

Optimization of RCC Dome

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Abstract - A Domes may be defined as a thin shell structure generated by the revolution of a regular curve about one of its axes. Domes are those space structures which provide large column free net precious areas. Sports stadiums, assembly halls, exhibition centers, shopping malls, industrial structures etc. are the some examples where dome is used. Dome encloses the maximum amount of space with minimum surface. This feature provides economy in terms of consumption of constructional material.

The dome is a structural system that consists of one or more layers of elements that are arched in all directions. The surface of a dome may be a part of a single surface such as a sphere or a paraboloid, or it may consists of patch work of different surfaces. In dome structure all members are subjected to tensile and compressive forces only. Even though dome has to design only for axial forces; the manual calculations involved in analysis, design and optimizing is complex and error prone. Hence an attempt is made to develop software for the analysis, design and weight optimization of a rcc dome. The software has been developed in MATLAB 9.0.

The membrane analysis method of shell theory is used for analysis of the dome and design is done by working stress method as per IS: 800-2007. For optimization of dome structure dynamic programming method is used. The effect of dead and live loads on dome is only considered for analysis purpose according to IS: 875 - 1987 part I-II. A study of parameters affecting the total weight of the domes is carried out.

For parametric study, domes of base diameters 6 m, 20 m, 45 m, are considered. The height of each dome varied between 1/5th to 10th of the base diameter. The results of parametric study are produced in the form of bar charts and discussion in detail. Based on the results conclusions are made.

Keywords—Optimization of weight of Steel, dome, MATLAB.9.0,

I. INTRODUCTION

Domes are space structures which provide large column free net precious area for utilization. Domes enclose the maximum amount of space with minimum surface. This feature provides economy in terms of consumption of constructional material. The availability of high speed computational tools and numerical software for analysis in recent years has facilitated structural engineers to develop many economical designs of space structures.

Optimization is the act of obtaining the best results under given circumstances. Optimization is being used in those design activities in which the goal is not merely to achieve a feasible design, but also a specified design objective. In the design of an engineering structure, engineer has to take many

technical decisions at several stages. The ultimate goal of all such decisions is to either minimize the effort required or maximize the desired benefit. Now a days, a number of optimization techniques are available. The mathematical programming techniques for optimization of steel dome structures can be used. Since economy can be easily assessed by total weight of structure, an attempt is hereby made to reduce the total weight of the structure.

The shape of the dome depends upon the type of the curve and the direction of the axis of revolution. When the segment of a circular curve revolves about its vertical diameter, a spherical dome is obtained. Similarly, conical dome is obtained by the revolution of a right angled triangle about its vertical axis, while an elliptical dome is obtained by the revolution of an elliptical curve about one of its axes. However, out of these, spherical domes are more commonly used. In the case of a spherical dome the vertical section through the axis of revolution in any direction is an arc of a circle.

Domes are used in variety of structures, such as (i) roof of circular areas (ii) circular tanks (iii) hangers, (iv) exhibition halls, auditoriums and planetariums' and (v) bottoms of tanks, bins and bunkers. Domes may be constructed of masonry, steel, timber and reinforced cement concrete. Stone and brick domes are one of the oldest architectural forms. However, reinforced concrete domes are more common now-a-days, since they can be constructed over large spans.

II. THEORY OF SHELLS OF REVOLUTION

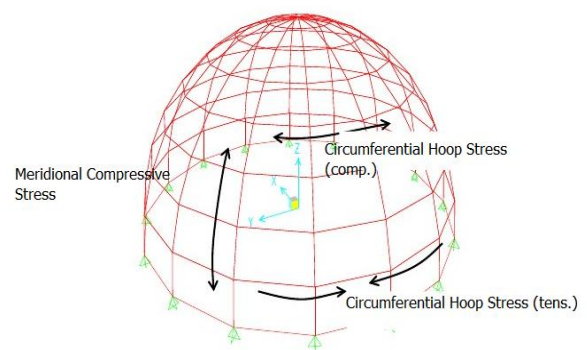


Figure 1 Stresses in dome

The Fig shows a typical shell of revolution, on which equilibrium of an element, obtained by intersection of meridian and latitude, is indicated. Forces along the circumference are denoted by N_ϕ and are called meridian stresses and forces at right angles to the meridian plane and along the latitude are horizontal and called the hoop stresses, denoted by N_θ . These two forces and the external force normal to the surface must be in equilibrium.

To summarize, therefore, two types of stresses are induced in a dome

- (i) Meridional thrust (T) along the direction of meridian.
- (ii) Hoop stress (H) along the latitudes.

Dome structure subject to various kinds of loads like dead load, imposed loads, wind loads, snow loads etc. But in this study work has been carried out for only dead load and imposed loads.

Summary of formulae:

$$\text{Meridional thrust } T = \frac{w R}{1 + \cos(\theta)}$$

$$H = \frac{W * R}{t} \left\{ \cos \theta - \frac{1}{(1 + \cos \theta)} \right\}$$

Dead load on the dome structure includes the self weight of the structure and weight of the roof covering if any. For calculation dead load recommendations of IS:875 (Part 1)-1987 is referred. For calculation and estimation of imposed loads IS: 875 (Part 2)-1987 is referred.

The aim of the present work is to develop a suite of computer programs in MATLAB programming language that will generate geometry and loading data, analyze and design spherical domes and to optimize steel weight and volume of concrete of the rcc dome to carry given loads for given topology using constrained optimization method.

A. Scope of the study

1. Simply supported RCC spherical dome will be considered.
2. Dead Load, Live Load as per IS: 875-1987 are considered
3. Analysis and design of Spherical R.C.C Dome for Gravity loads using E-tabs software.
4. Weight of the reinforcing steel and volume of concrete optimization using constrained optimization technique using MATLAB. The general program is written in order to solve the n number of numerical problems.

III. OPTIMIZATION METHODS

Optimization is the selection of a best element (with regard to some criteria) from some set of available alternatives. In the simplest case, an optimization problem consists of maximizing or minimizing a real function by systematically choosing input values from within an allowed set and computing the value of the function.

As the construction materials are getting extinct day by day it is important for the structural engineers to concentrate on

optimum designing of the structures. With a special reference to structural problem it is always one of the minimizing or maximizing a certain specific characteristic of structural system like cost, weight, performance capability of the system depends on the problem. This to be achieved without sacrificing any of the functional requirements like stresses deformation and load capabilities. Thus, the optimization procedure must only be used to those problems where there is a definite need of achieving a quality product or competitive product.

Definition of optimization problem

An optimization or a mathematical programming problem can be stated as follows.

Find $X = \{ X_1, X_2, \dots, X_n \}$ which minimizes $f(x)$

Subject to the constraints

$g_j(x) \leq 0, j=1, 2, \dots, m$ inequality constraint

$h_j(x) = 0, j=1, 2, \dots, p$ equality constraint

Where, X is an n-dimensional vector called the designed vector, $f(x)$ is termed the objective function. The number of variables n and the number of constraints m and/or p need not be related in any way.

There are three classes of optimization methods:

- 1) Simultaneous Mode of Failure
- 2) Optimality Criteria
- 3) Mathematical Programming

Types of search techniques:

- 1) Linear Programming
- 2) Nonlinear Programming

Various methods used in Nonlinear Programming are:

- 1) Feasible Direction Method
- 2) Sequential Unconstrained Minimization Method
- 3) Dynamic Programming
- 4) Geometric Programming
- 5) Sequential Linear Programming

Non traditional search techniques:

- 1) Genetic Algorithms
- 2) Simulated Annealing
- 3) Neural Networks Method Based Optimization

In the present study Dynamic Programming method has been used for optimization.

Dynamic Programming:

It is a very powerful optimization technique applicable to multistage decision problems. If the optimization problem can be conveniently decomposed into several stages of decision making by taking advantages of the serial relationship which may exist, dynamic programming method is useful. This method suggests decomposing a system into N-single stage problems; get a set of solution for each stage and finally pickup optimal solution of the system.

IV. METHODOLOGY

To develop the software, MATLAB version 9.0 has been used. The analysis of the dome structure is carried out by empirical formulas of Shell theory. Design is done by Working stress method and optimization is done by dynamic Search technique. All IS code provisions are incorporated in design process.

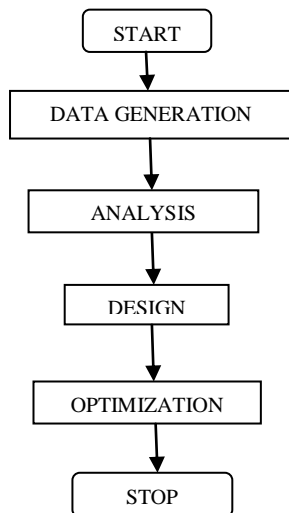


Fig.3 Master Flow chart

1. Data Generation

Data generation includes the generation of information pertaining to Radius of curvature of dome, angle θ , for a given base diameter, height and load. This reduces the error prone task of entering the information manually. The steps involved in the data generation of the dome structure are as follows.

1. Calculating Radius of Curvature & angle θ

The program requires the base diameter and height/Rise of dome. Based on these input parameters, Radius of Dome and angle θ is generated.

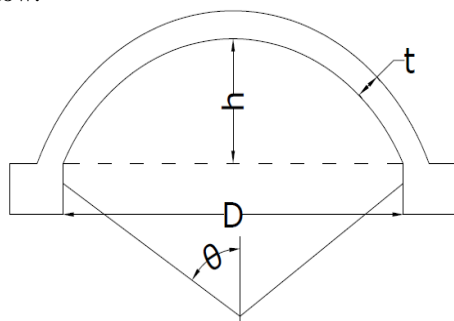
2. Boundary conditions

The analysis of the dome is performed assuming the dome to be simply supported at the the bottom circular segment.

3. Load application

The dome is analyzed for two independent load cases, namely, load and live load. Dead load consists of self weight of Dome for given thickness. Self weight of the dome is calculated using density of concrete, surface area of dome and thickness of the Dome. Initially the assumed cross section areas are used and subsequently during the optimization process, the designed area for the current optimization cycle is used. For design of dome structure, live load is calculated as per IS 456 4. Formulae used in data generation

Some of the formulae used in data generation process are listed below:



Radius of the dome curvature

$$R = \frac{(D/2)^2 + h^2}{2h}$$

$$\theta = \sin^{-1}(D / (2 * R))$$

2. Analysis

Analysis includes the calculation of forces in the Dome due to the applied loads. The analysis program is based on the empirical formulae derived from shell theory. It takes the output of the data generation program as its input.

Calculation of Meridional stress, $T = \frac{w R}{t * (1 + \cos \theta)}$

$$\text{Hoop stress, } H = \frac{W * R}{t} \left\{ \cos \theta - \frac{1}{(1 + \cos \theta)} \right\}$$

3. Design

The output of the analysis program serves as the input for the design program.

Area of Steel is calculated based on the stresses calculated in both Meridional direction and Hoop direction as below

$$A_{st_T} = \frac{T_{Max} * t}{\sigma_{st}}$$

$$A_{st_H} = \frac{H_{Max} * t}{\sigma_{st}}$$

Spacing of Rebar is calculated as below:

$$\text{Spacing_T} = \frac{(\pi / 4) * d^2}{(A_{st_T})}$$

$$\text{Spacing_H} = \frac{(\pi / 4) * d^2}{(A_{st_H})}$$

Number of bars in both calculated as below:

$$N1 = \frac{(2 * \pi * (D/2)) / 2}{\text{Spacing_T}}$$

$$N2 = \frac{h}{\text{Spacing_H}}$$

Length of bars in Meridional Direction:

$$LT = \pi * (D/2)$$

Length of bars in Hoop Direction:

$$\text{Bottom most bar } LH1 = 2 * \pi * (D/2)$$

$$\text{Radius @ second level, } r1 = \sqrt{(R^2 - (R - h + \text{Spacing_H}))^2}$$

Length of successive bars toward crown:

$$LH_n = 2 * \pi * (r_n)$$

Weight of Steel in Kg (Ws) :

$$W_s = 7850 * \pi/4 * d^2 * \left\{ (N1 * LT) + \sum_1^n LH \right\}$$

Volume of Concrete in m³ (Vc):

$$V_c = \pi * \left[h^2 + (D/2)^2 \right] * t$$

4. Optimization of dome.

Objective function

In the present study, the total cost of a RC frame is considered as the objective function F(x) which is given as follows,

(i) Weight of reinforcement

$$F(x) = \{A_{st} * \rho_s * l\}$$

Where A_{st} = Area of steel in mm²

ρ_s = density of steel = 7850 kg/m³ for Mild steel

l = length of bars in mm

Three variables are defined namely, area of reinforcement, length of bars and density of steel which is kept constant throughout the member, so two variables alone are sufficient to represent weight of reinforcing steel.

(ii) Volume of concrete

$$F(x) = \{A_c * t\}$$

Where, A_c = Surface Area of Dome in m²

$$= \pi * (h^2 + (D/2)^2)$$

t = Thickness of the Dome in m

Design constraints

This section describes constraints imposed on optimization of Domes as below

1) The first one specifies that the compressive stress of any point in the shell should not exceed the permissible value. Then,

$$g(1) = \frac{\sigma}{\sigma_{cb}} - 1 \leq 0$$

2) IS 2210 specifies the following two constraints to avoid buckling,

$$g(2.A) = \frac{\sigma_L}{\sigma_{ac}} - 1 \leq 0$$

$$g(2.B) = \frac{\sigma_T}{\sigma_{ac}} - 1 \leq 0$$

Where,

σ_L - Maximum Compressive stress in longitudinal direction

σ_T - Maximum Transverse compressive stress

σ_{ac} = Allowable buckling stress

$$= \frac{E_c * t}{R \sqrt{3}}$$

3) Following side constraint also to be provided for minimum thickness of Dome to be adopted

$$g(3) = \frac{40}{t} - 1 \leq 0$$

V. STEPS FOR THE PROGRAM

The computer program for the project is done using MATLAB version 9.0. The program is written for the manual design and later to optimize the area of steel using Non linear constrained optimization Technique.

1. User defined material data like Base diameter of Dome, load, grade of concrete and steel, initially assumed thickness are to be given as input.
2. The Radius of dome, angle θ , total load are calculated by the program in data generation part of the program.
3. Analysis part of the program calculates meridional and hoop stresses and checks for the constraints as mentioned in equations and retrieves the values of thickness which meets all the constraints, which gives the optimum thickness
4. In design part of the program, area of steel is calculated depending on the stresses generated in both meridional as well as hoop direction.

Program checks for minimum area of steel A_{st,min} provision.

If A_{st} calculated is less than A_{st,min}, minimum A_{st} will be assigned.

5. Further, number of bars and their spacing is calculated in order to calculate total weight of the steel reinforcement. Also the volume of concrete is calculated which gives the optimum values

Consider three domes of different spans viz 6m, 20m and 45 m for which

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$\sigma_{ST} = 230 \text{ N/mm}^2$$

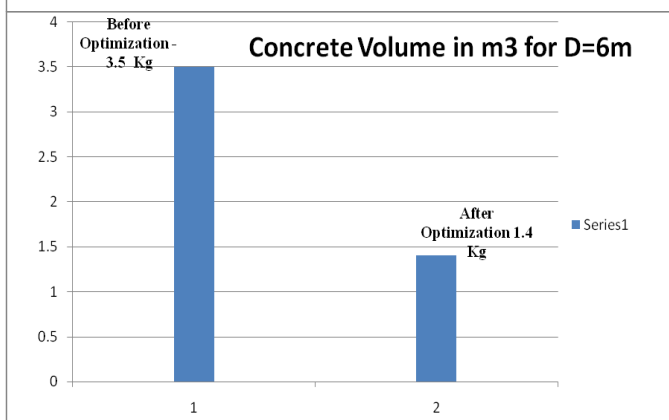
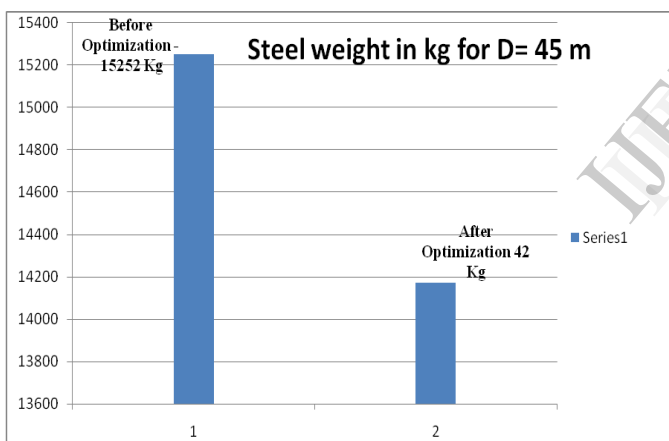
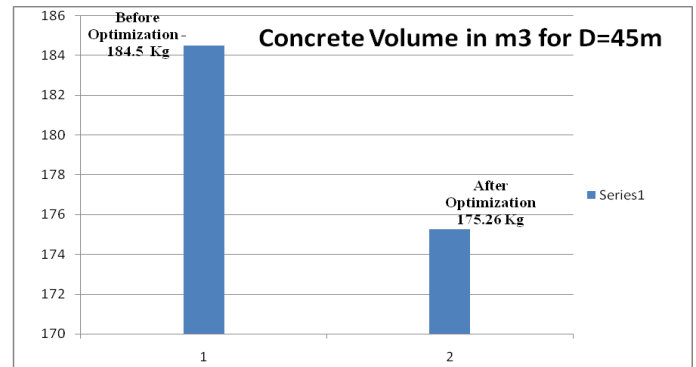
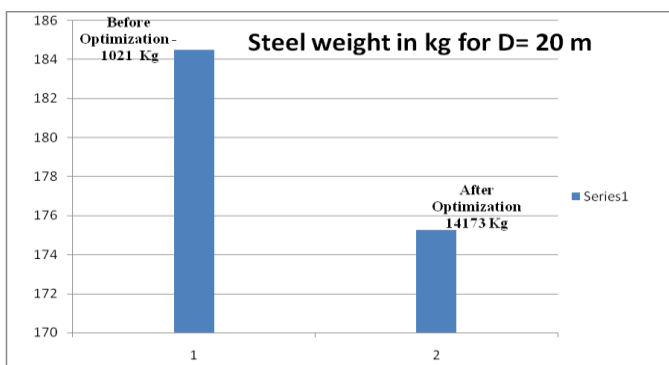
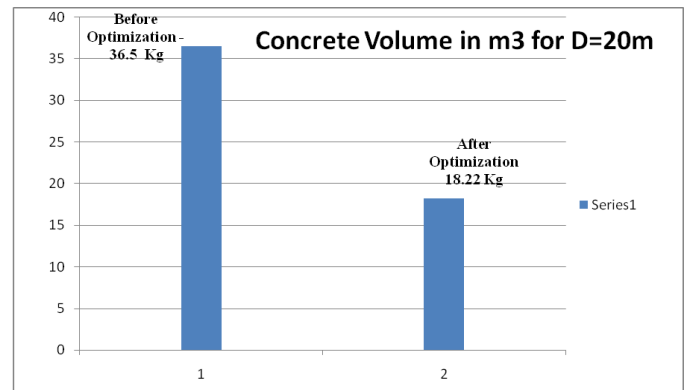
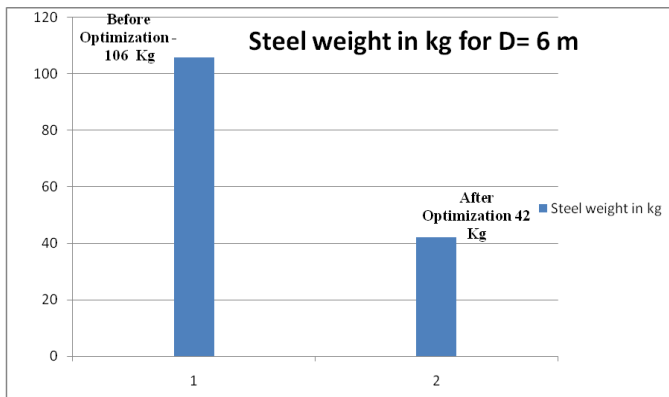
$$\text{Loads} = 2 \text{ kN/m}^2 \text{ (Live load)}$$

$$h = \text{generally considered as } 1/5^{\text{th}} \text{ of base dia, D}$$

Table The results obtained for above spans of domes

'D' in metres	Weight of Steel in Kg by empirical design	Weight of Steel in Kg after optimization
6	3.5	1.4
20	36.5	18.22
45	184.5	175.26

'D' in metres	Volume of concrete in m3 by empirical design	Volume of concrete in m3 after optimization
6	106	42
20	1021	956
45	15252	14173



In this problem the results are discussed based on the manual design and optimum design of the above tables it shows that the area of steel obtained for optimum design is less than the normal design

VI. CONCLUSIONS

The following conclusion may be made from the recent study:

1. It is possible to formulate and to obtain solution for the minimum cost design for R.C.C. Dome.
2. Significant savings in cost over the normal design can be achieved by the optimization. However the actual percentage of the saving obtained for optimum design for R.C.C. T-beam girder depend upon the span of slab and grade of material.

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