\beta+1)}\right)+\frac{b_2\alpha}{2(\beta+2)}\left(t_1^{(\beta+2)}-\gamma_2^{(\beta+2)}\right)-\frac{b_2\alpha}{(\beta+2)}t_1^{(\beta+2)}(t_1-\gamma_2)-\frac{b_2\alpha^2}{(\beta+2)}t_1^{(\beta+2)}\left(t_1^{(\beta+1)}-\gamma_2^{(\beta+1)}\right)+\frac{b_2\alpha}{(\beta+2)(\beta+3)}\left(t_1^{(\beta+3)}-\gamma_2^{(\beta+3)}\right)+\frac{b_2\alpha^2}{(\beta+2)(2\beta+3)}\left(t_1^{(2\beta+3)}-\gamma_2^{(2\beta+3)}\right)\right\}$$

The Total shortage cost per unit time during the interval $[t_1, T]$ let S.c.

$$\text{S.c.} =-C_3\left[\int_{t_1}^{T} I(t)dt\right]$$

$$\text{S.c.} =-C_3\left[\int_{t_1}^{T}\frac{b_2}{2}(t^2-t_1^2)dt\right]$$

$$\text{S.c.} =C_3\left\{t_1^2(T-t_1)-\frac{1}{3}(T^3-t_1^3)\right\} \qquad (43)$$

Then, the total cost per unit time under the condition $\gamma_2 \le t_1 \le T$ can be define

$$P_3(t_1) = \frac{1}{T}\{A_0 + D.c + H.c. + S.c.\}$$

Our main aim to minimize the total cost per unit time $P_3(t_1)$ .The necessary condition for minimize the total cost is $\frac{dP_3(t_1)}{dt_1} = 0$

$$\frac{dP_3(t_1)}{dt_1} = C_1\left\{b_1t_1\gamma_1+\frac{b_1\alpha\gamma_1^{(\beta+1)}}{(\beta+1)}\left[t_1-\alpha t_1^{(\beta+1)}\right]+b_1\alpha\gamma_1 t_1^{(\beta+1)}+Z_0(\gamma_2-\gamma_1)\alpha t_1^{\beta}-\frac{Z_0\alpha}{(\beta+1)}\left(\gamma_2^{(\beta+1)}+\gamma_1^{(\beta+1)}\right)(1+(\beta+1)t_1)+\frac{b_2}{2}t_1-\frac{b_2}{2}(3t_1^2-2t_1\gamma_2)-\frac{b_2}{2}\alpha+\frac{b_2}{2}\alpha t_1^{(\beta+1)}-\frac{b_2\alpha}{(\beta+2)}\left((\beta+3)t_1^{(\beta+2)}-(\beta+2)t_1^{(\beta+1)}\gamma_2\right)-\frac{b_2}{(\beta+2)}\alpha^2\left((\beta+1)t_1^{2(\beta+1)}+\left(t_1^{(\beta+1)}-\gamma_2^{(\beta+1)}\right)(\beta+2)t_1^{(\beta+1)}\right)+\frac{b_2\alpha}{(\beta+2)}t_1^{(\beta+2)}+\frac{b_2\alpha^2}{(\beta+2)}t_1^{2(\beta+1)}\right\}+C_2\left\{b_1t_1+b_1\alpha t_1^{(\beta+1)}+b_2t_1\right\}+C_3\{b_2t_1(T-1)\}$$

$$C_1 \left\{ b_1 t_1 \gamma_1 + \frac{b_1 \alpha \gamma_1^{(\beta+1)}}{(\beta+1)} \left[t_1 - \alpha t_1^{(\beta+1)} \right] + b_1 \alpha \gamma_1 t_1^{(\beta+1)} + Z_0 (\gamma_2 - \gamma_1) \alpha t_1^{\beta} - \frac{Z_0 \alpha}{(\beta+1)} \left(\gamma_2^{(\beta+1)} + \gamma_1^{(\beta+1)} \right) (1 + (\beta+1) t_1) + \frac{b_2}{2} t_1 - \frac{b_2}{2} (3t_1^2 - 2t_1 \gamma_2) - \frac{b_2}{2} \alpha \left((\beta+3) t_1^{(\beta+2)} - 2t_1 \gamma_2^{(\beta+1)} \right) + \frac{b_2}{2} \alpha t_1^{(\beta+1)} - \frac{b_2 \alpha}{(\beta+2)} \left((\beta+3) t_1^{(\beta+2)} - (\beta+2) t_1^{(\beta+1)} \gamma_2 \right) - \frac{b_2}{(\beta+2)} \alpha^2 \left((\beta+1) t_1^{2(\beta+1)} + \left(t_1^{(\beta+1)} - \gamma_2^{(\beta+1)} \right) \quad (\beta+2) t_1^{(\beta+1)} \right) + \frac{b_2 \alpha}{(\beta+2)} t_1^{(\beta+2)} + \frac{b_2 \alpha^2}{(\beta+2)} t_1^{2(\beta+1)} \right\} + C_2 \left\{ b_1 t_1 + b_1 \alpha t_1^{(\beta+1)} + b_2 t_1 \right\} + C_3 \{ b_2 t_1 (T-1) \} = 0 \qquad (44)$$

The minimum total average cost per unit time is obtained for those value of t_1 for which $\frac{d^2 P_3(t_1)}{dt_1^2} > 0$. As similar case first,

Using the software mathematica-5.1,we can calculate the optimal value t_1 from equation (44) and t_1 satisfies $\frac{d^2 P_3(t_1)}{dt_1^2} > 0$,

It means the total cost $P_3(t_1)$ per unit time will be minimum.

5. Conclusion

In this paper, we have developed a inventory lot size model for Weibull deterioration items with such as fruits, vegetables and food stuffs from depletion by direct spoilage while kept in store. The demand rate is assumed of time dependent. The shortages are allowed and shortages are completely backlogged. The deterioration cost, inventory holding cost and, shortage cost are considered in different three cases in this model. The model is solved analytically by minimizing the total inventory cost in different three cases in this model. In the analytical condition we found the minimum value of total inventory cost. For the more study this paper can be extended in several ways for example we may add exponential deterioration rate , stock dependent demand rate and price dependent demand rate.

References

[1] Dave, U., and Patel L.K., "(T, S) policy inventory model for deteriorating items with time proportional demand", Journal of the Operational Research Society. **32**(1981), 137-142.

[2] Cheng, Mingbao., & Wang , Guoqing. A note on the inventory model for deteriorating items with trapezoidal type demand rate. Computers & Industrial Engineering 56 (2009) 1296–1300

[3] Covert, R.P., and Philip, G.C., "An EOQ model for items with Weibull distribution deterioration. AIIE Transactions", **5**(1973), 323-326.

[4] Hill, R. M. Inventory model for increasing demand followed by level demand. Journal of the Operational Research Society, 46, 1250–1259.

[5] Ghare, P.M., and Schrader, G.F., "An inventory model for exponentially deteriorating items", Journal of Industrial Engineering, **14**(1963), 238-243.

[6] Aggarwal, S.P., "A note on an order-level inventory model for a system with constant rate of deterioration", Opsearch, **15**(1978), 84-187.

[7] Giri, B. C., Jalan, A. K., & Chaudhuri, K. S. Economic order quantity model with Weibull deterioration distribution, shortage and ramp-type demand. International Journal of Systems Science, 34,237–243.

[8] Wu, K. S., & Ouyang, L. Y. (2000). A replenishment policy for deteriorating items with ramp type demand rate. Proceedings of National Science Council ROC (A), 24, 279–286 .

[9] Burwell T.H., Dave D.S., Fitzpatrick K.E., Roy M.R., " Economic lot size model for price-dependent demand under quantity and freight discounts", International Journal of Production Economics, **48**(2)(1997), 141-155.

[10] Mandal, B., Bhunia, A.K., Maiti, M., "An inventory system of ameliorating items for price dependent demand rate", Computers and Industrial Engineering, **45**(3)(2003), 443-456.

[11] Deng, Peter Shaohua, Lin, Robert H.-J., & Chu, Peter. A note on the inventory models for deteriorating items with ramp type demand rate. European Journal of Operational Research, 178, 112–120.

[12] Panda, S., Senapati, S., & Basu, M. (2008). Optimal replenishment policy for perishable seasonal products in a season with ramp-type time dependent demand. Computers & Industrial Engineering, 54, 301–314.

[13] Chakraborti, T., and Chaudhuri, K.S., "An EOQ model for deteriorating items with a linear trend in demand and shortages in all cycles", International Journal of Production Economics, **49**(1997), 205-213.

[14] Goswami, A., & Chaudhuri, K. S. An EOQ model for deteriorating items with shortages and a linear trend in demand. Journal of the Operational Research Society, 42, 1105–1110.

[15] Chung, K., and Ting, P., "A heuristic for replenishment of deteriorating items with a linear trend in demand", Journal of the Operational Research Society,**44**(1993), 1235-1241.

[16] Benkherouf, L. On an inventory model with deteriorating items and decreasing Time-varying demand and shortages. European Journal of Operational Research, 86, 293–299.

[17] Wee, H.M., "A deterministic lot-size inventory model for deteriorating with shortages and a declining market", Computers and Operations, **22**), 345-356.

[18] Hariga, M.A., and Benkherouf, L., "Optimal and heuristic inventory replenishment models for deteriorating items with exponential time-varying demand", European Journal of Operational Research. **79**(1994), 123-137.

[19] Jalan, A.K., and Chaudhuri, K.S., "Structural properties of an inventory system with deterioration and trended demand", International Journal of systems Science, **30**(1999), 627-633.

[20] You, S.P., "Inventory policy for products with price and time-dependent demands", Journal of the Operational Research Society, **56**(2005), 870-873.