Optimization of the induction hardening process of Tow Axle Spindle

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Abstract

In the present paper, an investigation is performed to find out best combination of process parameters for induction hardening of Tow Axle Spindle with the help of design of experiments (DOE). Quality characteristics like effective case depth (ECD) and hardness are analyzed for various combinations of medium frequency power, feed rate, quench pressure and temperature. The experimental trials are conducted, based on the design matrix obtained from the rotatable central composite design (CCD) and D-optimal, with the help of 150 kW power converter equipped with an induction hardening station. To investigate the induction hardening process, for combined influence of all four above mentioned process parameters, significant regression modes are developed to predict the quality characteristics using response surface methodology (RSM). The mathematical model, developed during the course of research, helped in investigating induction hardening process with analytical technique like desirability. Desirability test showed its efficacy in finding out number of optimal strategies for hardening of spindle shaft axle to achieve the desired ECD and hardness values. The desirability index, in a multi-response process like induction hardening, suggested that selection of both, heating and quenching parameters is significant.

Keywords: Response Surface Methodology, Desirability, Induction hardening.

1. Introduction

Industrial hardening process, meant for specific applications, demand critical changes designed to be introduced in material properties. Manufacturing processes, which bring these critical changes, are classified as property-sensitive processes. Heat treatment, for example, is one such kind of extensively applied process. Processes like hardening, softening and grain refinement are often needed to be performed for very specific operations. Similar to other heat treatment processes, induction hardening brings changes into the properties of metal or alloy in the solid state, through a cycle of heating and cooling [1]. However, some of the distinctive functional objectives, which are desired to be obtained after induction hardening process, are as following:

a) Refined grain size.
b) Improved mechanical properties like strength, hardness, and toughness.
c) Increased wear resistance
d) Improved shock resistance.

The motivation of giving preference to induction hardening over other heat treatments processes is due to the fact that induction hardening has fast cycle time, better reproducibility and lower energy consumption. It is interesting to note that most of the time; induction hardening processes are set up using engineering experience and a trial-and-error procedure for achieving the above mentioned desired functional objectives. It also includes a series of hardening trials and metallographic testing. Therefore, besides the knowledge and experience, significant time is required to establish desired output quality out of the induction hardening process, resulting in the increased cost of the process itself.

An in-depth analysis of process planning for specifying the process parameters like hardening method, power, operating frequency, feed rate, quench temperature, etc, is presented by Cajner [2]. It is observed that the features of process setup, for induction hardening, do not influence the output of this process in a predictable manner. Therefore, it is necessary to develop more rigorous relationships between the system output and various parameters that control the process of Induction Hardening (IH). The present study focuses on optimizing those process parameters which are directly responsible for quality response i.e. required pattern of hardened layer, specified surface hardness and depth of surface hardening. A better planned and optimized IH process as described ahead can bring down material wastage significantly and reduce the cost and time of manufacturing.

1.1 Induction hardening process -a brief description

In the induction hardening process, the metal is heated up to a temperature till there is change of state or change in the structure. The change of structure does not occur at a constant temperature; therefore transformation range of temperature is critical for IH process. Austenite structure formation (above 750˚C) can be treated as the desired change of structure. It is then, followed by very rapid cooling of the structure to a temperature 205˚ to 215˚C or even lower than that. As a result of such rapid cooling, austenitic structure changes to martensite with a tetragonal crystal structure. The martensite exhibits hardness value of 50 ~ 65 on Rockwell scale depending on the grain structure and carbon content [3]. This martensite phase is only produced due to rapid quenching, where austenite is decomposed by entrapping up to 2% carbon. Therefore, it becomes extremely hard and brittle. IH process can be sub-divided into different phases depending upon heating and cooling cycle:

a) Heating of the metal,
b) Soaking of heat by metal and
c) Rapid cooling.

Favennec et al. (2003) [4] described that rapid cooling of steel generates thermal stresses due to uneven cooling, therefore, produces an unequal distribution of austenite and its decomposition product. This mixture of austenite and martensite, pearlite etc is formed at different rate of cooling in outer layers and central portion of steel component. Correspondingly it creates a soft core and hard outer surface of steel components, when induction hardening process terminates. Haimbaugh (2001) [5] described about desired quality parameters, of induction hardening process, i.e. effective case depth (ECD) and hardness value in details. Often, an improper IH process results in products with low hardness and insufficient ECD, therefore, resulting in non-conforming output. Following reasons can be put forth, which contributed towards, for low hardness value and insufficient ECD:

- Inadequate heating temperature or time or both;
- Unsatisfactory quenching conditions (temperature, pressure etc.).

It is, therefore, essential to maintain optimum process parameters to ensure the requisite quality characteristics of induction hardening process output. Mucha et al. (1989) [6] suggested following as those important process parameters which require definite attention, while considering the output quality characteristics:

a) Heating parameters (frequency, current, power)
b) Quenching parameters (pressure, temperature, flow rate)
c) Part handling (rotation, scanning, positioning)
d) Induction tooling (inductor coil)

Except inductor coil, all the other parameters are controlled by different settings in induction hardening equipment. Zinn and Semiatin (1988) [7] described that designing an inductor coil is a non trivial problem. It depends on various cross boundary phenomenon like effect of magnetism, electrical and thermal aspect on dynamic properties of steel components during induction hardening process. Induction hardening equipments are designed for producing a range of output therefore; only limited number of options is available to try for alteration in heating and quenching parameters. Since the parameters for IH process are interlinked, therefore, it is a perfect example of optimization of interdependent variables of a dynamic system. The presented paper discussed about finding all the possible optimum combinations of process parameters, which ensure the desired hardness values and ECD of steel component after IH process. Therefore, only heating and quenching parameters are considered for optimization to obtain correct ECD and hardness value. In the following section, motivations for optimizing the process parameters are discussed.

1.2 Optimization of processes with multiple variables

Response surface methodology (RSM) is a useful tool for predicting the behavior of any process especially in manufacturing and helps to analyze using different statistical and mathematical methods. Following are the requisites to utilize RSM for optimization of the manufacturing process:

a) Data collection for selection of independent variables which influence the system.
b) After selecting an experimental matrix based on design of experiment (DOE), prediction of suitable model of the system.
c) Perform optimization for this predicted model for already selected goals (maximizing, minimizing or for target value of response variables).

Bezerra et al. (2008) [8] also defined RSM as a powerful technique for optimization. Most of the work in RSM has been focused on the case where, there is only one response of interest. In a manufacturing process, like IH process, there are more than one response variables. Therefore, for the determination of optimum conditions, simultaneous consideration of all the responses on the input variables is necessary. In multi-response problem environment, to predict the combination of process parameters, a relatively robust and cost effective approach is an essential need. Method proposed by Taguchi (1987) [9], has proved very helpful amongst manufacturers. This conventional method is focused only on a single quality characteristic to optimize the parameter conditions, hence, when it comes across a multiple quality characteristics process, it sometimes may lead to serious degradation of the other critical quality characteristics. This limitation proved fatal when the characteristics are interlinked i.e. parameters have some definite weightage on quality parameters of process output like IH process.

Phadke (1995) [10] also mentioned that it is rather difficult to optimize responses simultaneously in a complex process by single-response method. Therefore, engineering judgment is primarily used to resolve such complicated problems. The major problem arises when this engineering decision increases the degree of uncertainty during decision-making process. This, over optimization, often makes it most critical to the quality of output and the validity and robustness of results and therefore, cannot be guaranteed using this approach alone.

Bainik and Mazumder [11] described the main objective of DOE as the selection of position where response is to be evaluated. This DOE results a mathematical model of the process or system, which can be in the form of polynomial. In order to construct a best suited model of IH process, specifically designed experiment matrix is required, which may be framed depending on the following methods:

a) Full factorial design
b) Central composite design
c) D-optimal Design.

Montgomery et al. [12] informed in-depth details on these methods.

Aman et al [13] presented their study on determining optimum parameters for CNC operation using desirability function. For optimization of tool life, cutting force, power consumption and surface roughness, four controllable factors i.e. cutting speed, feed, depth of cut and nose radius were selected. The desirability concept has been used for multi-response optimization owing to its better readability, acceptability and visualization as compared to other multi characteristic optimization techniques like utility concept, principal component analysis etc. Desirability function can be a better alternate for processes with multi-response objectives. The following section elaborates it further.

2. Desirability function for IH process optimization

Multi response optimization methods, including the conventional desirability function approach and loss function approach, are categorized into the prior method therefore; all the necessary information about preference should be made available before solving the problem. Desirability function proposed in 80's by Derringer and Suich, which is further modified as suggested by Del Castillo et al. (1996) [14]. Subsequently, Carlyle et al. (2000) [15] also informed that desirability function is a better approach for the optimization of multiple quality characteristics problems. In a significant research contribution, he has tried to address the limitation of Taguchi's method while encountering the effect of variability in multiple quality characteristics processes. Desirability function not only proved effective to optimizing inter-related multiple quality characteristics based processes but also represented a relatively simple method of scale free values between 0 and 1 for process characteristics. In this function, the most suitable parameter should be announced based on its proximity with the output response value. The highest geometric mean of the individual desirability for all the combinations considered as optimal parameter conditions. The desirability function D is defined as following in (1) as:

$$D=(d_1\times d_2\times \ldots\ldots\ldots\ldots d_n)^{1/n}=\left[\prod_{i=1}^{n} d_i\right]^{1/n} \quad (1)$$

where n denotes the number of quality characteristics. In our present study, we are targeting to optimize ECD and hardness value only; therefore, n will be 2 in our case, as mentioned in (2)

$$D=(d_1\times d_2)^{1/2} \quad (2)$$

Here desirability function d_i is assigned numbers between 0 and 1 to the possible value for each response. The value of d_i increases as the desirability of the corresponding response increases.

Kohli and Singh (2012) [16] have performed desirability test on induction hardening of AISI 1040 rolled steel by doing a set of experiments. However, process parameters taken into consideration were:

- Feed rate (mm/sec),
- Dwell time (sec),
- Current (amp.),
- Gap between work piece and inductor coil (mm).

As informed, desirability found to be 0.874 corresponds to maximum value of quality characteristics (Mean ECD, hardness value, total ECD) in a selected range of process parameters.

Since induction hardening process can only be terminated when both heating and quenching operations are completed, therefore, ignoring quenching parameters like temperature or pressure seems unreasonable. In his book, Rudnev et al. (2002) [17] presented state of the art in induction hardening process. It is suggested that while considering for optimization of Induction hardening process, it is necessary to comprehend heating as well as quenching process.

In order to validate the effect of heating parameters and quenching parameters on desirability function, both parameters are considered in our present research. Therefore, with this modified approach for predicting quality response functions like ECD and hardness values, a reasonable desirability can be established. To make this study more useful, modeling of IH process using two powerful strategies i.e. central composite design (CCD) and D-optimal is carried out. DOE matrices from CCD and D-optimal are then optimized for target values of ECD and hardness. The optimization results thus obtained, informed the efficacy of CCD and D-optimal Multi Response Optimization (MRO) processes.

The following section describes this new dimension for solving multiple response optimizations in the field of induction hardening process.

3. Experimental set up

3.1 Component presented for experiments

Steel is an alloy of iron and carbon, with carbon content maximum up to 1.7%. The carbon, which appears in the form of iron carbide, exhibits its ability to increase the hardness and strength to the steel. For the present study, a typical tow axle spindle, used in farm tractors, is considered. Total length of spindle is 200mm and the diameter presented for induction hardening is found to be as 35mm as shown in Figure 1.

Figure 1. Tow axle spindle

The carbon, which appears in the form of iron carbide, exhibits its ability to increase the hardness and strength to the steel. Apart from other elements e.g. silicon, sulphur, phosphorus and manganese are also present to greater or lesser amount to add desired properties to steel. The material for steel component is AISI 1045 (see Table 1 for specification as mentioned in Budinski and Budinski (2009)) [18] which is suitable for majority of automotive components like axle and spline shafts.

Table 1. Material specification of AISI 1045

C %	Mn %	P %	S %
0.43-0.5	0.6-0.9	0.04 max	0.05

3.2 Experimental plan for data collection

To perform the experiments, a medium frequency induction hardening equipment with 10 kHz, 150 kW rated power converter has been used. It is important to mention here that induction hardening equipments are manufactured to harden specific component or for specific range of steel components only, therefore, altering the heating and quenching parameters, beyond certain limits, is not possible. A careful selection of desired input process parameters with suitable range thus becomes an important task. This always helps to make the optimum usage of available resource, in our case; it is 150 kW power converter and attached hardening station. Process parameters, observed during the induction hardening process, were as follows:

1. Medium frequency power (in kW);
2. Feed rate (in % of rotation);
3. Quench temperature (in ◦ C); and
4. Quenching water pressure (in Kg/cm^2).

Literature review on properties achieved after the induction hardening process, along with engineering judgment and few preliminary trials on tow axle spindle for establishing quality parameters helped in deciding the range of parameters considered for experiments as shown in Table 2.

Table 2. Range of process parameters for experiments

Sl.	Parameter	Range
1.	MF power	75~90 kW
2.	Feed rate	75~90 %
3.	Quenching pressure	2.0~3.5 Kg/cm^2
4.	Quenching temperature	20~35 $^{\circ}C$

3.3 Design of experiment (DOE)

For obtaining an accurate response surface, identifying deserving data points and selection of a good set of points for carrying out experiments, proved vital. Therefore, design of experiments (DOE) is considered a useful tool for multi-response optimization (MRO) problem. To optimize the quality parameters, conducting large number of experimentation of different combination of process parameters is expansive. Here, software package design expert is used for obtaining DOE and collect data. As already discussed, DOE matrices are chosen based on Central Composite Design (CCD) with rotatable option and D-optimal with coordinate exchange. It helps to maintain the desired level of accuracy in less number of tests. As already informed, industrial IH equipments can only be operational in certain incremental steps (like 75, 80, 85 etc.). Therefore, certain modifications on DOE matrices are necessary, for obtaining experimental data of ECD and hardness value. DOE matrices thus generated, through design expert software, are then modified from the experimental results of IH process carried out on 50 no. of tow axle spindle. The complete sets of data, for conducting response surface methodology, with experimental results of chosen quality response parameters (ECD and hardness value) are presented in Table 3 & 4:

Table 3. DOE based on Central composite design

Run no.	MF Power kW	Feed Rate %	Quench Press. Kg/cm^2	Quench Temp. ° C	ECD mm	Hardness HRC
1	75	85	3.0	30	1.8	52
2	75	75	3.5	35	2.4	54
3	90	90	3.5	25	2.4	54
4	90	90	3.5	35	2.7	52
5	75	90	2.0	25	1.0	48
6	85	80	3.0	25	2.4	54
7	90	90	2.0	35	1.9	48
8	85	75	3.0	30	2.7	55
9	75	75	2.0	25	1.5	52
10	90	75	3.5	35	3.3	56
11	80	85	3.0	35	2.1	50
12	90	75	2.0	35	2.5	50
13	90	75	3.5	25	3.2	58
14	85	80	3.0	30	2.5	54
15	90	90	2.0	25	1.7	50
16	85	80	3.0	30	2.6	54
17	85	85	3.0	30	2.3	52
18	80	80	2.0	30	1.7	48
19	85	85	3.0	30	2.4	52
20	85	85	3.0	30	2.3	52
21	75	90	2.0	35	1.1	45
22	85	80	4.0	30	2.8	56
23	75	75	3.5	25	2.4	56
24	75	90	3.5	25	1.8	52
25	90	75	2.0	25	2.4	52
26	80	85	3.0	30	2.0	52
27	90	80	3.0	30	2.6	54
28	75	75	2.0	35	1.6	50
29	80	90	3.0	30	2.0	50
30	75	90	3.5	35	2.0	50

Table 4 DOE based on D-optimal

Run no.	MF Power kW	Feed Rate %	Quench Press. Kg/cm^2	Quench Temp. ° C	ECD mm	Hardness HRC
1	75	85	3.0	30	1.8	52
2	75	75	3.5	35	2.4	54
3	90	90	3.5	25	2.4	54
4	90	90	3.5	35	2.7	52
5	75	90	2.0	25	1.0	48
6	85	80	3.0	25	2.4	54
7	90	90	2.0	35	1.9	48
8	85	75	3.0	30	2.7	55
9	75	75	2.0	25	1.5	52
10	90	75	3.5	35	3.3	56
11	80	85	3.0	35	2.1	50
12	90	75	2.0	35	2.5	50
13	90	75	3.5	25	3.2	58
14	85	80	3.0	30	2.5	54
15	90	90	2.0	25	1.7	50
16	85	80	3.0	30	2.6	54
17	85	85	3.0	30	2.3	52
18	80	80	2.0	30	1.7	48
19	85	85	3.0	30	2.4	52
20	85	85	3.0	30	2.3	52
21	75	90	2.0	35	1.1	45
22	85	80	4.0	30	2.8	56
23	75	75	3.5	25	2.4	56
24	75	90	3.5	25	1.8	52
25	90	75	2.0	25	2.4	52
26	80	85	3.0	30	2.0	52
27	90	80	3.0	30	2.6	54
28	75	75	2.0	35	1.6	50
29	80	90	3.0	30	2.0	50
30	75	90	3.5	35	2.0	50

Those process outputs, which found crack or physically non- conforming, did not qualify for further analysis. Following section informed about the analysis of variance (ANOVA) on both the DOE matrices, and predicts the "best fit" model for IH process on presented steel component.

3.4 Prediction of Model using ANOVA

DOE matrices are analyzed for prediction of the mathematical models. The use of design expert software package allowed backward elimination of all the insignificant model terms in the initial polynomial equation. Table 5 & 6 show the analysis of variance (ANOVA) report:

Table 5. Model significance using ANOVA (based on CCD)

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
Model	8.28	8	1.03	241.08	< 0.0001
A-MF power	2.81	1	2.81	655.07	< 0.0001
B-Feed Rate	1.54	1	1.54	358.75	< 0.0001
C-Quench Press	3.07	1	3.07	716.08	< 0.0001
D-Quench Temp	0.09	1	0.09	20.05	0.0002
AB	0.03	1	0.03	5.96	0.0235
BD	0.01	1	0.01	3.37	0.0806
B^2	0.02	1	0.02	4.14	0.0547
C^2	0.05	1	0.05	11.75	0.0025
Residual	0.09	21	0	-	-
Lack of Fit	0.08	18	0	1.12	0.5357
Pure Error	0.01	3	0	-	-
Cor. Total	8.37	29	-	-	-

Std. Dev.	0.07	R-Squared	0.99
Mean	2.20	Adj R-Squared	0.98
C.V. %	2.97	Pred R-Squared	0.98
PRESS	0.16	Adeq Precision	63.42

ANOVA suggests that the predicted regression model is significant as model F-value found to be as 241.08 and there is only 0.01% chance that "Model F-Value" this large could occur due to noise. Values of Prob. > F less than 0.05 indicate model terms are significant. Therefore, medium frequency power, feed rate, quench pressure and temperature are all significant model terms. ANOVA analysis declared "Lack of Fit" as non-significant; therefore it is equally good as the model needed to be fit. Due to our consideration, only for those components whose hardness values were lying between 49 and 59 HRC, a quadratic regression model predicted is represented in the following equation (3):

$$ECD = 2.93791 + 0.10922 \times \text{MF power} - 0.18956 \times \text{Feed Rate} + 1.30988 \times \text{Quench Press} - 0.051947 \times \text{Quench Temp} - 0.0007 \times \text{MF power} \times \text{Feed Rate} + 0.0008 \times \text{Feed Rate} \times \text{Quench Temp} + 0.001 \times \text{Feed Rate}^2 - 0.14134 \times \text{Quench Press}^2 \quad (3)$$

Table 6. Model significance using ANOVA (based on D-optimal)

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
Model	8.8	4	2.2	1639.12	< 0.0001
A-MF power	4.1	1	4.1	3058.41	< 0.0001
B-Feed Rate	1.77	1	1.77	1320.52	< 0.0001
C-Quench Press	3.41	1	3.41	2538.73	< 0.0001
D-Quench Temp	0.08	1	0.08	61.43	< 0.0001
Residual	0.03	20	0	-	-
Lack of Fit	0.02	15	0	1.46	0.5355
Pure Error	0.01	5	0	-	-
Cor. Total	8.83	24	-	-	-

Std. Dev.	0.04	R-Squared	1.00
Mean	2.12	Adj R-Squared	0.99
C.V. %	1.72	Pred R-Squared	0.99
PRESS	0.04	Adeq Precision	147.42

When the experimental matrix, based on D-optimal strategy is analyzed, ANOVA suggests that predicted linear regression model is significant. Therefore, medium frequency power, feed rate, quench pressure and temperature are all significant model terms. ANOVA analysis brought out that the "Lack of Fit" as non-significant; therefore it is expected that linear regression model can be used to navigate the design space.

Predicted linear regression model for ECD is represented in following equation (4):

ECD = -0.149 +0.0419 x MF power -0.024 x Feed Rate +0.209 x Quench Press + 0.011 x Quench Temp (4)

Following section presents the results of optimization for both models i.e. quadratic regression model predicted using CCD approach and linear regression model as predicted, using D-optimal strategy.

4. Results and discussions

The induction hardening process is desired for improved mechanical properties in steel components. Tow axle spindle, steel component for our present study, is desired to serve in an industrial environment. Therefore the effective case depth of 2.0 mm with hardness value 56 HRC is assigned during the design stage. We have considered these designed values as target values for response functions (ECD and hardness value) in our study. Theoretical desirability function for such conditions, where response functions are assigned with target values, is described as following:
Goal-Target:

$d_i = 0$ if response < low value
$0 \le d_i \le 1$ as response varies from low to target
$1 \ge d_i \ge 0$ as response varies from target to high
$d_i = 0$ if response > high value.

In order to find out the desirability index value for comprehensive effect of both the quality response characteristics on varying values of input parameters, equal weightage is assigned. Following Table 7 shows the criteria and weight assigned to each process parameter and response characteristics.

Table 7. Criterion for process parameters and response characteristics

Constraint	Target	Range	Wt.	Impor-tance
MF power	In range	75~90	1	3
Feed rate	In range	75~90	1	3
Quench Pressure	In range	2~3.5	1	3
Quench Temp.	In range	25~35	1	3
ECD	Target =2.0	1.3~3.3	1	3
Hardness	Target =56	49~59	1	3

For target value of 2.0 mm as ECD and 56 HRC as hardness value, optimum values for mf power, feed rate, quenching pressure and temperature are found as 75 kW, 82%, 3.5Kg/cm^2 and 25˚C respectively. These process parameters are resultant of optimization of regression model predicted using CCD approach. The desirability index found for predicted input conditions as 0.916. (See Fig. 2).

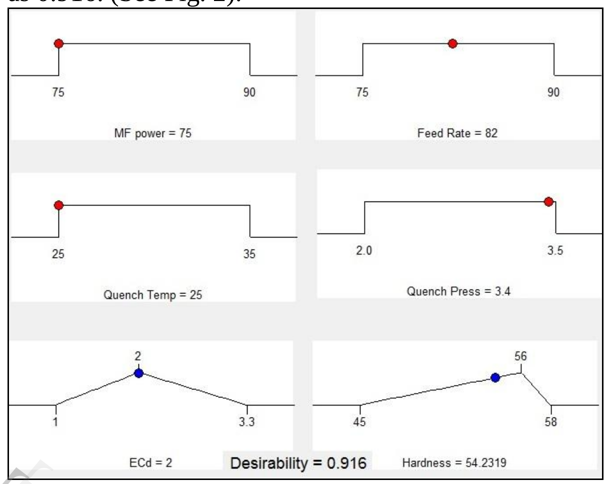

Figure 2. Ramp function graph for desirability (optimization using CCD)

While investigating the ramp function graph obtained for optimization of linear model, desirability index found to be 0.969 for the same set of process parameters. This graph is generated using linear model of IH process, which is predicted with the help of D-optimal approach, (See Fig. 3 shown below)

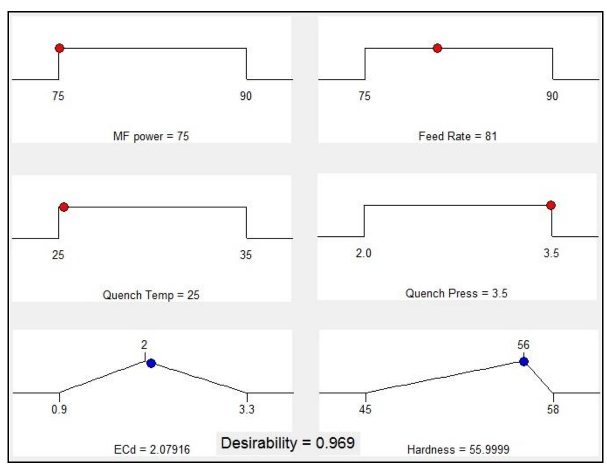

Figure 3 Ramp function graph for desirability (optimization using D-optimal)

Therefore considering heating as well as quenching parameters for quality characteristics evaluation, proved as a useful strategy. This is very significant contribution for multi response optimization in the field of processes like induction hardening where heating and quenching both are equally important to achieve desired quality characteristics (ECD and hardness) of output.

5. Conclusions

In this study, an optimization of process parameters for the induction hardening of Tow Axle Spindle is presented. The statistical analysis using ANOVA indicated that the effect of process parameters like quench pressure and temperature along with MF power and feed rate, too are significant factors for process response like ECD and hardness of IH process output. The optimum range of input variables (i.e. IH process parameters) that generated desired process output was determined through the desirability function. With the selection of a D-optimal design, the substantial reduction of experimental effort can be produced. The predicted regression model, showed maximum desirability index as 0.969. The methodology shown in this paper is an empirical methodology that can be adapted to MRO processes of similar nature problem.

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