

## OPTIMIZATION OF RETAINING WALL BY USING OPTIMTOOL IN MATLAB

1.Prof.Sable K.S., *PHD, Principal in Sayadri College of Engg.Alephata, Pune*

2.Miss Patil Archana A. *M.E.Structure in Amrutvahini College of Engg.,Sangamner,Pune*

### Abstract

*Optimization of concrete retaining walls is an important task in geotechnical and structural engineering. However other than stability considerations very often in such design the settlement aspects is neglected. As such, attention to various aspects of geotechnical engineering design needs to be considered. However, consideration of all these aspects makes the design complicated. To economize the cost under such situation needs to vary the dimensions of the wall several times making it very tedious and monotonous. As it is extremely difficult to obtain a design satisfying all the safety requirements, it is necessary to cast the problem as one of the mathematical non linear programming techniques. A program is developed for analysis and designing low-cost or low-weight cantilever reinforced concrete retaining walls with and without base shear key in matlab for optimtool. The optimtool is used to find the minimum cost and weight for concrete retaining walls. Illustrative cases of retaining wall are solved, and their results are presented and discussed by using Interior point method from optimtool. The optimum design formulation allows for a detailed sensitivity analysis to be made for variation in top thickness of stem, surcharge load and internal angle of friction with different height.*

### 1. Introduction

Optimization is the act of obtaining the best result under given circumstances. In design process, engineers have to take many technological and managerial decisions at several stages. The ultimate goal of all such decisions is either to minimize the effort required or to maximize the desired benefit. The objective of the optimization is to minimize the total cost or total weight per unit length of the retaining structure subjected to constraints based on stability, bending moment and shear force capacities, and the requirements of the IS 456-2000. The improvements in

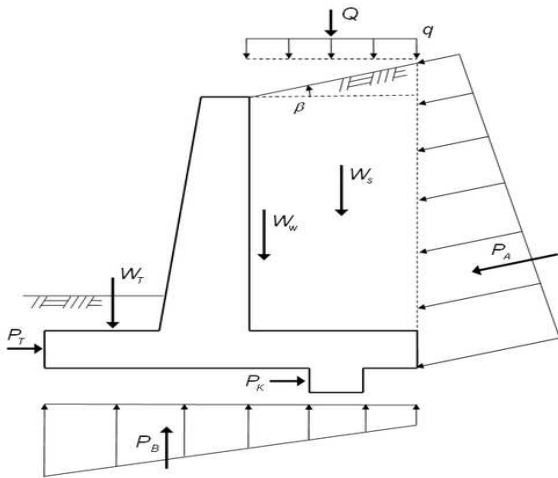
numerical methods and computer technology have given impetus to this concept of optimization.

### 2. Methodology

Several studies have been done to develop methodologies for the analysis and design of cantilever retaining walls. However, limited work has been undertaken to develop methods for their optimum cost design. In general, the forces acting on this model of a retaining structure are consistent; formulation includes both passive forces on the front of the toe and base shear key sections and the bearing force of the base soil. Figure1 shows the general forces acting on the retaining wall:  $W_W$  is the combined weight of all the sections of the reinforced concrete wall;  $W_S$  is the weight of backfill acting on the heel;  $W_T$  is the weight of soil on the toe;  $Q$  is the surcharge load;  $P_A$  is force due to the active earth pressure;  $P_K$  and  $P_T$  are the forces due to passive earth pressure on the base shear key and front part of the toe section, respectively; and  $P_B$  is the force due to the bearing stress of the base soil. Three failure modes are considered in the analysis of the retaining structure: overturning, sliding, and bearing stress. The overturning moment about the toe of the wall is a balance of the force due to the active soil pressure of the retained soil weight and the self weight of the concrete structure, the soil above the base, and the surcharge load. The passive forces on the front of the toe and the base shear key section are not considered in the overturning moment. The factor of safety for overturning  $FS_O$  about the toe is defined as:

$$FS_O = \frac{\Sigma M_R}{\Sigma M_O}$$

where  $\Sigma M_R$  is the sum of the moments about toe resisting overturning and  $\Sigma M_O$  is the sum of the moments about toe tending to overturn the structure.



**Figure 1. "General Forces Acting on the Retaining Wall"**

The active earth pressure coefficient ka is:

$$K_p = \frac{1 - \sin\phi}{1 + \sin\phi}$$

where  $\phi$  is the angle of internal friction.

The passive earth pressure coefficient Kp is:

$$K_p = \frac{1 + \sin\phi}{1 - \sin\phi}$$

For the sliding mode of failure, only the horizontal component of the active force is considered. Horizontal resisting forces are due to the weight of wall and soil on the base, surcharge load, friction between soil and base of wall, and passive force due to soil on the toe and base shear key sections. The factor of safety against sliding FS<sub>S</sub> is defined as:

$$FS_S = \frac{\sum F_R}{\sum F_D}$$

where  $\sum F_R$  is the sum of the horizontal resisting forces and  $\sum F_D$  is the sum of the horizontal sliding forces.

In the bearing analysis of the structure, the base of retaining wall is considered to be a shallow foundation. The minimum and maximum applied bearing stresses on the base of the foundation are:

$$q_{Max} = \frac{\sum V}{B} \left( 1 \pm \frac{6 \cdot E}{B} \right)$$

where q<sub>min</sub> and q<sub>max</sub> are the bearing stresses on the toe and heel sections, B is the width of the base,  $\sum V$  is

the sum of the vertical forces (due to the weight of wall, the soil above the base, and surcharge load), and E is the eccentricity of the resultant force system expressed as:

$$E = \frac{B}{2} - \frac{\sum M_R - \sum M_O}{\sum V}$$

The eccentricity is determined from the ratio of the summation of overturning moments about the toe to the sum of vertical forces. The factor of safety for the bearing capacity FS<sub>B</sub> is:

$$FS_B = \frac{q_u}{q_{max}}$$

where qu is the ultimate bearing capacity of the foundation.

### 3. Formulation of Optimum Design Problem

The general three phases considered in the optimum design of any structure are: structural modelling, optimum design modelling, and the optimization algorithm. In the structural modelling, the problem is formulated as the determination of a set of design variables for which the objective of the design is achieved without violating the design constraints. For the optimum design modelling, study the problem parameters in depth, so as to decide on design parameters, design variables, constraints, and the objective function. In the search for finding optimum design starts from a design or from a set of designs to proceed towards optimum. For economic design of retaining wall, optimization methodology and above parameters are discussed in the following sections.

#### Structural Modelling

In optimal design problem of retaining wall the aim is to minimize the construction cost and weight of the wall under constraints. This optimization problem can be expressed as follows:

minimize f(X)

subject to

$$g_i(X) \leq 0 \quad i=1,2, \dots, p$$

$$h_j(X) = 0 \quad j=1,2, \dots, m$$

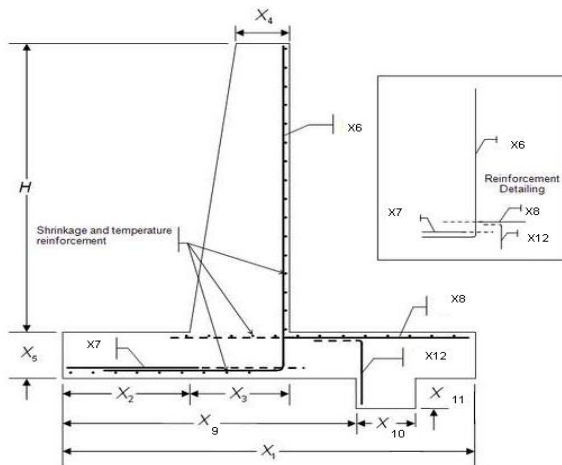
$$L_k \leq X_k \leq U_k \quad k=1, 2 \dots, n$$

where f(X) is the objective function g<sub>i</sub>(X), h<sub>j</sub>(X) are inequality and equality constraints respectively and L<sub>k</sub>, U<sub>k</sub> are lower and upper bound constraints. To economic design of retaining wall, the objective function, design variables and design constraints should be defined explicitly.

## Optimum Design Modelling

### A) Design Variables

The design variables are divided into two categories: those that prescribe the geometric dimensions of wall cross-section, and those that model the steel reinforcement. In general, there are eight geometric design variables representing the dimensions of the retaining wall:  $X_1$  is the width of the base,  $X_2$  is the toe projection,  $X_3$  is the thickness at the bottom of the stem,  $X_4$  is the thickness at the top of the stem,  $X_5$  is the thickness of base slab,  $X_9$  is the distance from toe to the front of the base shear key,  $X_{10}$  is the width of the key, and  $X_{11}$  is the depth of the key. There are four additional design variables related to the steel reinforcement of the various sections of the retaining wall:  $X_6$  is the vertical steel reinforcement in the stem,  $X_7$  is the horizontal steel reinforcement in the toe,  $X_8$  is the horizontal steel reinforcement in the heel, and  $X_{12}$  is the vertical steel reinforcement in the base shear key. While the geometric design variables may be either continuous or discrete values, the steel reinforcement design variables are modeled as a set of discrete values. In this formulation, where the retaining structure is designed for a unit length, the number of bars in a unit meter length of the retaining wall conforms to the minimum and maximum amount of steel allowed.



**Figure 2. "Mathematical Model used for Optimum Design of Reinforced Concrete Cantilever Retaining Wall"**

### B) Constraints

The typical design philosophy for retaining structures seeks designs that provide safety and stability against

failure modes and comply with concrete building code requirements. These requirements may be classified into four general groups of design constraints: stability, capacity, reinforcement configuration, and geometric limitations. Each of the design constraints are posed as penalties on the overall objective functions of the design and are non-zero only when violated. In other words, if the design is feasible, the sum of the constraint penalties will be zero. According to IS code 456:2000, the design constraints may be classified as geotechnical and structural requirements. These requirements represent the failure modes as a function of the design variables. Feasible retaining wall designs should provide minimum factor of safety coefficients for overturning, sliding, and bearing capacity failure modes. Failure modes is summarized in table are as follows-

**Table1. "Failure Modes of Retaining Wall"**

Inequality constraints	Failure mode
$g_1(\mathbf{X})$	Overturning stability
$g_2(\mathbf{X})$	Maximum bearing capacity
$g_3(\mathbf{X})$	Minimum bearing capacity
$g_4(\mathbf{X})$	Sliding stability
$g_5(\mathbf{X})$	No tension condition
$g_6(\mathbf{X})$	Moment at bottom of stem
$g_7(\mathbf{X})$	Moment at toe
$g_8(\mathbf{X})$	Moment at heel
$g_9(\mathbf{X})$	Moment at shear key
$g_{10}(\mathbf{X})$	Shear at bottom of stem
$g_{11}(\mathbf{X})$	Shear at Toe
$g_{12}(\mathbf{X})$	Shear at heel
$g_{13}(\mathbf{X})$	Shear at shear key
$g_{14}(\mathbf{X})$	Minimum area reinforcement criteria
$g_{15}(\mathbf{X})$	Maximum area reinforcement criteria
$g_{16}(\mathbf{X})$	Additional geometric constraints

**1. Overturning failure mode-** The stabilizing moments, due to vertical forces must be greater than the overturning moments, due to horizontal forces to prevent rotation of the wall around its toe. The stabilizing moments result mainly from the self-weight of the structure, whereas the main source of overturning moments is the active earth pressure. Overturning failure is a result of excessive lateral earth pressures with relation to retaining wall resistance thereby

causing the retaining wall system to topple or rotate (overturn).

$$g_1(X) = FSo - (Mvtotal/Mhtotal) \leq 0$$

where Mvtotal = Total vertical moment of forces that tends to resist overturning about toe.

Mhtotal = Total horizontal moment of forces that tends to overturn about toe.

FSo = Factor of safety against overturning.

**2. Bearing failure mode-** The bearing capacity of the foundation must be large enough to resist the stresses acting along the base of the structure.

$$g_2(X) = Pmax - S.B.C \leq 0$$

where S.B.C. = Safe bearing capacity of soil

Pmax=Maximum contact pressure at the interface between the wall structure and the foundation soil.

$$g_3(X) = -Pmin \leq 0$$

Pmin = Minimum contact pressure at the interface between the wall structure and the foundation soil.

**3. Sliding failure mode-**The net horizontal forces must be such that the wall is prevented from sliding along its foundation. The most significant sliding force component usually comes from the lateral earth pressure acting on the active (backfill) side of the wall. Sliding failure is a result of excessive lateral earth pressures with relation to retaining wall resistance thereby causing the retaining wall system to move away (slide) from the soil it retains.

$$g_4(X) = FSs - ((Vtotal*\mu + \text{Horizontal force from passive pressure})/Htotal) \leq 0$$

where (Vtotal\*\mu + Horizontal force from passive pressure)=Resistance to sliding

Htotal=Total horizontal driving forces.

FSs=Factor of safety against sliding.

**4. Tension failure mode-** For stability, the line of action of the resultant force must lie within the middle third of the foundation base.

$$g_5(X) = E - (B/6) \leq 0$$

Where B = Base width of the wall

E = Eccentricity of the resultant force.

**5. Moment failure mode-** The maximum bending moment at the face of the Support should be less than the resistance moment of stem:

The flexural strength Mrs is calculated as:

$$Mrs = 0.87 * As * fy * (ds - 0.416 * Xu)$$

Where Xu is the location of neutral axis for provided steel,

$$Xu = (0.87 * fy * As) / (0.36 * fck * b)$$

As is the cross-sectional area of steel reinforcement at stem, fy is the yield strength of steel,

ds is the effective depth at stem.

$$g_6(X) = Ms - Mrs \leq 0$$

Where Mrs = Flexural strength of stem

Ms = Maximum bending moment at the face of the wall

A critical section for the moment is considered at the junction of stem with toe slab. So maximum bending moment at a vertical section at the junction of the stem with toe slab should be less than the moment of resistance of toe slab:

$$g_7(X) = Mt - Mrt \leq 0$$

Where Mrt = Flexural strength of the toe slab

Mt = Maximum bending moment at a vertical at the junction of the stem with toe slab.

A critical section for the moment is considered at the junction of stem with heel slab. So maximum bending moment at a vertical section at the junction of the stem with heel slab should be less than the resistance moment of the heel slab:

$$g_8(X) = Mh - Mrh \leq 0$$

Where Mrh = Flexural strength of the heel slab

Mh= Maximum bending moment at a vertical at the junction of the stem with heel slab.

$$g_9(X) = Mk - Mrk \leq 0$$

where Mk = Moment at base shear key

Mrk = Maximum bending moment at base shear key.

**6. Shear failure mode -** The retaining wall has to be designed as a cantilever slab to resist moments and shear forces.

$$g_{10}(X) = Vs - Vus \leq 0$$

Where Vus = Shear capacity of concrete at stem

As = Area of reinforcement at stem.

ds = Effective depth at stem.

Vs = design shear carrying capacity at stem

The net loading acts upwards and flexural reinforcement has to be provided at the bottom of the toe slab. To prevent toe shear failure, nominal shear stress at the junction of stem with toe slab should be less than shear strength of concrete at toe.

$$g_{11}(X) = Vt - Vut \leq 0$$

Where Vut = Shear capacity of concrete at toe

At = Area of reinforcement at toe.

dt = Effective depth at toe.

$V_t$  = Design shear carrying capacity of toe.

The net loading acts downwards and flexural reinforcement has to be provided at the top of the heel slab. To prevent toe shear failure, critical section for the shear is considered at the junction of stem with heel slab and nominal shear stress at the junction of stem with heel slab should be less than shear strength of concrete.

$$g_{12}(X) = V_h - V_{uh} \leq 0$$

Where  $V_{uh}$  = Shear capacity of concrete at toe

$A_h$  = Area of reinforcement at heel.

$d_h$  = Effective depth at heel.

$V_h$  = design shear carrying capacity of toe.

$$g_{13}(X) = V_k - V_{uk} \leq 0$$

Where  $V_{uk}$  = Shear capacity of concrete at shear key

$A_k$  = Area of reinforcement at shear key.

$d_k$  = Effective depth at shear key.

$V_k$  = design shear carrying capacity of shear key.

#### 7. Minimum area of reinforcement criteria-

$$g_{14}(X) = (0.12/100) * b * D_s - A_s \leq 0$$

$b$  = Base width of retaining wall

$D_s$  = Thickness at the bottom of stem

$A_s$  = Area of reinforcement in stem

$$g_{15}(X) = (0.12/100) * b * D - A_t \leq 0$$

$D$  = Thickness of the base slab

$A_t$  = Area of reinforcement in toe

$$g_{16}(X) = (0.12/100) * b * D - A_h \leq 0$$

$A_h$  = Area of reinforcement in heel

#### 8. Maximum area of reinforcement criteria-

$$g_{17}(X) = A_s - (4/100) * b * D_s \leq 0$$

$$g_{18}(X) = A_t - (4/100) * b * D \leq 0$$

$$g_{19}(X) = A_h - (4/100) * b * D \leq 0$$

**9. Additional geometric criteria-** There is several additional geometric constraints that are applied to combinations design variables to prevent infeasible retaining wall dimensions.

$$g_{20}(X) = X_2 + X_3 - X_1 \leq 0$$

$$g_{21}(X) = X_9 + X_{10} - X_1 \leq 0$$

#### 10. Lower and upper bound constraints-

The derived constraint expressions are found to be highly nonlinear in the design variables.

##### C) Objective Function

The objective function is a function of design variables the value of which provides the basis for choice between alternate acceptable designs. The objective of

**Table2." Lower and upper bounds of design**

#### Variables"

Note-  $h$  = Height of stem in m

Design variables	Lower bounds	Upper bounds
Width of the base $X_1$	$X_1 = 0.4 * h * (12/11)$	$X_1 = (0.7 * h) / 0.9$
Toe projection $X_2$	$X_2 = [0.4 * h * (12/11)] / 3$	$X_2 = [(0.7 * h) / 0.9] / 3$
Thickness at the bottom of the stem $X_3$	$X_3 = 0.2$	$X_3 = (h / 0.9) / 10$
Thickness at the top of the stem $X_4$	$X_4 = 0.2$	$X_4 = 0.2$
Thickness of base slab $X_5$	$X_5 = [h * (12/11)] / 12$	$X_5 = (h / 0.9) / 10$
The vertical steel reinforcement in the stem, per unit length of wall $X_6$	$X_6 = 0.0012 * X_3$	$X_6 = 0.04 * X_3$
Horizontal steel reinforcement in the toe, per unit length of wall $X_7$	$X_7 = 0.0012 * X_5$	$X_7 = 0.04 * X_5$
Horizontal steel reinforcement in the heel, per unit length of wall $X_8$	$X_8 = 0.0012 * X_5$	$X_8 = 0.04 * X_5$
The distance from toe to the front of the base shear key $X_9$	$X_9 = 0.4 * h * (12/11)$	$X_9 = (0.7 * h) / 0.9$
Width of the shear key $X_{10}$	$X_{10} = 0.3$	$X_{10} = (h / 0.9) / 10$
Depth of the shear key $X_{11}$	$X_{11} = 0.3$	$X_{11} = (h / 0.9) / 10$
The vertical steel reinforcement in the base shear key $X_{12}$	$X_{12} = 0.0012 * X_{10}$	$X_{12} = 0.04 * X_3$

design may be minimization of weight /cost/stress concentration factor. In structural designs the objective function is usually weight or cost minimization. The forms of the two objective functions for this optimization are consistent, i.e. the cost of concrete and reinforcing steel (both include the cost of the material per unit volume and costs associated with labour and installation). The cost function  $f$  (cost) is:

$$f(\text{cost}) = C_s * W_{st} + C_c * V_c$$

where  $C_s$  is the unit cost of steel,  $C_c$  is the unit cost of concrete,  $W_{st}$  is the weight of steel per unit length of the wall, and  $V_c$  is the volume of concrete per unit.

length of the wall. The second objective function is based solely on the weight of the materials. The weight function  $f$  (weight) is:

$$f(\text{weight}) = W_{st} + 100 * V_c * \gamma_c$$

where  $\gamma_c$  is the unit weight of concrete and a factor of 100 is used for consistency of units.

**Table3."Optimum Values of Design Variables for 3.2m Height Retaining Wall"**

Design variables	Unit	Lower Bounds	Upper Bounds	Optimum values Minimum Cost	Optimum values Minimum Weight
X <sub>1</sub>	m	1.396	2.488	1.7594	1.7569
X <sub>2</sub>	m	0.466	0.8296	0.5979	0.6291
X <sub>3</sub>	m	0.2	0.356	0.301	0.201
X <sub>4</sub>	m	0.2	0.2	0.2	0.2
X <sub>5</sub>	m	0.291	0.356	0.291	0.2730
X <sub>6</sub>	m <sup>2</sup>	0.00024	0.0142	0.000519	0.000884
X <sub>7</sub>	m <sup>2</sup>	0.000349	0.0142	0.000349	0.000328
X <sub>8</sub>	m <sup>2</sup>	0.000349	0.0142	0.000349	0.000335*

**Table4."Values of behavioural Constraints at Optimum values of Design Variables for 3.2 m retaining Wall"**

Symbol	Minimum Cost	Minimum Weight
g <sub>1</sub> (X)	0.7214	0.7281
g <sub>2</sub> (X)	146.2108	146.4938
g <sub>3</sub> (X)	0.0020	0.0002
g <sub>4</sub> (X)	0.2610	0.2486
g <sub>5</sub> (X)	0.0000	0.0000
g <sub>6</sub> (X)	0.2649	0.3602
g <sub>7</sub> (X)	19.8976	13.3762
g <sub>8</sub> (X)	8.1518	0.2704
g <sub>10</sub> (X)	49.1923	52.8123
g <sub>11</sub> (X)	10.6466	2.6787
g <sub>12</sub> (X)	18.5880	12.9409

**Table5."Input Parameter for design example"**

Input Parameter	Unit	Symbol	Design Value	
			For 3.2m	For 6.3m
Internal angle of friction	Degree	Phi	35	35
Surcharge load	kN/m <sup>2</sup>	Q	10	10
Backfill slope	Degree	beeta	0	0
Height of stem	m	H	3.2	6.3
Yield strength of steel	kN/m <sup>2</sup>	Fy	500*10 <sup>3</sup>	500*10 <sup>3</sup>
Characteristic strength of concrete	kN/m <sup>2</sup>	Fck	25*10 <sup>3</sup>	25*10 <sup>3</sup>
Density of soil	kN/m <sup>3</sup>	rhosoil	17	17
Unit weight of concrete	kN/m <sup>3</sup>	D	25	25
Concrete cover	m	cover	0.025	0.025
Safe bearing capacity of soil	kN/m <sup>2</sup>	S.B.C.	250	250
Coefficient of friction under base		mue	0.55	0.55
Factor of safety for overturning stability		FSo	1.4	1.4
Factor of safety against sliding		FSs	1.4	1.4
Factor of safety for bearing capacity		FSb	3	3
Cost of steel	Rs/kg	Cs	60	60
Cost of concrete	Rs/m <sup>3</sup>	Cc	8000	8000
% minimum steel	%	ρ <sub>min</sub>	0.12	0.12
% maximum steel	%	ρ <sub>max</sub>	4	4

**Table6."Optimum Values of Objective Functions, Weight of steel, and Volume of Concrete for 3.2 m retaining Wall"**

Objective Function	Unit	Optimum value	Weight of Steel (kg/m)	Volume of Concrete (m <sup>3</sup> /m)
Min. cost	Rs/m	13585	51.4396	1.3136
Min.weight	kg/m	2883.2	80.1159	1.1212

**Example1- Optimization for 3.2m Height Cantilever Retaining Wall**

In general relation between linear and nonlinear is found to be difficult. The numbers of cases of retaining wall are optimized for one meter length of retaining wall. The design variables described as  $X_1$  is base width of retaining wall,  $X_2$  is toe width of retaining wall,  $X_3$  is bottom thickness of stem,  $X_4$  is top thickness of stem,  $X_5$  is base thickness of retaining wall,  $X_6$  is vertical reinforcement area provided in stem,  $X_7$  is horizontal steel area in toe of retaining wall, and  $X_8$  is horizontal reinforcement area in heel of the retaining wall. As shown in Table 3, out of first five design variables that describe the shape of the optimum retaining wall and remaining three describe the area of reinforcement in stem, toe, and heel respectively. For a specified set of design parameters used more iterative method from optimtool i.e. interior point method the derived constraints expressions are found to be highly nonlinear in the design variables. In addition, all the design variables have lower bounds and upper bounds, which are considered as per given by Saribus and Erabatur [1] and IS 456-2000. In analysis process design considerations are related to total height of retaining wall but in optimization, taking initial assumptions i.e. lower bound and upper bound of the design variables are related to the height of stem and thickness of base is as design variable.. Lower bound and upper bound constraints are considered for all examples as per Table 2

#### Example2 - Optimization for 6.3m Retaining Wall without Shear Key

In optimization of 6.3m retaining wall, changing height of retaining wall in program coding and in objective function, as per diameter of steel bar obviously changed weight of steel. From constraint equations and Table 7 gives idea, the vertical reinforcement area of stem( $X_6$ ) is depends on Bottom width of stem( $X_3$ ). In cost optimization model shows that  $X_3$  increases with  $X_6$  and vice versa in weight optimization model. In weight optimization model volume of concrete reduces and weight of steel increases than that of cost optimization model.

**Table7."Optimum Values of Design Variables for 6.3m Height Retaining Wall without Shear Key"**

Design variable	Unit	Lower Bounds	Upper Bounds	Optimum values Minimum Cost	Optimum values Minimum Weight
$X_1$	m	2.749	4.90	3.3008	3.3108
$X_2$	m	0.916	1.633	1.0305	1.1057
$X_3$	m	0.2	0.70	0.6284	0.3060
$X_4$	m	0.2	0.2	0.2	0.2
$X_5$	m	0.573	0.70	0.5910	0.5910
$X_6$	m <sup>2</sup>	0.00024	0.028	0.0015	0.004639
$X_7$	m <sup>2</sup>	0.000688	0.028	0.0007092	0.0007092
$X_8$	m <sup>2</sup>	0.000688	0.028	0.001139	0.001252

**Table8."Values of behavioural Constraints at Optimum values of Design Variables for 6.3m Height Retaining Wall without Shear Key"**

Symbol	Minimum Cost	Minimum Weight
$g_1(X)$	0.7031	0.7252
$g_2(X)$	61.246	58.6384
$g_3(X)$	0.0050	0.0050
$g_4(X)$	0.1370	0.1630
$g_5(X)$	0.0000	0.000
$g_6(X)$	2.0534	0.0175
$g_7(X)$	106.07	97.2584
$g_8(X)$	68.3736	34.0904
$g_{10}(X)$	85.302	133.7170
$g_{11}(X)$	28.147	17.9790
$g_{12}(X)$	0.0243	0.0028

**Table9." Optimum values of Objective Functions, Weight of Steel, and Volume of Concrete"**

Objective Function	Unit	Optimum value	Optimum Weight of Steel(kg/m)	Optimum Volume of Concrete (m <sup>3</sup> /m)
Min. cost	Rs/m	52589	269.296	4.5602
Min. weight	kg/m	9615.8	739.390	3.5506

#### Example3- Optimization for 6.3m Height Retaining Wall with Shear Key

In this set of retaining wall designs, a base shear key is included in the design variables. Four

additional variables are added to optimization program,  $X_9$  is the location of shear key from toe,  $X_{10}$  is width of shear key,  $X_{11}$  is height of the shear key and  $X_{12}$  is vertical reinforcement area provided in shear key. In this program, additional geometric constraints and moment capacity, shear capacity constraints are included. In objective function equations included weight of steel for key with respective length and spacing between the reinforcement and volume of concrete required for shear key with respective to design variables. Provision of shear key for greater height is best solution.

**Table10." Optimum Values of Objective Functions, Weight of Steel, and Volume of Concrete for 6.3m Height Retaining Wall with Shear Key"**

Objective Function	Unit	Optimum Value	Optimum Weight of steel(kg/m)	Optimum Volume of concrete (m <sup>3</sup> /m)
Min. cost	Rs/m	52262	264.153	4.5516
Min.weight	kg/m	9652.73	717.322	3.5742

#### 4) Sensitivity Analysis

A sensitivity analysis adds quality to a design and supplies very important information on the work being designed from the view point of cost and reliability. The sensitivity analysis is very useful to (a) the designer, who can know which data values are more influential on the design, (b) to the builder, who can know how changes in prices influence the total cost, and (c) to the code maker, who can know the costs and reliability changes associated with an increase or decrease in the required safety factors or failure probabilities. The design parameters considered in above cases having wide range of parameters that are related to loading, geometry, soil properties, code specifications, unit cost, and other characteristics of construction materials. Sensitivity of the optimum solution to changes in these parameters is an important issue as far as practical design is concerned. The analysis results include the sensitivities of the optimum weight and optimum cost as objective functions and the

**Table11." Optimization for 6.3m Height Retaining Wall with Shear Key"**

**Table12." Values of behavioural Constraints at Optimum values of Design Variables for 6.3m Height Retaining Wall with Shear Key"**

Design variables	unit	Lower bounds	Upper bounds	Optimum values minimum cost	Optimum values minimum weight
$X_1$	m	2.749	4.90	3.295	3.3044
$X_2$	m	0.916	1.633	1.0418	1.1046
$X_3$	m	0.2	0.7	0.6170	0.3050
$X_4$	m	0.2	0.2	0.2	0.2
$X_5$	m	0.573	0.70	0.5730	0.5730
$X_6$	m <sup>2</sup>	0.00024	0.028	0.001524	0.004692
$X_7$	m <sup>2</sup>	0.000688	0.028	0.000688	0.000688
$X_8$	m <sup>2</sup>	0.000688	0.028	0.001169	0.001283
$X_9$	m	2.749	4.9	1.5	1.5
$X_{10}$	m	0.3	0.7	0.3	0.3
$X_{11}$	m	0.3	0.7	0.3	0.3
$X_{12}$	m <sup>2</sup>	0.00024	0.028	0.000360	0.000360

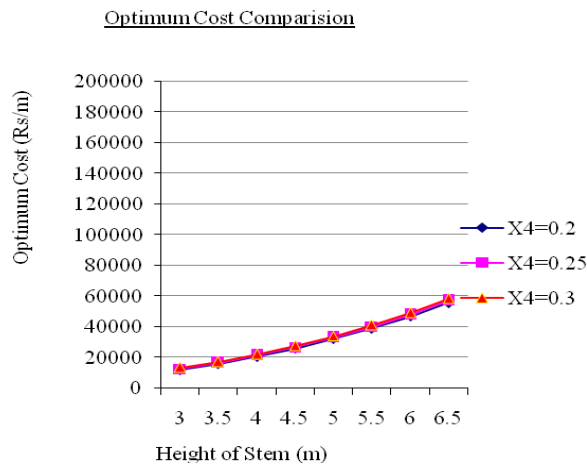
Symbol	Minimum Cost	Minimum Weight
$g_1(X)$	0.6976	0.7225
$g_2(X)$	62.8021	59.5761
$g_3(X)$	0.0263	0.0003
$g_4(X)$	0.3878	0.4233
$g_5(X)$	0.0002	0.0000
$g_6(X)$	0.1085	0.0144
$g_7(X)$	95.4298	87.3916
$g_8(X)$	62.5473	32.7742
$g_9(X)$	40.7251	40.7251
$g_{10}(X)$	84.8838	134.8692
$g_{11}(X)$	23.4356	13.5859
$g_{12}(X)$	0.0083	0.0568
$g_{13}(X)$	76.724	76.7238

optimum values of the several design variables. As a representative of such analyses, results concerned with the sensitivity of optimum solutions with respect to height and top thickness of stem, surcharge load, internal angle of friction of retained soil. In sensitivity analysis, sensitivity of the objective functions explained for all design parameters which are considered in number of above not cases. Both objective functions are considered and their optimum

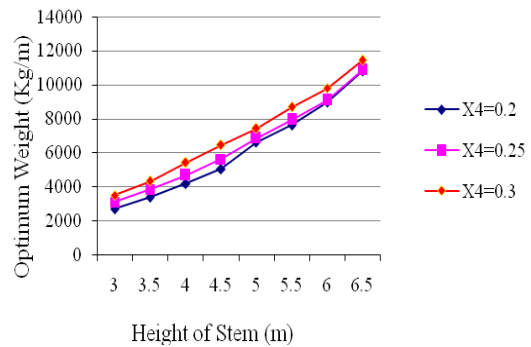


value variations with respect to changes in height of the stem for various top thickness values are given in Graph1 and 2. In the range of the stem height (3-6.5m) shows that for higher values of height, the optimum cost and weight become more sensitive to variations in the stem height. This is more apparent for the cost function. For example, when top thickness of stem ( $X_4$ )=0.2m and height changes from 3.0 to 6.5m, the cost of the wall increases 3.6524 times, whereas the weight increases 3.0 times. Smaller top thickness values of the stem produce more favourable optimum solutions for both objective functions. From table, shows that the optimum values for a given height, the first four design variables are not much affected by increases of  $X_4$ , from  $X_4=0.20$  to 0.30. For both minimization models, the optimum values of the last three design variables corresponding to reinforcing steel areas show sensitivity to changes in height, but in general to shifts in  $X_4$ . Only horizontal steel reinforcing area in the cost and weight minimization model is influenced by changes in  $X_4$ .

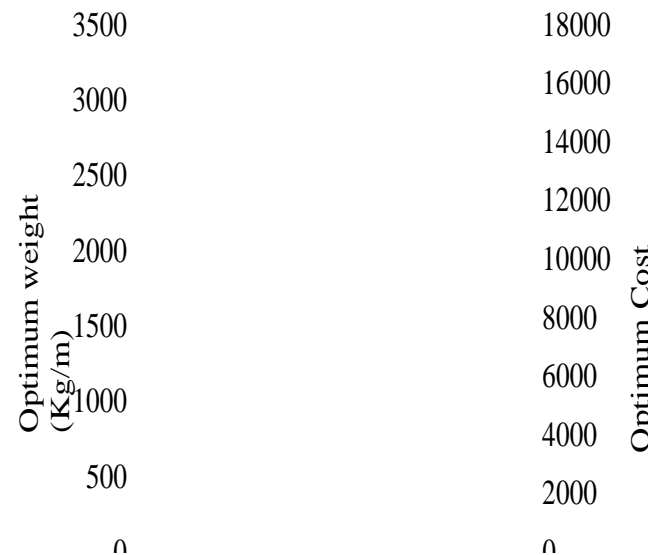
**Graph1."Comparison of Optimum Cost and Height of Stem for different Top Thickness of Stem"**



**Optimum Weight Comparison**



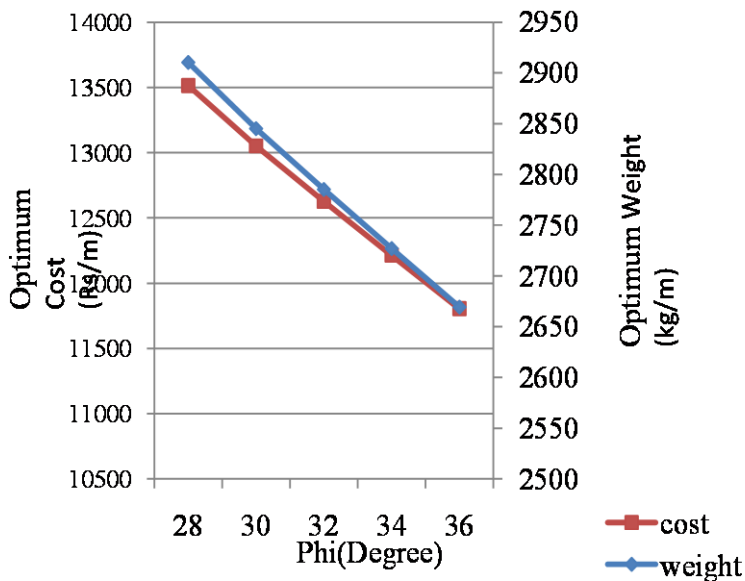
**Graph2." Comparison of Optimum Weight and Height of Stem for different Top Thickness of Stem"**



**Graph3." Comparison of Optimum Weight, Optimum Cost and Surcharge Load for 3.0 m Height of Stem"**

The input equation in terms of height of stem and surcharge load are prepared for the study of sensitivity for surcharge loads corresponding to input parameter. The optimum values for the objective functions are shown in Graph 3 for the surcharge load varying from 0 to 50 kPa. In this, as  $q$  changes from 0

to 50 kPa, the optimum cost increases 0.613 times, and the optimum weight increases by 0.23 times. The cost the surcharge load increases, in both of the optimization models. The optimum value of horizontal reinforcement area in heel ( $X_8$ ) increases with an increase in the surcharge load, yet after 10kPa in the cost and weight minimization model. Minimization model is more sensitive to variations in surcharge load compared to the weight minimization model. As for the sensitivity of the design variables, significant sensitivity is observed in base width( $X_1$ ) to stem thickness at bottom ( $X_3$ ) and vertical reinforcement in stem( $X_6$ ). These variables increase as the surcharge load increases, in both of the optimization models. The optimum value of horizontal reinforcement area in heel ( $X_8$ ) increases with an increase in the surcharge load, yet after 10kPa in the cost and weight minimization model.



**Graph4." Comparison of Optimum Weight, Optimum Cost and Internal Angle of Friction for 3.0 m Height of Stem"**

From graph 4, sensitivity analysis related to the internal friction angle of retained soil, prepared different constrained equations in terms of H and  $\emptyset$  for finding active pressure and passive pressure. The minimum weight model is more sensitive in the range of  $\emptyset$  is 28 to 36 degree when compared to the minimum cost model. As far as the design variables are concerned, in the cost minimization as well as weight minimization model Base thickness ( $X_5$ ) and toe steel area ( $X_7$ ) are insensitive to change in the internal angle of friction. The retaining wall base width ( $X_1$ ), toe width ( $X_2$ ), stem bottom thickness ( $X_3$ ), reinforcement area in stem ( $X_6$ ), heel reinforcement area ( $X_8$ ), are decreases with an increase in the angle of internal friction.

### 5) Conclusion

The purpose of optimization is to choose the best one of the many acceptable designs available. For achieving economy in conventional analysis optimization programming made by considering geometric, moment and shear constraints, getting optimum value for number of cases of retaining wall with and without shear key. Sensitivity of the optimum solutions with respect to surcharge load, internal angle of friction gives the idea about effect on geometric parameters of retaining wall related to geotechnical and structural requirements.

### References

- [1] Saribaş, A. and Erbatur, F. (1996). "Optimization and Sensitivity of Retaining Structures." Journal of Geotechnical Engineering, 122(8), 649-656.
- [2] Kaveh A. and Abadi A. S. M. (2010). "Harmony Search Based Algorithm for the Optimum Cost Design of Reinforced Concrete Cantilever Retaining Walls." International Journal of Civil Engineering, 4(8), 336-357.
- [3] Charles V. Camp and Alper Akin "Design of retaining walls using big bang-big crunch optimization Optimum Design of Cantilever Retaining Walls"
- [4] Sivakumar babu & b. m. basha Title "inverse reliability based design optimization of cantilever retaining walls" Indian institute of science of bangalore, india
- [6] M. Asghar Bhatti Title "Retaining Wall Design Optimization with MS Excel Solver" Department of Civil & Environmental Engineering, University of Iowa, Iowa City, IA
- [7] M. Ghazavi, V. Salavati ,Title "Sensitivity analysis and design of reinforced concrete cantilever retaining walls using bacterial foraging optimization algorithm"

