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Optimal PID Tuning of a Plant Based on Frequency Domain Specifications

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***Abstract*—The widespread application of the proportional integral-derivative (PID) control in industry, because of their simplicity, robustness it can still be a challenge to find a general and effective PID tuning method. In this paper, a simple PID controller tuning method based on nonlinear optimization is developed to satisfy both robustness and performance and the objective is to achieve a fast response to set point changes .In proposed method, constraint on overshoot ratio the closed-loop bandwidth is maximized for specified gain and phase margins . The closed-loop amplitude ratio is given from the frequency analysis of PID controller in parallel form with for the first-order plus time delay system. Simulation examples demonstrated by the proposed design method gives the better closed-loop system performances than existing design methods.**

Key terms - proportional integral-derivative (PID) Tuning, Closed loop performance, Non linear optimization, Phase and gain margin, process control, Peak overshoot (M_T).

I. INTRODUCTION

During the 1930s three mode controllers with proportional, integral, and derivative (PID) actions became commercially available and gained widespread industrial acceptance. These types of controllers are still the most widely used controllers in process industries. Large amount of work has been done from 1942 with various tuning Methods [1]. Control system design using pole-placement is well-known technique. But it yields a unique solution for the controller. However, a unique solution does not allow any flexibility [2]. Robustness in process control design is important as the process model used is often an approximation of the system dynamics [3]-[5].

Robust control design is an area of intensive research. The common approach is to provide better performance index. The popular performance index is the integral square error (ISE) [6], [9], [14]. The closed-loop control system with sufficient gain and phase margin provides robustness as well as better closed-loop performance. One of the frequent practical uses of controller design is to tune a controller of fixed structure (e.g. a PID controller) in such a way that the step response of the closed-loop system has a minimal settling time with a small overshoot [7], [8], [10]. Numerical methods cannot solve frequency domain equations because of five unknowns from four equations [11]-[15] .But the IMC-PID design is examined from the frequency domain point of view. Equations for typical frequency domain specifications such as gain and phase margins and bandwidth are derived for the IMC-PID design. Equations for real-time monitoring of the gain and phase margins of a PID control system are also derived. But robustness criterion cannot be exactly met.

The main contribution of this work is to formulate the PID tuning into a nonlinear optimization problem to with constraints on both GPM and M_T and maximize the band width. So that closed loop performance and criteria on robustness are both satisfied simultaneously. The closed-loop response as fast as possible (minimized settling time) for given bound on the overshoot ratio and robustness criteria.

This paper is organized as follows. In the next section, the closed-loop amplitude ratio equation is derived to calculate the bandwidth and the overshoot ratio from the open-loop amplitude ratio and the phase equations for PID in parallel and first order plus time-delay (FOPTD) model form are explicitly given. Further more. In the following section, the new tuning method to meet both performance criteria and robustness is described and the resulting optimization problem is formulated.

II. CALUCULATION OF GAIN AND PHASE MARGINS

Gain margin and phase margin are calculated from open loop frequency analysis and closed loop amplitude ratio is calculated from closed loop frequency analysis.

A. Open-Loop Frequency Analysis

The transfer function of the PID controller in parallel form is given by

$$G_c(s) = K_c\,(1+\tau_D s+\frac{1}{\tau_I s}) \qquad (1)$$

and

The transfer function of a FOPTD process is given by

$$G_p(s) = \frac{K_p}{\tau s+1} e^{-\theta s} \qquad (2)$$

Then the open-loop transfer function is given by

$$G_{ol}(s) = G_c(s)\, G_p(s)$$

$$= K_c\,(1+\tau_D s+\frac{1}{\tau_I s})\,\frac{K_p}{\tau s+1} e^{-\theta s} \qquad (3)$$

$$= K_c K_p(\frac{1+\tau_I s+\tau_I\tau_D s^2}{\tau_I s(1+\tau s)})\, e^{-\theta s} \qquad (4)$$

by using frequency analysis on each term in (4), i.e. Replacing $e^{-\theta s}$ = 1-θs and s= jω.

The amplitude ratio AR_{ol} and phase change φ_{ol} are given by

$$= K_c K_p(\frac{1+\tau_I j\omega-\tau_I\tau_D\omega^2}{j\tau_I\omega(1+j\tau\omega)})(1-\theta s) \qquad (5)$$

$$AR_{ol} = K_c K_p\sqrt{(\frac{(1-\tau_I\tau_D\omega^2)^2+(\tau_I\omega)^2}{(\omega\tau_I)^2(1+(\tau\omega)^2)}} \qquad (6)$$

$$\Phi_{ol} = \{\wedge(\omega) - \omega\theta - \tan^{-1}(\omega\tau) - \pi/2, \text{ if } \wedge(\omega) \geq 0; \quad \wedge(\omega) - \omega\theta - \tan^{-1}(\omega\tau) + \pi/2, \text{ if } \wedge(\omega) < 0 \qquad (7)$$

Where

$$\wedge(\omega) = \tan^{-1}(\omega\tau_I/1 - \omega^2\tau_I\tau_D) \qquad (8)$$

B. Closed loop frequency analysis

For open-loop system Gol , the closed-loop transfer function is given by

$$G_{cl}(s) = \frac{G_{ol}(s)}{1+G_{ol}(s)} \qquad (9)$$

the amplitude ratio of closed-loop system can be calculated by manipulating the above equation is given by

$$AR_{cl} = \frac{1}{\sqrt{(\frac{1}{AR_{ol}}+\cos\Phi_{ol})^2+\sin^2\Phi_{ol}}} \qquad (10)$$

Thus, the amplitude ratio of the closed-loop system AR_{cl} can be calculated directly from the open-loop amplitude ratio AR_{ol} and phase change φ_{ol} .

The maximum closed-loop amplitude ratio M_T can be obtained by calculating

$$M_T = \max\,(AR_{cl}(\omega)) \quad \forall\, \omega \qquad (11)$$

The bandwidth ωb is then can be calculated by solving the equation

$$AR_{cl}(\omega) = 0.707 \qquad (12)$$

III. OPTIMAL PID DESIGN BASED ON GPMS

Gain margin and phase margin are calculated by the following equations

$$A = \frac{1}{|G_{ol}(j\omega_p)|} \qquad (13)$$

$$\varnothing = \angle G_{ol}(j\omega_g) + \pi \qquad (14)$$

Where

$$|G_{ol}(j\omega_p)| = 1 \qquad (15)$$

$$\angle G_{ol}(j\omega_g) = -\pi \qquad (16)$$

Substituting (6) and (7) into (13)–(16), we have

$$A = \frac{\omega_p\tau_I}{K_c K_p}\sqrt{(\frac{1+(\tau\omega_p)^2}{(1-\tau_I\tau_D\omega_p^2)^2+(\tau_I\omega_p)^2})} \qquad (17)$$

$$\Phi = \{\wedge(\omega_g) - \omega_g - \tan^{-1}(\omega_g) + \pi/2, \text{ if } \wedge(\omega_g) \geq 0; \quad \wedge(\omega_g) - \omega_g - \tan^{-1}(\omega_g) + 3\pi/2, \text{ if } \wedge(\omega_g) < 0 \qquad (18)$$

And

$$K_c K_p\sqrt{(\frac{(1-\tau_I\tau_D\omega_g^2)^2+(\tau_I\omega_g)^2}{(\omega_g\tau_I)^2(1+(\tau\omega_g)^2)}} = 1 \qquad (19)$$

$$\{\wedge(\omega_p) - \omega_p - \tan^{-1}(\omega_p) = -\pi/2, \text{ if } \wedge(\omega_p) \geq 0; \quad \wedge(\omega_p) - \omega_p - \tan^{-1}(\omega_p) = -3\pi/2, \text{ if } \wedge(\omega_p) < 0 \qquad (20)$$

The above four equations (17)–(20), cannot solve them directly. Because there are five unknowns ωg, ωp, Kc, τ_i, and τ_D in four equations For given gain margin A and phase margin φ. However, the extra degree-of-freedom can be used to maximize the closed loop bandwidth. The optimization problem with constraints on gain margin, phase margin, and maximum closed-loop amplitude ratio can be formulated as

$$\max_{\omega_g,\omega_p,K_c,\tau_i,\tau_D} \omega_b \qquad (21)$$

s.t

$$AR_{cl}(\omega_b) = 0.707 \qquad (22)$$

$$A \geq A^* \qquad (23)$$

$$\varphi \geq \varphi^* \qquad (24)$$

$$M_T \leq M_T^* \qquad (25)$$

where A* and ∅* are given GPM criterions, respectively, M_T*is the upper bound of the maximum amplitude ratio.

C. Analysis

a) To get M_T we need to find the maximum of ARcl(ω) in the entire frequency range (0,∞), but it is difficult because of the nonlinearity of function ARcl(ω).
b) So consider the corresponding frequency for M_T is actually in the range (0,ωb) in this problem. Since ω_b is unknown, an extra parameter ω_{max} is adopted in solving the optimization problem, and ARcl is actually evaluated in a limited range $(0,\omega_{max}]$.
c) The constrained nonlinear optimization problem for proposed method is solved by fmincon function from MATLAB optimization toolbox.
d) F zero function in MATLAB is used to evaluate the constraint functions (21)–(24) in the optimization solver, the gain margin A, phase margin φ, and bandwidth ωb are calculated by solving (17)–(20) and (21) with f zero function in MATLAB.

IV. SIMULATION EXAMPLE

Simulation example of FOPTD system is illustrated in this section. The Closed-loop responses to step change at time 0 in both set-point and load disturbance are analyzed and compared with previous work. The Simulink model to any process model for set point and load changes are shown in Fig.1 and Fig. 2 respectively. Let us consider the plant model given by

$$G_{p1}(s) = \frac{1}{s+1} e^{-0.1s} \quad (26)$$

Different PID tuning methods are used for specified gain margin of A* = 3 and phase margin Φ* = 30° and ω_{max}=100, such as ISE-GPM-load and ISE-GPM set point method (existed. In [15], Ho et al. use ISE as the objective function in the optimization problem with constraints on GPM). The proposed method gives results of optimization with and without the constraint on M_T are both illustrated for set point and disturbance changes are compared with ISE-GPM method. Closed-loop responses to unit step change on set-point for proposed method with different M_T* values and ISE-GPM-set point are shown in Fig.3. The corresponding PID controller parameters and key simulation results, such as ISE, settling time T_S, actual gain A, and phase margin φ, are compared in Table I.

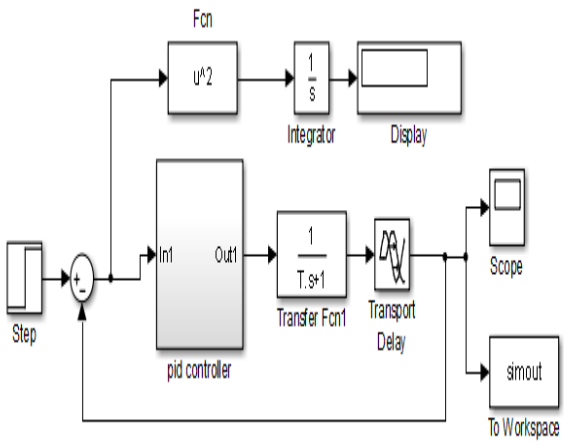


Fig. 1 Simulink model for set-point response

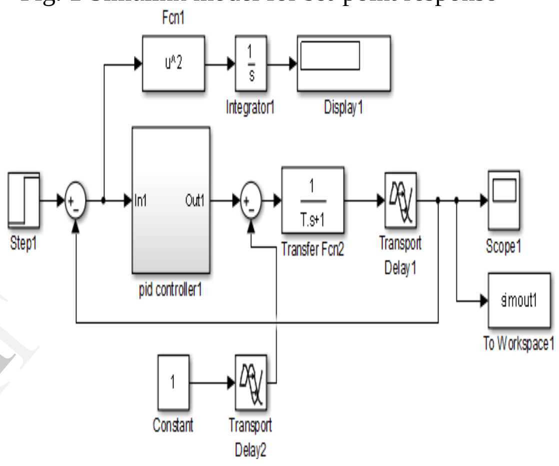
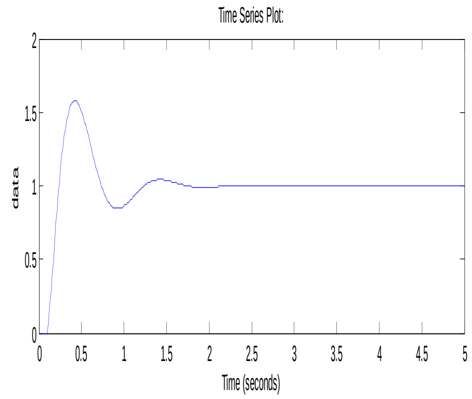
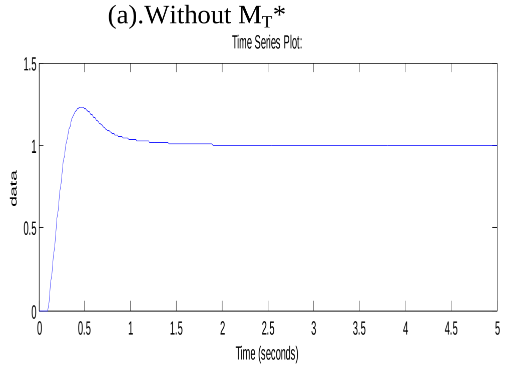


Fig .2Simulink model for disturbance response

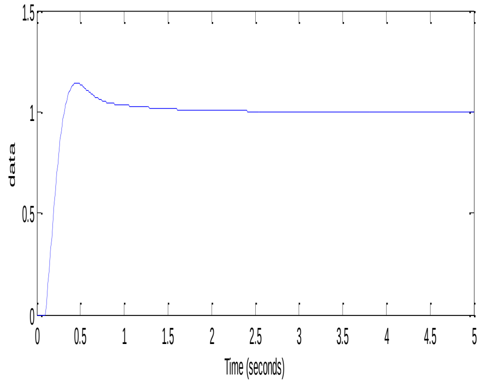


(a).Without M_T*



(b).With M_T*=1.2

Time Series Plot:



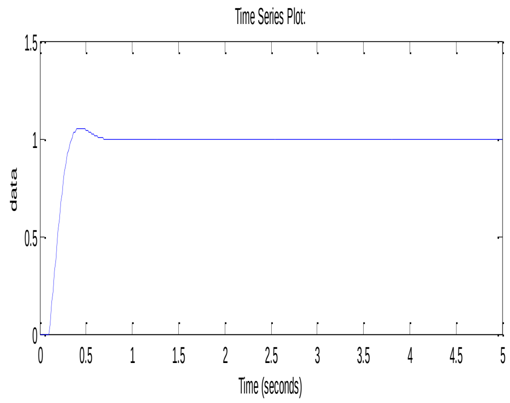
(c).With M_T^* = 1.1

TABLE I :SIMULATION RESULTS FOR SET-POINT-RESPONSES

Parameter	Kc	τ i	τ d	A	Ø	Ts	ISE
Proposed w/o M_T^*	6.1448	0.1902	0.0307	3	30	1.6800	0.2325
Proposed with M_T^*=1.2	5.2613	0.4770	0.0224	3	30.0735	1.4300	0.1759
Proposed with M_T^*=1.1	5.2758	0.6553	0.0180	3	30.0035	1.5200	0.1680
Proposed with M_T^*=1.0	5.3699	1.0323	0.0027	3	30.1735	0.6800	0.1638
ISE-GPM-set point	5.7474	0.2082	0.0382	3	30.000	1.7000	0.2219

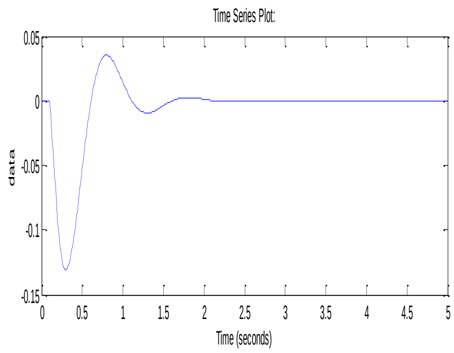
TABLE II :SIMULATION RESULTS FOR STEP LOAD DISTURBANCE RESPONSES

Parameter	Kc	τ i	τ d	A	Ø	ISE
Proposed w/o M_T^*	6.1448	0.1902	0.0307	3	30	0.0043
Proposed with M_T^*=1.8	5.3522	0.4770	0.0224	3	30.005	0.1757
ISE-GPM-load	5.8789	0.2082	0.0382	3	30	0.0045

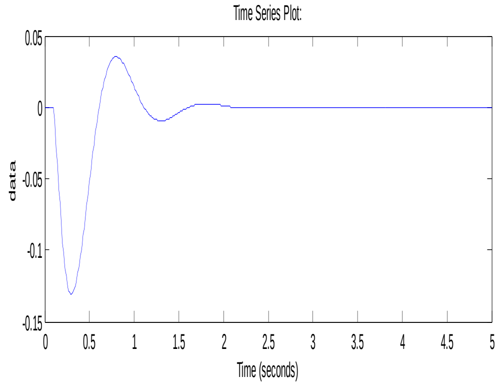


(d) With M_T^*= 1.0

Fig. 3 Step set -point responses for different M_T^* values



(a) With out M_T^*



(b) With M_T* =1.8

Fig. 4 Load disturbance responses for different M_T* values

We can see that the proposed method with an overshoot constraint M_T*= 1.0 gives the best performance in both settling time and ISE and also the proposed method gives the worst closed-loop performance on step set-point change in terms of ISE without constraint on overshoot ratio M_T*. From the above results by changing constraint on M_T* the better tradeoff obtain between closed loop performance and Robustness.

The unit disturbance responses for proposed Method for different M_T* values are shown in Fig. 4. The corresponding PID controller parameters and results on closed-loop performance are given in Table II. In this case, the proposed method for a smaller M_T* leads to a smoother response but larger ISE and the proposed method gives the smallest ISE value without constraint on overshoot but it gives more oscillations. From the above results the constraint on overshoot ratio basically set a balance between set-point tracking and disturbance rejection because of smaller overshoot constraints lead to better set-point responses but worse load disturbance rejection in terms of ISE in this example.

VI. FUTURE SCOPE

Future work is to extend the proposed method to second-order and higher order systems to obtain better performance and more general applications.

V. CONCLUSION

An alternative PID tuning approach has been presented to the popular step response. The new approach to satisfy both robustness and closed-loop performance criteria simultaneously the PID tuning problem is formulated as a nonlinear optimization problem. In this proposed method the bandwidth was maximized with constraints on gain margin, phase margin, and maximum closed-loop amplitude ratio. The GPM serve as robustness criteria, while bandwidth and maximum amplitude ratio serve as closed-loop performance criteria. Simulation results showed that the proposed method better than existing GPM-based method. The proposed method with process control for first order plus time delay system still leads to comparable performance to ISE-GPM method for set-point change response, and better performance for load disturbance response. Moreover, a unique advantage of proposed method is the flexibility brought by the constraint on maximum closed loop amplitude ratio M_T.

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