Optimization of Generalized Discrete Fourier Transform for CDMA

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ABSTRACT

Generalized Discrete Fourier Transform (GDFT) with non-linear phase is a complex valued, constant modulus orthogonal function set. GDFT can be effectively used in several engineering applications, including discrete multi-tone (DMT), orthogonal frequency division multiplexing (OFDM) and code division multiple access (CDMA) communication systems. The constant modulus transforms like discrete Fourier transform (DFT), Walsh transform, and Gold codes have been successfully used in above mentioned applications over several decades. However, these transforms are suffering from low cross-correlation features. This problem can be addressed by using GDFT transform. This paper describes the optimization of Generalized Discrete Fourier Transform with Non-Linear Phase for code division multiple access (CDMA). We have also implemented Gold, Walsh and DFT codes. Their performance is analyzed and compared on the basis of various parameters such as Maximum Value of Out-of-Phase Auto-Correlation, Maximum Value of Out-of-Phase Cross-Correlation, Mean-Square Value of Auto-Correlation, Mean-Square Value of Cross-Correlation and merit factor. The result of simulation in the form of comparison of Bit-error-rate (BER) and Signal-to-noise (SNR) ratio for various spreading codes is presented.

Keywords

CDMA, Simulation, Orthogonal, BER, SNR, correlation

1. INTRODUCTION

CDMA uses unique spreading codes to spread the baseband data before transmission. The signal is transmitted in a channel, which is below noise level. The receiver then uses a correlator to dispread the wanted signal, which is passed through a narrow band pass filter. Unwanted signals will not be dispread and will not pass through the filter. Codes take the form of a carefully designed one/zero sequence produced at a much higher rate than that of the baseband data. The rate of a spreading code is referred to as chip rate rather than bit rate. The concept of CDMA is based around the fact that a data sequence is multiplied by a spreading code or sequence which increases the bandwidth of the signal. Then within the receiver the same spreading code or sequence is used to extract the required data. Only when the required code is used, does the required data appear from the signal. The process of extracting the data is called correlation. When a code exactly the same as that used in the transmitter is used, then it is said to have a correlation of one and data is extracted. When a spreading code that does not correlate is used, then the data will not be extracted and a different set of data will appear. This means that it is necessary for the same spreading code to be used within the transmitter and receiver for the data to be extracted.

1.1 CDMA CODE TYPES

There are several types of codes that can be used within a CDMA system for providing the spreading function:

- ***PN codes:*** Pseudo-random number codes (pseudo-noise or PN code) can be generated very easily. These codes will sum to zero over a period of time. Although the sequence is deterministic because of the limited length of the linear shift register used to generate the sequence, they provide a PN code that can be used within a CDMA system to provide the spreading code required. They are used within many systems as there is a very large number that can be used.

A feature of PN codes is that if the same versions of the PN code are time shifted, then they become almost orthogonal, and can be used as virtually orthogonal codes within a CDMA system.

- ***Truly orthogonal codes:*** Two codes are said to be orthogonal if when they are multiplied together the result is added over a period of time they sum to zero. For example a codes 1 -1 -1 1 and 1 -1 1 -1 when multiplied together give 1 1 -1 -1 which gives the sum zero. An example of an orthogonal code set is the Walsh codes used within the IS95 / CDMA2000 system.

Among known binary spreading code families, Gold codes have been successfully used for asynchronous communications in direct sequence CDMA (DS-CDMA) systems due to their low cross-correlation features. Walsh, Gold, Walsh-like [3] and several other binary spreading code sets are designed to optimize so called even correlation functions. However, the odd correlations are as important as even correlations. Therefore, Fukumasa, Kohno and Imai proposed a new set of complex pseudo-random noise (PN) sequences, called equal odd and even (EOE) sequences, with good odd and even correlations [4]. EOE sequences are generated by using one of the binary code sets like Gold or Walsh.
More recently, research has refocused on constant modulus spreading codes for radio communications applications due to the efficiency limitations of non-linear gain characteristics of commonly used power amplifiers in transceivers. Hence, the complex roots of unity are widely proposed as complex spreading codes by several authors in the literature. All codes of such a set are placed on the unit circle of the complex plane. Frank–Zadoff, Chu and Oppermann have forwarded a variety of complex spreading codes [5]–[8]. Moreover, Oppermann has shown that Frank–Zadoff and Chu sequences are special cases of his family of spreading sequences [9].
This paper introduces generalized discrete Fourier transform (GDFT) with nonlinear phase. GDFT provides a unified theoretical framework where popular constant modulus orthogonal function sets including DFT and others are shown to be special solutions. Therefore, GDFT provides a foundation to exploit the phase space in its entirety in order to improve correlation properties of constant modulus orthogonal codes. We present GDFT and demonstrate its improved correlations over the popular DFT, Gold, Walsh and Oppermann families leading to superior communications performance for the scenarios considered in the paper.

2. DISCRETE FOURIER TRANSFORM (DFT)

Let $\{e_r(n)\}$ be a periodic, constant modulus, complex sequence which can be expressed as the r^{th} *power* of the first *primitive N*th *root of unity* z_1 raised to the n^{th} *power* as [1]

$$e_r(n) = \{(z_1^r)^n\} = \cos 2\pi rn/N + j\sin 2\pi rn/N \qquad (1)$$
$$n = 0,1,2,\ldots\ldots\ldots, N\text{-}1 \quad \text{and} \quad r = 0,1,2,\ldots\ldots\ldots\ldots\ldots, N\text{-}1$$

The complex sequence (1) over a finite discrete-time interval in a geometric series is expressed as follows [10],[11]:

$$\frac{1}{N}\sum_{n=0}^{N-1} e_r(n) = \frac{1}{N}\sum_{n=0}^{N-1}(z_1^r)^n = \frac{1}{N}\sum_{n=0}^{N-1} e^{j(2\pi r/N)n} = \begin{cases} 1, & r = mN \\ 0, & r \neq mN \\ & m = integer \end{cases} \qquad (2)$$

Then, (2) is rewritten as the definition of the discrete Fourier transform (DFT) set $\{e_k(n)\}$ satisfying the orthonormality conditions

$$(e_k(n), e_l^*(n)) = \frac{1}{N}\sum_{n=0}^{N-1} e_k(n)e_l^*(n)$$
$$= \frac{1}{N}\sum_{n=0}^{N-1} e^{j(2\pi k/N)n} e^{-j(2\pi l/N)n}$$
$$= \frac{1}{N}\sum_{n=0}^{N-1} e^{j(2\pi/N)(k-l)n}$$
$$\begin{matrix} 1, & r = k - l = mN \\ 0, & r = k - l \neq mN \\ & m = integer \end{matrix} \qquad (3)$$

The notation (*) represents the complex conjugate function of a function. One might rewrite the first primitive Nth root of unity as where , and it is called *the fundamental frequency* defined in radians. We are going to extend the phase functions in (3) in order to define the nonlinear phase GDFT in the following section.

3. GENERALIZED DISCRETE FOURIER TRANSFORM (GDFT)

Let us generalize (2) by rewriting the phase as the difference of two functions

$$\varphi_{kl}(n) = \varphi_k(n) - \varphi_l(n) = r\forall n,$$

and expressing a constant modulus orthogonal set as follows,

$$\frac{1}{N}\sum_{n=0}^{N-1} e^{j(2\pi r/N)n}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} e^{j(2\pi\varphi_{kl}(n)/N)n} \qquad (4)$$

Therefore, by inspection

$$\frac{1}{N}\sum_{n=0}^{N-1} e^{j(2\pi(\varphi_k(n)-\varphi_l(n))/N)n}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} e^{j(2\pi\varphi_k(n)/N)n} e^{-j(2\pi\varphi_l(n)/N)n}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} e_k(n) e_l^*(n)$$

$$= (e_k(n), e_l^*(n)) \qquad (5)$$

Hence, the basis functions of the new set are defined as

$$\{e_k(n)\} \triangleq e^{j(2\pi/N)\varphi_k(n)n} k, n = 0,1,\ldots,N-1 \qquad (6)$$

We call this orthogonal function set as the *Generalized Discrete Fourier Transform* (GDFT).

4. GDFT DESIGN METHODS

Let us define the DFT matrix of size N × N as

$$A_{DFT} = [A_{DFT(k,n)}]$$
$$= [e^{j(2\pi/N)kn} \; k, n = 0,1,2,\ldots,N-1. \quad (7)$$

The square GDFT matrix as a product of the three orthogonal matrices as follows

$$A_{GDFT} = G_1 A_{DFT} G_2,$$
$$A_{GDFT} A_{GDFT}^{-1} = I$$
$$A_{GDFT}^{-1} = A_{GDFT}^{*T},$$
$$G_1 G_1^{*T} = I \quad G_1 G_1^{*T} = I \qquad (8)$$

Where G_1 and G_2 are constant modulus diagonal matrices and written as follows:

$$G_{1(k,n)} = \begin{cases} e^{j\theta_{kk}}, & k = n \\ 0, & k \neq n \\ k, n = 0,1,\ldots,N \end{cases} \qquad (9)$$

and

$$G_{2(k,n)} = \begin{cases} e^{j\gamma_{nn}}, & k = n \\ 0, & k \neq n \\ k, n = 0,1,\ldots,N \end{cases} \qquad (10)$$

The notation indicates that conjugate and transpose operations applied to the matrix, and is the *identity matrix*. Therefore, the transform kernel generating A_{GDFT} matrix through this methodology is expressed as follows:

$$\{e_k(n)\} \triangleq e^{j[(2\pi k/N)n+\theta_{kk}+\gamma_{nn}]}$$
$$k, n = 0,1,\ldots,N-1 \qquad (11)$$

4.1 ALGORITHM

Step 1) Find N X N DFT

$$A_{DFT} = [A_{DFT(k,n)}]$$
$$= [e^{j(2\pi/N)kn}]_{k,n=0,1,\ldots\ldots\ldots\ldots,N-1}$$

Step 2) Find G1 & G2

$$G_{1(k,n)} = \begin{cases} e^{j\theta_{kk}}, & k = n \\ 0, & k \neq n \\ k, n = 0,1,\ldots,N \end{cases}$$

$$G_{2(k,n)} = \begin{cases} e^{j\gamma_{nn}}, & k = n \\ 0, & k \neq n \\ k, n = 0,1,\ldots,N \end{cases}$$

Step 3) Calculate GDFT

$$A_{GDFT} = G_1 A_{DFT} G_2$$

Step 4) Find dam, dcm, dmax, RAC, RCC, F

a. Maximum value of out-of-phase Auto-correlation:

$$d_{am} = \max_{\substack{0\le k<M \\ 1\le m<N}} \{|d_{k,k}(m)|\},$$

b. Maximum value of out-of-phase Cross-correlation:

$$d_{cm} = \max_{\substack{0\le k<M \\ 1\le m<N}} \{|d_{k,l}(m)|\},$$

$$d_{max} = \max\{d_{am}, d_{cm}\}$$

c. Mean –square Value of Auto-Correlation, $R_{AC.}$

$$R_{AC} = \frac{1}{M}\sum_{k=1}^{M}\sum_{\substack{m=1-N \\ m\neq 0}}^{N-1} |d_{k,k}(m)|^2$$

d. Mean –square Value of Cross-Correlation, R_{CC}

$$R_{CC} = \frac{1}{M(M-1)} \sum_{k=1}^{M} \sum_{\substack{l=1 \\ l \neq k}}^{M} \sum_{m=1-N}^{N-1} |d_{k,l}(m)|^2$$

e. The Merit Factor.

$$F_K = \frac{d_{k,k}(0)}{2 \sum_{m=1}^{N-1} |d_{k,k}(m)|^2}$$

Step 5) Simulate using AWGN

Step 6) Plot BER vs SNR

5. Optimal Design of GDFT

The optimal design of phase shaping function ψ(n), based on a performance metric is presented in this section. The phase function $\{\psi_k(n)\, n\}$, of (8) is now decomposed into two functions in the time variable as follows:

$$\hat{\varphi}_k(n) = \varphi_k(n)n = kn + \psi(n)$$

$$\psi(n) = \hat{\varphi}_k(n) - kn = [\varphi_k(n) - k]n$$

$$k \in \{0,1,\ldots,N-1\}, \quad n \in \{1,\ldots,N-1\}$$

$$\psi(0) \in \mathbb{R} \quad \hat{\varphi}_k(0) = \psi(0).$$

The linear term of phase function *kn* is highlighted due to its significance in the orthogonality requirements. The GDFT framework offers us the flexibility to define the phase shaping function according to the design requirements. Note that any function will give us an orthogonal GDFT.

The cross-correlation sequence of a GDFT basis function pair *(k,l)* with length N is defined as

$$R_{\hat{\varphi}_k \hat{\varphi}_l}(m) = \sum_{n=0}^{N-1} e^{j\left(\frac{2\pi}{N}\right)\hat{\varphi}_k(n)} e^{-j\left(\frac{2\pi}{N}\right)\hat{\varphi}_l(n+m)}$$

$$= \sum_{n=0}^{N-1} e^{j\left(\frac{2\pi}{N}\right)[-lm+(k-l)n+\psi(n)-\psi(n+m)]}$$

The auto-correlation function of a GDFT basis function as

$$R_{\hat{\varphi}_k \hat{\varphi}_k}(m) = \sum_{n=0}^{N-1} e^{j\left(\frac{2\pi}{N}\right)\hat{\varphi}_k(n)} e^{-j\left(\frac{2\pi}{N}\right)\hat{\varphi}_k(n+m)}$$

$$= \sum_{n=0}^{N-1} e^{j\left(\frac{2\pi}{N}\right)[-km+\psi(n)-\psi(n+m)]}$$

6. RESULT & DISCUSSION

The GDFT and other codes are implemented in Matlab and the results are shown below

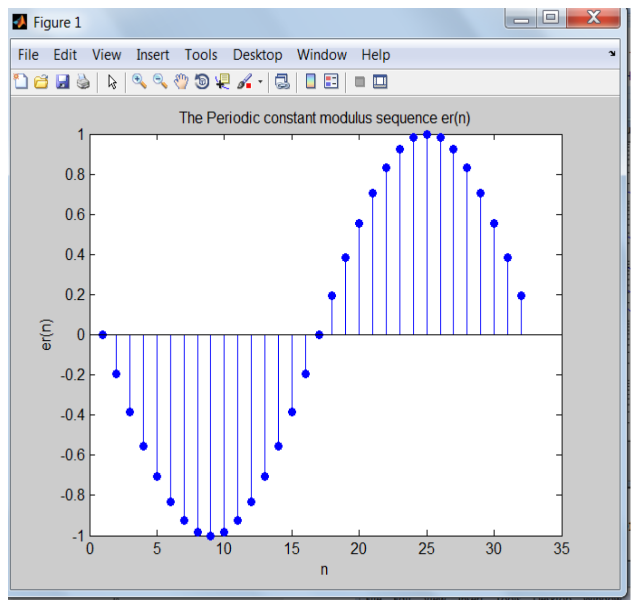

Fig 1 : The Periodic constant modulus sequence $e_r(n)$

Fig 1 shows *er(n)* which is a periodic constant modulus sequence and plays an important role in Generalized Discrete Fourier Transform. It is a complex valued sequence as the r^{th} *power* of the first *primitive Nth root of unity.* Hence it has zeros on the unit circle in z-plane as shown in fig. 2

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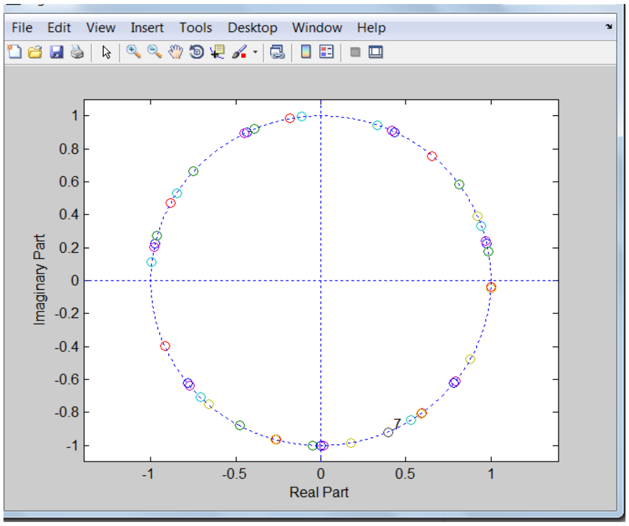

Fig 2 : Pole/zero plot for primitive Nth root of unity

As spreading codes are to be dispread after transmission, the correlator function is required for it. It has been found that the GDFT provides good Auto-correlation and Cross-correlation values. Following figures depicts the results of simulation of different spreading codes such as Walsh, Gold, DFT and GDFT. Their Correlation is displayed along with Bit-error rate and Signal-to-Noise ratio.

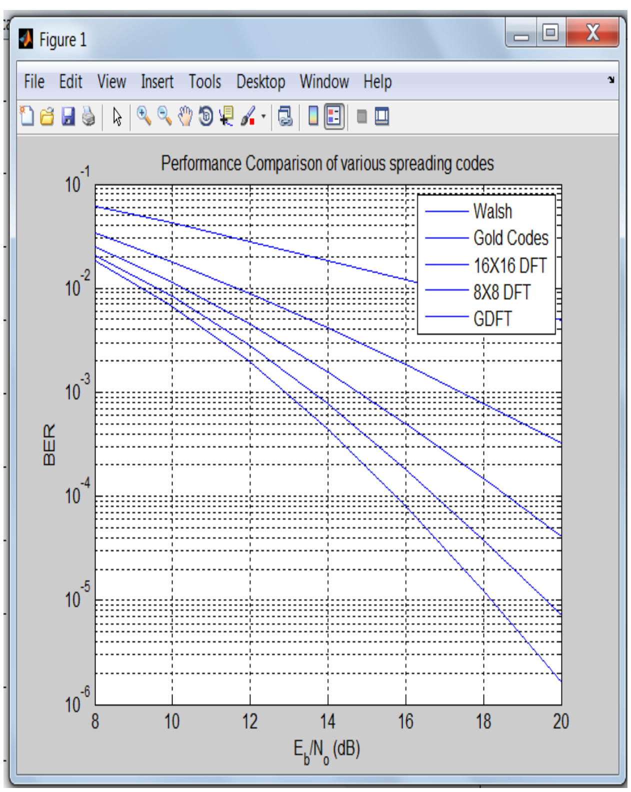

Fig 3 : Performance comparison of various codes

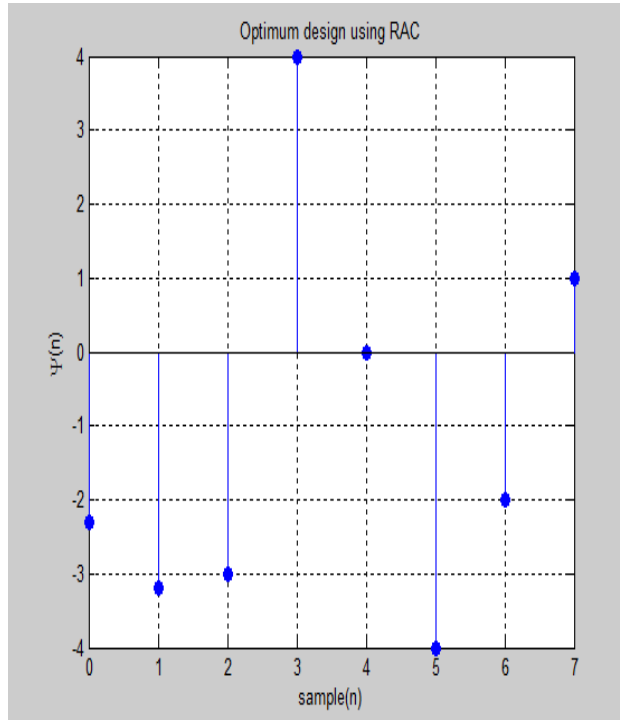

Fig 4 : Optimum Design of ψ(n) using R_{AC}

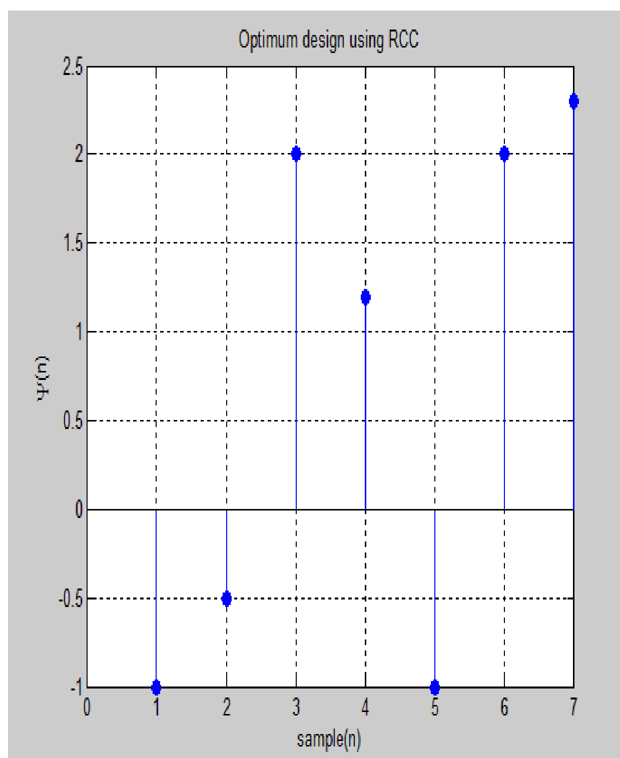

Fig 5 : Optimum Design of ψ(n) using R_{CC}

The main advantage of the proposed method is the ability to design a wide collection of constant modulus orthogonal code sets based on the desired correlation performance mimicking the specs of interest. Moreover, the proposed GDFT technique can also be considered as a natural enhancement to DFT to obtain improved performance. Note that the auto-correlation magnitude functions of the codes in any GDFT set are the same. displays auto-correlation function of a size 16 GDFT code optimized based on along with size 16 DFT set for comparison purposes.

Similarly, cross-correlation functions (CCF) of the first and second codes of optimal GDFT design based on metric and DFT set for are displayed in Fig. 6. These figures highlight the merit of the proposed GDFT framework over the traditional DFT.

In order to compare performance of code families, several objective performance metrics are used. All the metrics used in this study depend on aperiodic correlation functions (ACF) of the spreading code set

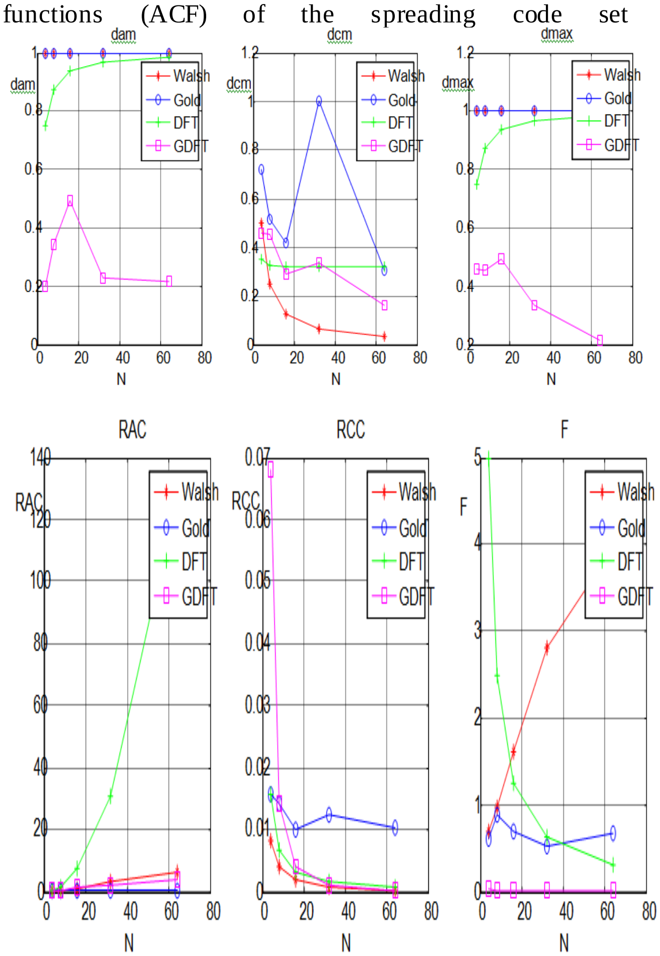

Table 1 : Performance comparison of various codes

Code	dam	Dcm	Dmax	RAC	RCC	F
Walsh	1.000	0.031	1.000	1.960	0.002	2.213
Gold	1.000	0.306	1.000	0.247	0.007	0.786
8 x 8 DFT	0.875	0.326	0.875	1.637	0.006	0.002
16 x 16 DFT	0.937	0.088	0.937	7.318	0.002	0.001
GDFT	0.342	0.454	0.454	0.335	0.014	0.006

7. CONCLUSION

In this paper, we have discussed the MATLAB implementation of Walsh, Gold, DFT and GDFT codes. These codes are simulated and their results are presented. It has been observed from the results that GDFT provides better and effective correlation functions which can be exploited in optimum way in asynchronous CDMA communication systems. However, there is further scope for optimization.

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