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Optimal Solution of Pentagonal Fuzzy Transportation Problem Using an Innovative Method

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ABSTRACT:
In this article, we propose a ranking method based on a rank value of fuzzy numbers to find initial basic feasible solution for a pentagonal fuzzy transportation problem. Firstly, the proposed ranking method, which is based on rank value of fuzzy number, is applied. This converts the pentagonal fuzzy transportation problem into a crisp transportation problem, after this numerous methods are applied to determine an initial basic feasible solution (IBFS). We also provide a numerical example of the innovative algorithm and contrast its result with the answer found by other approaches. The innovative method is easy to comprehend and implement to real life transportation challenges. Furthermore, a number of additional fuzzy operations research problems can be resolved using the ranking method.

***KEYWORDS*: fuzzy number; Pentagonal fuzzy number; Fuzzy transportation problem; ranking function; initial basic feasible solution.**

INTRODUCTION:

Uncertainty is a serious issue in any decision-making process. Many techniques and tools have been developed to address the unclear group of decision-making environment. One of the newest strategies for dealing with imprecision is fuzzy set theory. A popular network-planned linear programming problem that arises in many different contexts and has gained a lot of attention lately is the Fuzzy Transportation Problem (FTP). Writers to define and solve the fuzzy transportation problem often use fuzzy numbers such as trapezoidal or triangular fuzzy numbers. However, real-world issues usually include more than four factors. To address these issues, the pentagonal fuzzy number is applied to problems.

A generalization of standard sets theory that permits circumstances in between the entire and nothing are called fuzzy sets. Zadeh [2] introduced the idea of a fuzzy set with a membership function in order to take into consideration the uncertainty involved in decision-making. For the objective of demonstrating the degree of belonging to the set under discussion, a value from the unit interval [0, 1] is assigned to every member of the discussion world. A membership function in a fuzzy set is used to determine how much an element belongs to a class. The membership value is a number between 0 and 1, where 0 means the element is not a member of a class, 1 means it is, and further values indicate the class membership degree. Fuzzy numbers have values that are not clear, as opposed to ordinal numbers, which are precise. Fuzzy numbers representing the opinions of different decision-makers are frequently represented as triangles. Fuzzy numbers with their membership functions have previously been studied for trapezoidal, triangular, pentagonal, heptagonal, diamond, and pyramid shapes. Applications for these numbers include reliability, risk analysis, and non-linear equations. Uncertain numbers were used in many processes.

One optimization problem, Transportation Problem (TP) [1], is determining the most effective cost for transporting products from various sources to different locations. In traditional decision-making scenarios, parameters are concisely taken into account. However, estimating the actual values of problem parameters, such as transportation cost, demand and supply values, becomes challenging in real-life scenarios due to a variety of factors like incomplete input information, poor statistical analysis, fluctuations in the financial market, state of the roads, etc. Decision makers' (DM) opinions are sought in order to forecast the values of parameters in order to address this. DMs frequently use language to describe the values of the parameters. Parameters that can handle and reflect uncertainty, including fuzzy numbers [2] and generalized trapezoidal-valued intuitionistic fuzzy numbers [3], are taken into consideration while addressing these concepts. Numerous types of decision-making issues arise from this situation.

The Fuzzy Transportation Problem (FTP) is one of these issues where fuzzy numbers are taken into consideration for at least one parameter. When transportation constraints were based on crisp values, Hitchcock [1] proposed the fundamental transportation model in 1941. Transportation issues might arise, such as those involving production, scheduling, investments, plant positioning, inventory control, and staff scheduling in many different situations. Numerous writers have created a mathematical model for transportation issues in many contexts. A two-step method for deciphering a fuzzy transportation problem with triangular fuzzy integers was proposed by Aldi Kane [4]. Using the ASM and Zero Suffix methods, Aurora Nur Aini [5] described how to explain the transportation problem without coming up with a workable solution at first. Nirbhay Mathur [6] to describe the fuzzy transportation problem with trapezoidal fuzzy numbers put the lowest demand-supply approach forth. Numerous writers [7–10] used triangular or trapezoidal fuzzy numbers, which are widely used in fuzzy logic, to define and resolve the fuzzy transportation problem. Real-world issues, however, typically involve more than four variables. The pentagonal fuzzy number is utilized to find solutions to certain problem.

In this paper, a new method is innovative to solve FTP with pentagonal fuzzy specifications, which is based on a "Sub interval Average" method and a new method to find IBFS. Numerical examples are solved using the innovative algorithm and the IBFS obtained is compared with the solutions obtained using existing methods to illustrate the advantage of this method. The main contributions of the paper are

- An approach for Pentagonal Fuzzy Numbers using "Sub interval Average".
- Innovatively, a new simplified method for determining IBFS is presented; this method yields an IBFS that is more optimally solved than the IBFS obtained from certain other methods in the literature.
- We have a novel approach to solving the problem where all the parameters are taken into consideration in fuzzy form, unlike some of the methods that are currently found in the literature.

PRELIMINARIES

Definition 1 (Fuzz Number)

A normal and convex fuzzy subset of a real line $\mathbb{R}$ with a membership function $\mu_{\tilde{A}}:\mathbb{R}\rightarrow[0,1]$ that is piece-wise continuous inside its domain is referred to as a fuzzy number $\tilde{A}$.

Definition 2 (Pentagonal Fuzzy Number (PFN))

A pentagonal fuzzy number $\langle \ddot{N}_p \rangle = (a,b,c,d,e)$, should satisfy the following condition

- In the interval $[0,1]$, $\mu_{\langle \ddot{N}_p \rangle}(x)$ is a continuous function.
- In $[a,b]$ and $[b,c]$, $\mu_{\langle \ddot{N}_p \rangle}(x)$ is steadily increasing and continuing function.
- In $[c,d]$, $\mu_{\langle \ddot{N}_p \rangle}(x)$ is strictly decreasing and continuous function.

Definition 3 (Pentagonal Fuzzy Transportation Problem (PFTP))

When dealing with real-world issues, data is not always available in the crisp form for a variety of reasons, including measurement errors, cost fluctuations over time, and environmental variables. Instead, it can have some fuzziness. The term "PFTP" refers to a TP that has at least one parameter expressed as a PFN. It is defined as follows:

$$Min\ Z = \sum_{u=1}^{M}\sum_{v=1}^{N} C_{uv}X_{uv}$$

$$\text{Subject to } \sum_{v=1}^{N} X_{uv} \le S_u; \qquad \text{u} = 1,2,3,\ldots,M \tag{1}$$

$$\sum_{u=1}^{M} X_{uv} \ge D_v; \qquad \text{v} = 1,2,3,\ldots,N \tag{2}$$

$$X_{uv} \ge 0;\ \text{u} = 1,2,3,\ldots,M\ \ and\ \ \text{v} = 1,2,3,\ldots,N \tag{3}$$

Here,

M : Number of sources;

N : Number of locations;

S_u : Pentagonal fuzzy supply at u^{th} origin;

D_v : Pentagonal fuzzy demand at v^{th} destination;

C_{uv} : Pentagonal fuzzy cost of transportation of unit product from u^{th} origin to v^{th} destination;

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X_{uv} : Amount to be transported from u^{th} origin to v^{th} destination such that the total transportation cost is minimized;

$\sum_{v=1}^{N} S_u$: Total pentagonal fuzzy availability of the product;

$\sum_{v=1}^{M} D_u$: Total pentagonal fuzzy demand of the product;

A sufficient and essential requirement for existence of solution is $\sum_{u=1}^{M} S_u = \sum_{v=1}^{N} D_v$ i.e., the problem needs to be balanced. If the problem is unbalanced, a dummy source or origin must be introduced in order to make it balanced.

	1	2	…	N	Supply
1	C_{11}	C_{12}	…	C_{1n}	S_1
:	:	:	…	:	:
M	C_{M2}	C_{M2}	…	C_{MN}	S_M
Demand	D_1	D_2	…	D_N	

Transportation Table

Ranking Methods for Pentagonal Fuzzy Numbers

"Sub interval Average" method for Pentagonal Fuzzy Numbers:

$$R(a,b,c,d,e) = \left(\frac{6(a+b+c+d+e)}{30} \right)$$

Definition

Using the above ranking function, comparison of two PFNs B_1^i and B_2^j can be done in the following way:

If $\Re(B_1^i)$ and $\Re(B_2^j)$ are two fuzzy numbers, then

- $\Re(B_1^i) < \Re(B_2^j) \Rightarrow B_1^i \prec B_2^j$
- $\Re(B_1^i) > \Re(B_2^j) \Rightarrow B_1^i \succ B_2^j$
- $\Re(B_1^i) = \Re(B_2^j) \Rightarrow B_1^i \approx B_2^j$

A New Method to Find IBFs (Innovative Method)
The procedures to find the IBFS are as follows:

- Construct the table of transportation.
- To solve the above TP, we convert fuzzy cost values into crisp values by applying the ranking function.
- Analyze the problem to ensure that it is balanced.
- Move on to the next step if the problem is balanced. If it is unbalanced, transform it to a balanced TP.
- The number of columns in the cost matrix divides the row-wise difference between the largest and smallest value in each row.
- To calculate the column-wise difference between the smallest and largest values in each cost matrix column, divide the total number of rows by the column-wise difference.
- Assign a specific cell in a given matrix, find the highest of the resultant values, and get the matching least cost value. Let's suppose the maximum result involves more than one value. Anyone is up for selection.
  Steps are continuing until all allocations are completed.

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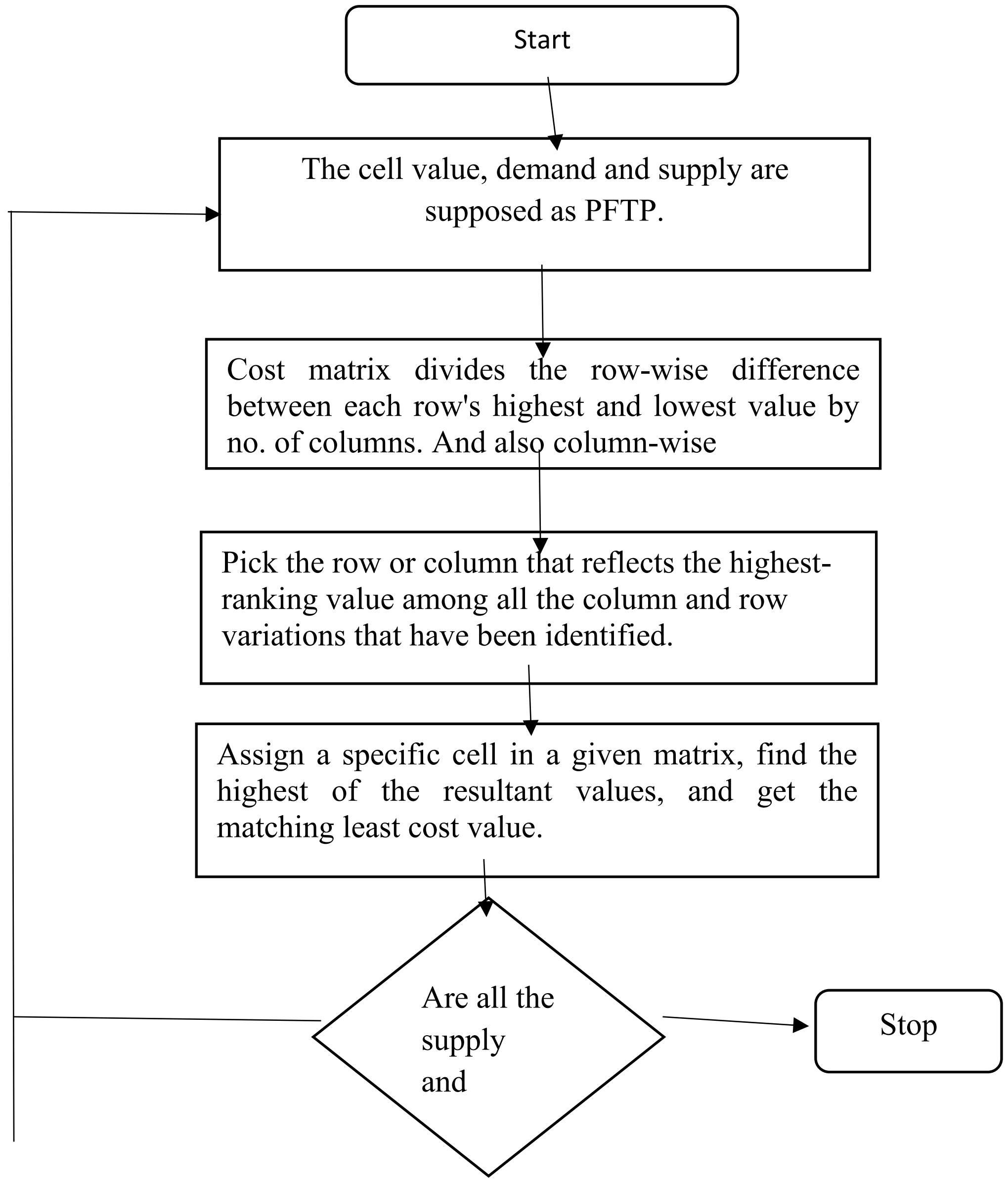
Proposed Algorithm

The following steps are used in proposed algorithm

- Write down the problem in the form of Table 1.
- Use the "Sub interval Average" ranking technique to transform the fuzzy problem to crisp transportation problem.
- Apply the above proposed product method to obtain IBFS of the problem.
- Use MODI method to check if the IBFS obtained is optimal or not.
- If not, repeat MODI method until we arrive at optimal solution.
- Calculate optimum (minimum) transportation cost.

Flow Chart of an Innovative Method



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NUMERICAL ILLUSTRATIONS

Example 1

Three factories A, B, C, D of a company has availabilities 32; 42; 48 and 25, respectively. These factories supply to four warehouses W, X, Y, Z with demands 53; 34; 41 and19, respectively. The transportation cost is given in Table.

Factories/Warehouses	W	X	Y	Z
A	(3,5,6,7,8)	(5,7,8,9,10)	(4,6,810,12)	(3,4,5,8,9)
B	(1,2,4,6,7)	(4,5,6,7,8)	(1,2,3,4,5)	(2,3,8,9,10)
C	(3,4,5,7,8)	(2,4,6,8,10)	(1,2,4,5,6)	(7,8,9,10,11)
D	(2,4,5,6,7)	(1,5,9,10,11)	(2,5,6,7,9)	(2,5,7,11,12)

Solution:

Step 1: Problem is converted to tabular form

Factories/Warehouses	W	X	Y	Z	Availability
A	(3,5,6,7,8)	(5,7,8,9,10)	(4,6,810,12)	(3,4,5,8,9)	32
B	(1,2,4,6,7)	(4,5,6,7,8)	(1,2,3,4,5)	(2,3,8,9,10)	42
C	(3,4,5,7,8)	(2,4,6,8,10)	(1,2,4,5,6)	(7,8,9,10,11)	48
D	(2,4,5,6,7)	(1,5,9,10,11)	(2,5,6,7,9)	(2,5,7,11,12)	25
Demand	53	34	41	19	

Step 2: The reduced crisp TP on converting the fuzzy data to crisp values using "Sub interval Average" ranking technique is shown in Table:

	W	X	Y	Z	Availability
A	5.80	7.80	8	5.80	32
B	4	6	3	6.40	42
C	5.40	6	3.60	9	48
D	4.80	7.20	5.80	7.40	25
Demand	53	34	41	19	147

Step 3: The problem is balanced (Total availabilities=Total demand). Therefore, we go to the next step and find the IBFS.

Final allocation table of the PFTP.

	W	X	Y	Z	Availability
A	5.80	7.80 $\boxed{13}$	8	5.80 $\boxed{19}$	32
B	4 $\boxed{21}$	6 $\boxed{21}$	3	6.40	42
C	5.40 $\boxed{7}$	6	3.60 $\boxed{41}$	9	48
D	4.80 $\boxed{25}$	7.20	5.80	7.40	25
Demand	53	34	41	19	147

In Final Table , the total number of source (m) is 4, the total number of destination (n) is 4, and total number of non-negative allocation 7 is equal to m + n − 1 = 4 + 4 − 1 = 7. Therefore, it has a basic feasible solution. The overall cost can be calculated by

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multiplying the transportation cost of each cell by the units assigned to its assigned value. Thus, a basic feasible solution to the problem =13*7.80 + 19*5.80 + 21*4 + 21*6 + 7*5.40 +41*3.60 + 25*4.80 =727.

Optimal solution:

Final allocation table of MODI method

	W	X	Y	Z	Availability
A	5.80 13	7.80	8	5.80 19	32
B	4 15	6	3 27	6.40	42
C	5.40	6 34	3.60 14	9	48
D	4.80 25	7.20	5.80	7.40	25
Demand	53	34	41	19	147

The minimum total transportation cost:

=5.80*13 + 5.80*19 + 4*15 + 3*27 + 6*34 + 3.60*14 + 4.80*25= 701.

Example 2

Three factories L, M, N and O of a company has availabilities 51; 43; 36 and 25, respectively. These factories supply to four warehouses T, U, V and W with demands 35; 48; 30 and 42, respectively. The transportation cost is given in Table:

Factories/ Warehouses	T	U	V	W
L	(10,11,12,13,14)	(6,7,8,10,12)	(12,14,15,18,20)	(17,18,19,20,22)
M	(17,18,19,20,22)	(3,4,5,7,9)	(11,12,14,16,18)	(6,7,8,10,12)
N	(9,15,18,21,22)	(15,16,17,18,19)	(17,18,20,22,24)	(3,4,5,7,9)
O	(12,14,15,18,20)	(10,11,12,13,14)	(6,7,8,10,12)	(9,15,18,21,22)

Solution

Step 1: Problem is converted to tabular form

Factories/ Warehouses	T	U	V	W	Availability
L	(10,11,12,13,14)	(6,7,8,10,12)	(12,14,15,18,20)	(17,18,19,20,22)	51
M	(17,18,19,20,22)	(3,4,5,7,9)	(11,12,14,16,18)	(6,7,8,10,12)	43
N	(9,15,18,21,22)	(15,16,17,18,19)	(17,18,20,22,24)	(3,4,5,7,9)	36
O	(12,14,15,18,20)	(10,11,12,13,14)	(6,7,8,10,12)	(9,15,18,21,22)	25
Demand	35	48	30	42	

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Step 2: The reduced crisp TP on converting the fuzzy data to crisp values using "Sub interval Average" ranking technique is shown in Table:

Factories/ Warehouses	T	U	V	W	Availability
L	12	8.60	15.80	19.20	51
M	19.20	5.60	14.20	8.60	43
N	17	17	20.20	5.60	36
O	15.80	12	8.60	17	25
Demand	35	48	30	42	

Step 3: The problem is balanced (Total availabilities=Total demand). Therefore, we go to the next step and find the IBFS.

Final allocation table of the PFTP.

Factories/Warehouses	T	U	V	W	Availability
L	12 35	8.60 11	15.80 5	19.20	51
M	19.20	5.60 37	14.20	8.60 6	43
N	17	17	20.20	5.60 36	36
O	15.80	12	8.60 25	17	25
Demand	35	48	30	42	

In Final Table , the total number of source (m) is 4, the total number of destination (n) is 4, and total number of non-negative allocation 7 is equal to m + n − 1 = 4 + 4 − 1 = 7. Therefore, it has a basic feasible solution. The overall cost can be calculated by multiplying the transportation cost of each cell by the units assigned to its assigned value. Thus, a basic feasible solution to the problem =35*12 + 11*8.60 + 5*15.80 + 37*5.60 + 6*8.60 + 36*5.60 + 25*8.60 =1269.

Optimal solution**:**

Final allocation table of MODI method

Factories/ Warehouses	T	U	V	W	Availability
L	12 35	8.60 11	15.80 5	19.20	51
M	19.20	5.60 37	14.20	8.60 6	43
N	17	17	20.20	5.60 36	36
O	15.80	12	8.60 25	17	25
Demand	35	48	30	42	

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The minimum total transportation cost:

=35*12 + 11*8.60 + 5*15.80 + 37*5.60 + 6*8.60 + 36*5.60 + 25*8.60 =1269.

COMPARISON OF THE RESULTS

Comparison with existing methods of finding IBFS:
Table 1 and Table 2 present the comparison of the solutions obtained by an innovative method with some existing methods. The algorithm put forward by us first uses an innovative method to find IBFS followed by MODI method. The advantage of using this combination is that the product method gives IBFS closer to the optimal solution (in most of the problems), which minimizes the number of iterations required to find the optimal solution and MODI method ensures the optimality of the solution. These methods when applied successively, eventually leads us to optimal solution of the TP in lesser time and involving lesser computations. The method proposed by us gives optimal solution in crisp form. Different authors have expressed contrasting point of views in this matter. Although, it has some limitations, but obtaining a crisp optimal solution makes its comparison with the solutions obtained using different methods, easier. In addition, due to this, the solution can be interpreted easily as it is free of uncertainty. As a result, decision-making process becomes less complicated.

Table 1: Comparison Table: Example 1

Methods	**IBFS**	**Optimal Solution**	**Iteration**
North-West corner method	775.00	701	4
Least cost method	730.40	701	3
Row-minima method	772.40	701	4
Column minima method	770.60	701	4
An Innovative method	727.00	701	2

Table 2: Comparison table: Example 2

Methods	**IBFS**	**Optimal Solution**	**Iteration**
North-West corner method	1797.00	1269	4
Least cost method	1314.60	1269	2
Row-minima method	1664.00	1269	4
Column minima method	1314.60	1269	2
An Innovative method	1269.00	1269	1

CONCLUSIONS:

This research article proposes an algorithm to solve PFTP in which the ranking technique and then an innovative method to find IBFS of crisp valued transportation problem is applied. The merits of the method proposed in this paper are as follows:

1. The proposed ranking technique easily converts the pentagonal fuzzy numbers to crisp numbers.
2. The solution is obtained as a crisp number, which makes its comparison, with existing methods, easier.
3. The solution obtained by an innovative method is very close to the optimal solution.

Hence, a number of iterations to obtain optimal solution is comparatively less. In addition, it can be deduced from the comparison of the solution with other methods that this method is more effective and less tedious than the existing methods, since the IBFS obtained by our method is found to be very close to the optimal solution. Thus, this method is of great importance in industrial field.

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