Optimal J.I.T Epq Model With Items Of Imperfect Quality, Exponential Declining Demand And Regular Preventive Maintenance.

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Abstract:

This paper attempts to establish an Economic Production Quantity (EPQ) model where items produced are not considered to be of good quality. Defective items are sold at a low price after the items are being screened.

It is highly necessary to start preventive maintenance as soon as the production stops to ensure smooth running and long lasting of machine. Idea of JIT is also considered to avoid shortages. The effectiveness and utility of this model is illustrated with numerical example and sensitivity analysis has been performed.

Key words: Preventive maintenance, Inventory, Just-in-time, Imperfect, Declining demand.

Introduction: In EOQ or EPQ models of traditional method, it often happens that some of the inventories are either excess or short resulting a penalty cost. To overcome this problem, use of just-in-time production method of Japanese has proved fruitful and effective to minimize the cost. Salameh etal [14] proposed a JIT buffer inventory model in order to provide regular preventive maintenance. A comparative cost function between inventory ordering under an economic quantity (EOQ) system or a just –in –time system was presented by Fazal et al [5]. Schonberger [16] and Wantuck [19] have clearly proved that the conversion to JIT will reduce space in plants and factories. In this paper conception of JIT is assumed to avoid shortages.

Olorunniwo et al [12] pointed out that preventive maintenance is based on one of the two extreme assumptions. The production unit is restored to either a bad as old or good as new condition .Actually due to preventive maintenance the condition of equipment may be enhanced at a level between those of two extremes. In developing the mathematical model of this paper it is assumed that regular preventive maintenance will keep the production unit to such condition that prevents sudden failure as will as maintains the quality as before .Groenevelt et al [6] showed the effects of the machine breakdown and corrective maintenance on the economic lot –size decisions. An approach for preventive maintenance scheduling in light of production plan is discussed by Balasubramanian [2].Valdez-Flores [18] and Nakagawa [11] show in their survey papers that the preventive maintenance improves to a certain extent the health condition of the equipment reducing the effective ages of the machine. Monga et al [10] presented optimal system which incorporated design model the effects of preventive maintenance and minimal repair. An algorithm for preventive maintenance policy is presented by Lie et al [7].Schniederjans et al [16] concluded JIT purchasing is preferable to EOQ with price discount. From the above discussions it may be pointed that none of them has taken into consideration JIT and items of imperfect quality practically it is seen that all the items produced in a machine are not of the some quality. Therefore, it is necessary to separate the defective items. Naturally after being screened the defective items are sold at a low price Zhang and Gerchak [20] proposed an EOQ model where defective items are replaced by nondefective ones. Rosenblatt et al [13] come to a conclusion that presence of defective products leads to the motivation of smaller lot sizes. The effect of defective items on the lot sizing policy is amply illustrated by chakraborty et al [3] ; Moinzadheh et al [9] , urban et al [17] , Anily et al [1] Salameh et al [15] considered an EPQ model for items with imperfect quality .

The demand for an item naturally deelines over time due to various reason such as introduction of better products, change in trend or perception about the product and so on. An EOQ model with demand dependent unit cost is developed by Cheng [4].

In the present paper an EPQ model is suggested when all the items are not of same quality. The items having some defects are sold at a low price. Here we would like to emphasize on the regular preventive maintenance for maintaining the same quantity just-in-time production quantity is also solved here .The demand declines exponentially with time in this model.

.Lau et al [8] developed a multi-item newsboy problem with capacity constraints. Storage capacity of items in this model is limited .Now a numerical example is taken and is solved by penalty function method and sensitivity analysis has been performed.

Notations:

The following notations are used

S=number of produced units in each cycle.

p=rate of defective items among S.

N(S,p)=S-pS=(1-p)S=Number of good items among the produced items.

V= unit selling price of defective items.

d= unit screening cost.

h= inventory holding cost per unit per unit time.

c_2= shortage cost per unit per unit time.

λ= a constant (>0)

T_0= preventive maintenance time per cycle.

Q= number of produced unit+ the amount of shortage

t_1= time of screening.

t_2= time of production

$p^/$= defective items/unit.

x= production /unit time.

Assumptions:

The following assumption are made

(1) The items produced are not of the same quality, some of those are defectives.

(2) Defective items are sold at a low price.

(3) The regular preventive maintenance of machine guarantees that the probability of a break down of the production unit during T is approximately zero.

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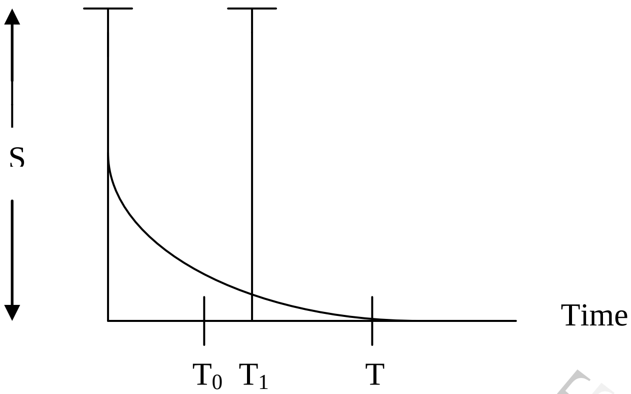
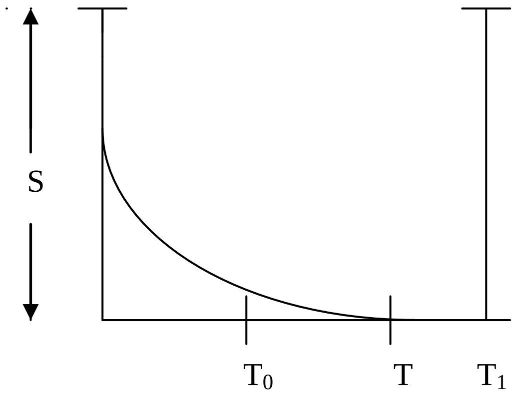
(4) The just-in-time inventory is also determined.

(5) Exponential declining demand with time is considered.

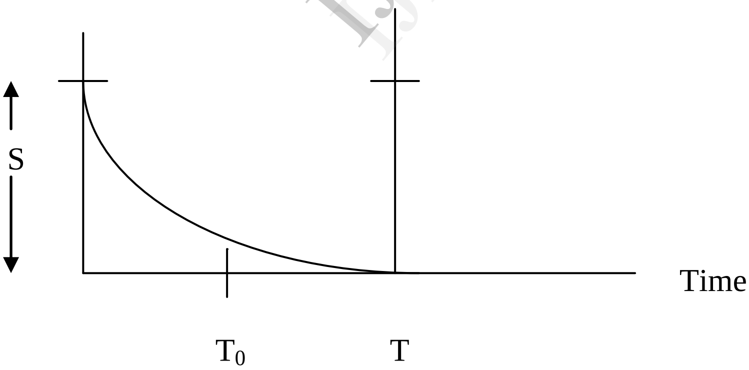
(6) Limiting storage capacity is also considered here



Case I (T_1<T-T_0)



Case II (T_1>T-T_0)



Case(III) (T_1=T-T_0)

Inventory Inventory

$$\frac{dq}{dt} = -e^{-\lambda t} \qquad (1)$$

$$q(0) = (1-p)S \text{ ....}(1a)$$

where $q(T) = 0$ ....(1b)

Integrating and using the boundary condition (1a) we get from (1)

$$q(t) = \frac{D_0 e^{-\lambda t}}{\lambda} + (1-p^{/})S - \frac{D_0}{\lambda} \qquad (2)$$

The boundary condition (1b) gives

$$T = \frac{1}{\lambda}\ln\left[\frac{D_0}{D_0 - \lambda(1-p^{/})S}\right] \qquad (3)$$

Let $T_1=(T-T_0) = t_1+t_2$

Where t_1=Time of screening

& t_2=Time of production.

Now $p.t_1=S$ and $x.t_2=S$, Therefore $T_1= t_1+t_2 = \frac{S}{p} + \frac{S}{x} = \frac{(p+x)S}{px}$ (4)

The total variable cost is comprised of the sum of the holding cost, stock out cost and screening cost. In case JIT stock-out cost does not occur.

The defective items among S =p.S

Case I

In this case since T_1<T-T_0, inventories are accumulated during the period (T-T_1) which will be used for the next cycle. Therefore an extra holding cost is added to each cycle. The holding cost is

$$HC = h\int_0^T q(t)dt + \int_{T_1}^T q(t)dt = h\int_0^T \{\frac{e^{-\lambda t}}{\lambda} + (1-p^/)S - \frac{1}{\lambda}\}dt + h\int_{T_1}^T \{\frac{D_0 e^{-\lambda t}}{\lambda} + (1-p^/)S - \frac{D_0}{\lambda}\}dt$$

$$= h[\frac{1-e^{-\lambda T}}{\lambda^2} + (1-p^/)ST - \frac{T}{\lambda} + -\frac{D_0}{\lambda^2}(e^{-\lambda T} - e^{-\lambda T_1}) + (T-T_1)(S - p^/ S - \frac{D_0}{\lambda})]$$

The screening cost is SRC= d.S

The selling price of defective items DP= v.$p^/$.S

The production cost PC = C^*.S

The total variable cost function is defined as

TVC= HC+SRC + PC – S.P. of defective items

$$TVC = h[\frac{D_0(1-p^/)S}{\lambda} + \ln[\frac{D_0}{D_0 - \lambda(1-p^/)S}](\frac{(1-p^/)S\lambda - D_0}{\lambda^2}) + \frac{D_0}{\lambda^2}(e^{-\frac{\lambda(p+x)S}{px}} - 1 + \frac{\lambda(1-p^/)S}{D_0})$$
$$+ \{\frac{1}{\lambda}\ln[\frac{D_0}{D_0 - \lambda(1-p^/)S}] - \frac{(p+x)S}{px}\}\{S - p^/ S - \frac{D_0}{\lambda}\}]$$
$$+ d.S + C^* S - vp^/ S \quad \text{.......................................} (6)$$

.

The variable cost per unit time $TCU_1 = \frac{TVC_1}{T}$ (7)

The problem is

Min TCU(S)

s.t. $aS \leq A$

Case II

In this case since $T_1 > (T - T_0)$,shortages occur during the period $(T_1 - T)$.Therefore a stock out cost is added to each cycle.

Stock out time = $T_1 - T$

Therefore, the number of unit short per preventive maintenance cycle can be expressed as

$$W(t) = e^{-\lambda t}(T_1 - T)$$

Stock-out cost SHC is

$$\begin{aligned} &= C_2 \int_{T}^{T_1} (T_1 - T) e^{-\lambda t dt}; (T_1 > T) \\ &= \frac{C_2 (T_1 - T)}{\lambda} (e^{-\lambda T} - e^{-\lambda T_1}) \end{aligned} \quad (8)$$

The holding cost

$$
\begin{aligned}
HC_2 &= \int_0^T q(t)\,dt \\
&= h\int_0^T \{\frac{D_0 e^{-\lambda t}}{\lambda} + (1-p^/)S - \frac{D_0}{\lambda}\}dt \\
&= h\{\frac{(1-p^/)SD_0}{\lambda} + \frac{(1-p^/)S}{\lambda}\ln[\frac{D_0}{D_0-\lambda(1-p^/)S}] - \frac{D_0}{\lambda^2}\ln[\frac{D_0}{D_0-\lambda(1-p^/)S}]\}
\end{aligned} \tag{9}
$$

The screening cost is SRC= d.S

The selling price of defective items DP= $v.p^/.S$

The production cost PC = $C^*.S$

The total variable cost function is defined as

TVC= HC+SRC + PC+SHC – S.P. of defective items

$$
\begin{aligned}
TVC &= h\{\frac{D_0(1-p^/)S}{\lambda} + \ln[\frac{D_0}{D_0-\lambda(1-p^/)S}](\frac{(1-p^/)S\lambda - D_0}{\lambda^2})\} \\
&+ \frac{C_2}{\lambda}\{\frac{(p+x)S}{px} - \frac{D_0}{\lambda}\ln[\frac{D_0}{D_0-\lambda(1-p^/)S}]\}\{1-\frac{\lambda(1-p^/)S}{D_0} - e^{-\frac{\lambda(p+x)S}{px}}\} \\
&+ d.S + C^*S - vp^/S
\end{aligned} \tag{10}
$$

.

The variable cost per unit time $TCU_1 = \frac{TVC_1}{T}$ (11)

The problem is

$$\text{Min } TCU(S)$$

$$\text{s.t.} \quad aS \le A$$

Case III

In case of JIT since T_1=T-T_0 i.e. the perfect inventories are accumulated just at the end of the cycle period.Therefore neither an extra holding cost nor the stockout cost is added to each cycle.

The holding cost is

$$HC_3 = h\int_0^T q(t)dt$$

$$= h\int_0^T \{\frac{D_0 e^{-\lambda t}}{\lambda} + (1-p^/)S - \frac{D_0}{\lambda}\}dt \qquad (12)$$

$$= h\{\frac{D_0(1-p^/)S}{\lambda} + \ln[\frac{D_0}{D_0 - \lambda(1-p^/)S}](\frac{(1-p^/)\lambda\lambda - D_0}{\lambda^2})\}$$

The screening cost is SRC= d.S

The selling price of defective items DP= v.$p^/$.S

The production cost PC = C^*.S

The total variable cost function is defined as

TVC= HC+SRC + PC – S.P. of defective items

In this case the total cost is

$$TVC = h[\frac{D_0(1-p^/)S}{\lambda} + \ln[\frac{D_0}{D_0 - \lambda(1-p^/)S}](\frac{(1-p^/)S\lambda - D_0}{\lambda^2}) + d.S + C^*S - vp^/S \qquad (13)$$

.

TCU(S)=

$$= \frac{h[\frac{(1-P)s}{\lambda} - \frac{(1-p)s}{\lambda}\ln[1-\lambda(1-p)s] + \frac{1}{\lambda^2}\ln[1-\lambda(1-p^/)S] + d.S - v.p^/.S.}{\frac{1}{\lambda}\ln[\frac{1}{1-\lambda(1-p)s}]} \qquad (14)$$

s.t. aS ≤A

The problem is

Min TCU(S)

s.t. aS ≤ A

5. Numerical example & Sensitivity Analysis

In this paper the optimal production quantity is determined under the situation of exponential declining demand, quantities with defective items, regular preventive maintenance of machine. A numerical example is considered to illustrate the developed

The following parametric values

λ=0.15

p=$ 0.25/unit

h=$ 5.00/unit/time

d=$ 0.5/unit

c_2=$ 1.00

v=$ 20.00/unit/time

A=20

a=0.5

Case –I (T_1<T-T_0)

Changing parameters	Change in parameters	S	TCU(in $)
λ	0.50	310.3	74912.19
	0.60	309.3	56984.15
	0.75	389.7	37683.20
$p^{/}$	0.25	310.3	74912.19
	0.50	309.3	71019.45
	0.75	310.3	67176.73
D_0	200	310.3	2.19
	150	310.3	47949.32
	100	389.3	15090.13

Case II (In Case Of Stock Out Occurs—(T_1>T-T_0)

Changing parameters	Change in parameters	S	TCU(in $)
λ	0.50 0.60 0.75	310.3 309.3 389.7	75053.72 57032.15 37690.23
p'	0.25 0.50 0.75	310.3 309.3 310.3	75053.72 71375.91 67788.15
D_0	200 150 100	310.3 310.3 389.3	75053.72 47966.98 14824.49

CASE-III (In Case Of JIT) (T_1=T-T_0)

Changing parameters	Change in parameters	S	TCU(in$)
λ	0.50	310.3	49273.38
	0.60	310.3	41560.66
	0.75	309.3	33856.34
$p^{/}$	0.25	310.3	49273.38
	0.50	309.3	33394.89
	0.75	309.3	17541.41
D_0	200	310.3	49273.38
	150	309.3	37681.47
	100	310.3	26204.11

Conclusions:

In this study we consider are JIT EPQ model with defective items. Regular maintenances of machine are also considered to avoid break down and to produce the same quality. The demand declines with time exponentially. The assumptions are very realistic as it will help producer to determine the optimal product and total cost under capacity constraint. In this paper the optimum JIT inventory is determined by trading off the HC, SRC, and SHC.

The results of the sensitivity analysis are also consistent.

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