Optimized Unsupervised Image Classification Based on Neutrosophic Set Theory

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Abstract: **In this paper, a new technique is used to an unsupervised learning image classification based on integration between neutrosophic sets and optimization linear programming. Neutrosophic sets are used to segment the image into three main components namely objects (*O*), edges (*E*), and Background (*B*). The neutrosophic image components (O, E, B) are corresponding to the neutrosophic sets components (T, I, F). The components of neutrosophic image valued in $\left[^-0,1^+\right]$ are representing the association intensities degree of pixel for each image components. Netrosophic image components are contributed to solving one of the important problems in image classification known as "overlapping" within cluster. While, the problem of overlapping between clusters is solved by using optimization linear programming.**

Key words: Neutrosophic set, image classification, linear programming optimization

I. INTRODUCTION

Since several decades, the world is witnessing a remarkable development in the science of computer vision. The principle of computer vision is based on deal with the images and methods of treatment. Hence the interest of researchers in the computer vision with image processing, which is concerned, in essence, on the methods and many different algorithms. Among these algorithms are image classification algorithms.

Classification is the field devoted to the study of methods designed to categorize data into distinct classes. This categorization can be divided to distinct labeling of the data (supervised learning [1]), division of the data into classes (unsupervised learning [2]), selection of the most significant features of the data (feature selection [3]), or a combination of more than one of these tasks [4].

Unsupervised image classification (UIC) starts by partitioning the image data into groups (or clusters). The classes in UIC are unknown, according to similarity measure, groups of image data can be compared with reference to data by an analyst [5]. UIC can be categorized into two main groups namely Hierarchical [6] and Partitional [7] algorithms.

In hierarchical clustering algorithms (HCA) a sequence of clustering with each clustering being a partition of the data set are showing as a tree [8]. HCA is characterized by two advantages, first the number of classes does not need be specified a priori and the others they are independent of the initial condition. However, HCA is suffers from be a static algorithm and its inability to solve the overlapping clusters problem [9]. HCA are divided according to the clusters construction methods or according to the similarity measure. For methods construct the clusters by recursively partitioning the instances in either a top-down or bottom-up fashion. These methods can be subdivided as agglomerative [10] and divisive [11] methods. Whereas, the merging or division of clusters is performed according to some similarity measure, chosen so as to optimize some criterion (such as a sum of squares). The hierarchical clustering methods could be further divided according to the manner that the similarity measure is calculated [12].

On the other hand, partitional clustering algorithms (PCA) are based on image data set segmentation into a specified number of clusters. PCA can be treated as an optimization problem as a result of reliance on the square error function to minimize certain criteria. Both HCA and PCA algorithms are participate in advantages and drawbacks. There are two categories from PCA namely Iterative [13] and non-iterative [14] algorithms. K-means algorithm [15] is the most widely used in iterative partitional algorithms. The basic idea for k-means algorithm is to find a clustering structure that minimizes a certain error criterion which measures the distance of each instance to its representative value. The most well-known criterion is the Sum of Squared Error (SSE) [16], may be globally optimized by exhaustively enumerating all partitions, which is very time-consuming, or by giving an approximate solution using heuristics. Another partitioning algorithm, which attempts to minimize the SSE is the *K*-medoids [17] or partition around medoids (PAM) [18].

Lillesand and Kiefer [19] presented a non-iterative approach to unsupervised clustering with a strong dependence on the image texture. Researches [20-21] have shown that the iterative algorithms are more efficient than its counterpart non-iterative, where it does not rely too much on data points order.

There are other unsupervised classifications methods are used recently represented in Density-based Methods [22] which assume that the points that belong to each cluster are drawn from a specific probability distribution. Model-based Clustering Methods [23], these methods attempt to optimize the fit between the given data and some mathematical models. Unlike conventional clustering, which identifies groups of objects; model-based clustering

methods also find characteristic descriptions for each group, where each group represents a concept or class. The most frequently used induction methods are decision trees [24] and neural networks [25]. Grid-based Methods [26], these methods partition the space into a finite number of cells that form a grid structure on which all of the operations for clustering are performed. The main advantage of the approach is its fast processing time [27]. Soft-computing Methods, In addition to neural networks, there are some methods that belong to soft computing methods such as Fuzzy Clustering [28], Evolutionary Approaches for Clustering [29] and Simulated Annealing for Clustering [30].

In this paper, a new an unsupervised image classification technique is used based on neutrosophic sets [31] and optimization linear programming [32]. Neutrosophic set $(N_s S)$ considered a part from neutrosophy theory which interested by studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra. The idea of neutrosophy theory depends on event or entity, where between an idea $\langle A \rangle$ and its opposite $\langle Anti - A \rangle$, there is a continuum power spectrum of neutralities $\langle Neut - A \rangle$. Truth value (T), indeterminacy value (I) and falsehood value (F) are represented neutrosophic components referring to neutrosophy, neutrosophic logic, neutrosophic set, neutrosophic probability, neutrosophic statistics [33]. In neutrosophic set, the indeterminacy is quantified explicitly and the truth-membership, indeterminacy-membership and falsity-membership are independent. The neutrosophic set is a generalization of an intuitionistic set [34], classical set [35], fuzzy set [36], paraconsistent set [37], dialetheist set [38], paradoxist set [39], and tautological set [40].

Linear programming is constrained optimization, where the constraints and the objective function are all linear. It is called "programming" because the goal of the calculations help you choose a "program" of action [41]. The linear programming model, for neutrosophic image classification problem, involves on two main parts called constraints and objective function. Constraints are describing the query images as lower and upper weights for neutrosophic query image components. On neutrosophic image clustering classification to be maximized a linear objective function means that categorization of similar images in clusters with out overlapping within or between clusters.

The rest of the paper is organized as follows: section 2 presents general framework for proposed technique. neutrosophic image processing is given in section 3. feature extraction for neutrosophic image is presented in section 4. Neutrosophic image cluster is illustrates in section 5. Section 6 illustrates accuracy evaluation for the technique. Section 7 presents experimental results to illustrate the efficiency of the algorithm. Section 8 concludes the paper, and outlines future research.

II. GENERAL FRAMEWORK

This paper presents a novel system to image clustering namely Optimization neutrosophic image classification system (ONsICS). As shown in figure 1, ONsICS consists of two techniques are neutrosophic image processing and optimization image clustering. Neutrosophic image processing is used to convert gray image to enhanced binary image (EBI) based on object, edge and background of image components. Each image can be represented as neutrosophic components (T, I, F) and stored the extracted image components feature as a vector in database. All similar image features are gathered together in a one category by using neutrosophic image clustering (NsIC) technique. Image clusters are optimized by using linear programming to solve image overlapping problem as shown in figure 1.

III. NEUTROSOPHIC IMAGE PROCESSING:

Let I_{img} be a Universe of discourse represents image and I_{comp} is a set of I_{img} represents image components (as object, edge, background) which is composed by bright pixels. Aim of the neutrosophic image domain $(N_S D)$ is transferring image I_{img} to neutrosophic domain by describing the pixel by three membership sets T, I and F as $P_{Ns}(T, I, F)$ [43]. The pixel can represents as:

$$P_{Ns}(i, j) = \{T(i, j), I(i, j), F(i, j)\} \quad where,$$

$T(i, j)$ is the probability belonging to white pixels set. It is defined as:

$$T(i, j) = \frac{\overline{g}(i, j) - \overline{g}_{min}}{\overline{g}_{max} - \overline{g}_{min}}$$

$$\overline{g}(i, j) = \frac{1}{w \times w} \sum_{m=i-w/2}^{i+w/2} \sum_{n=j-w/2}^{j+w/2} g(m, n)$$

Where, $\overline{g}(i, j)$ is the local mean value of pixels of the window.

$I(i, j)$ is indeterminate set. It is defined as:

$$I(i, j) = \frac{\delta(i, j) - \delta_{min}}{\delta_{max} - \delta_{min}}$$

$$\delta(i, j) = abs(g(i, j) - \overline{g}(i, j))$$

Where, $\delta(i, j)$ is the absolute value of difference between intensity $g(i, j)$ and its local mean value $\overline{g}(i, j)$.

$F(i, j)$ is non white pixels set. It is defined as:

$$F(i, j) = 1 - T(i, j)$$

A.   Neutrosophic Image Entropy:

Neutrosophic image entropy [44] is defined as the summation of the entropies of three sets $T, I, and\ F$, which is employed to evaluate the distribution of the elements in the neutrosophic domain:

$$En_{Ns} = En_T + En_I + En_F$$

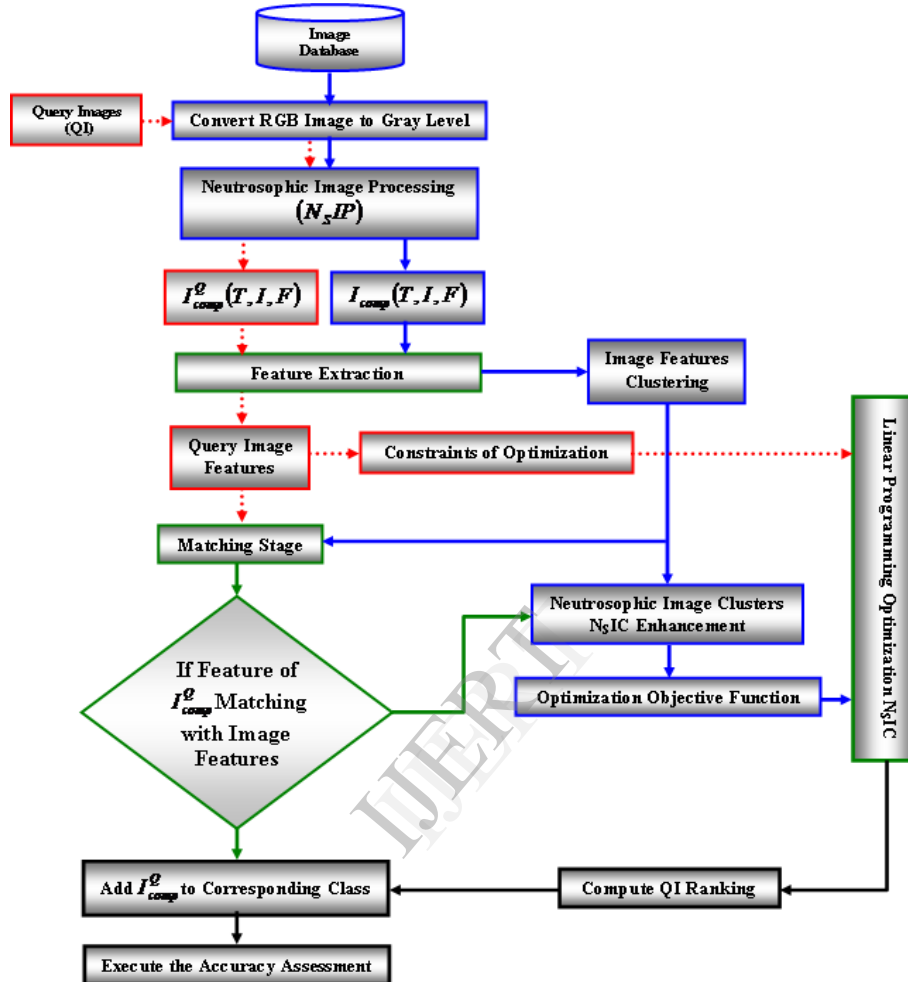$$En_T = - \sum_{i=\min\{T\}}^{\max\{T\}} p_T(i)\ln p_T(i)$$

Fig. 1: Optimization image clustering flowchart.

$$En_I = - \sum_{i=\min\{I\}}^{\max\{I\}} p_I(i)\ln p_I(i)$$

$$En_F = - \sum_{i=\min\{F\}}^{\max\{F\}} p_F(i)\ln p_F(i)$$

where $En_T, En_I, and\ En_F$ are the entropies of the sets T , I and F , respectively. $p_T(i), p_I(i), and\ p_F(i)$ are the probabilities of elements in T , I and F, respectively, whose values equal to i .

IV.   NEUTROSOPHIC IMAGE FEATURE EXTRACTION:

Image feature Extraction is the first step to image retrieval system. Neutrosophic image $(N_S\ \text{Im})$ is divided into three matrices are represented as images called object, edge and background. Each image is consisting of matrix representing the probability white pixel values for object component and probability of non white pixel values for background component while the intermediate matrix expresses the probability of the boundary between the white and non-white pixels. The combinations of pixel brightness value in $(N_S\ \text{Im})$ components are calculated by using a widely method namely Gray Level Co-occurrence Matrix (GLCM) []. The spatially related in various directions with reference to distance and angular relationships for co-occurring pairs of pixels is one of the most important advantages for GLCM calculations.

The feature extraction for $(N_S \text{ Im})$ components by GLCM is based on pixel and its next neighbor pixel. The Contrast, Energy, Homogeneity and Correlation are the parameters of GLCM which calculated by:

$$Contrast = \sum_{i=1}^{m}\sum_{j=1}^{n}(i-j)^2 I_{comp}(i,j), \ where$$

$$0 \le i \le m \ , \ 0 \le j \le n$$

$$(m,n) \, is \, image \dim entions$$

$$Energy = \sum_{i=1}^{m}\sum_{j=1}^{n}\left(I_{comp}(i,j)\right)^2$$

$$Homogeneity = \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{I_{comp}(i,j)}{1+|i-j|}$$

$$Correlation = \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{\{i \times j\} \times I_{comp}(i,j) - \{\mu_x \times \mu_y\}}{\sigma_x \times \sigma_y}, \quad where$$

$$\mu_x, \mu_y \, are \, mean \, of \, probability \, matrix.$$

$$\sigma_x, \sigma_y \, are \, s \tan dard \, deviations \, of \, probability \, matrix.$$

V. NEUTROSOPHIC IMAGE CLUSTERING:

Image clustering can classify similar images into the same group. Let image data set be $\text{Im} = \{I_{comp}^i, i=1,2,...,n\}$, and I_{comp}^i be an image in a d-dimensional space. Image clustering problem is to find image category $Cl_{im} = \{Cl_{im_1}, Cl_{im_2},..., Cl_{im_n}\}$, which satisfies:

$$\text{Im} = \bigcup_{i=1}^{m} Cl_{im_i}$$

$$Cl_{im_i} \ne \Phi \quad for \ i=1,2,...,m$$

$$Cl_{im_i} \bigcap Cl_{im_j} = \Phi \quad for \ i,j=1,2,...,m, \ i \ne j$$

Among clustering methods, the fuzzy c-means algorithm is widely used. An objective function for a clustering method is important to define. The objective function of fuzzy c-means is defined as:

$$J_m(U,E) = \sum_{k=1}^{n}\sum_{k=1}^{c}(\mu_{ik})^m \|x_k - e_i\|^2$$

Where m is constant, and $m > 1$. Cluster i is expressed as $e_i (i=1,2,...,c)$. The membership between sample k and cluster is expressed as;

$$\mu_{ik} \begin{pmatrix} i=1,2,...,c, \\ k=1,2,...,n \end{pmatrix}, \ where,$$

$$\mu_{ik} \in \{0,1\}, \ \forall i,k; \ \sum_{i=1}^{c}\mu_{ik}=1, \forall k$$

can be computed by:

$$\mu_{ik} = \frac{c}{c + \sum_{j=1}^{c}\left(\dfrac{\|x_k - e_i\|}{\|x_k - e_j\|}\right)^{\frac{1}{m-1}}}$$

where e_i is the cluster center and can be computed by:

$$e_i = \frac{\sum_{k=1}^{n}(\mu_{ik})^m x_k}{\sum_{k=1}^{n}(\mu_{ik})^m}, \ 1 \le i \le c$$

The mean and the variance of μ_{ik} for the cluster are computed as:

$$\overline{\mu_i} = \frac{\sum_{k=1}^{n_i}\mu_{ik}}{n_i} \ , \ \sigma_i^2 = \frac{\sum_{k=1}^{n_i}\left|\mu_{ik} - \overline{\mu_i}\right|^2}{n_i}$$

Where, the fuzzy partition matrix is $\mu_i = \{\mu_{i1}, \mu_{i2},..., \mu_{in_i}\}$.

As shown in figure 2, $(N_S \text{ Im})$ clustering method based on fuzzy c-means is used.

A. Neutrosophic image clusters enhancement:

The indeterminacy of image pixel $I_{comp}(i,j)$ is determined by its intensity value $I(i,j)$. Strength of the correlation between neutosophic image components T and F with I are influenced by the distribution of the pixels and the entorpy of I.

The set $I_{\text{Im}} \subseteq [0,1]$ may represent not only indeterminacy but also vagueness, uncertainty, imprecision, error, etc. So, the overlapping problem will appear within and between neutrosophic image cluster as shown in figure 3.

Threshold processing will solve the overlapping problem within and between neutrosophic image clusters by determine the mysterious region between background and objects. In gray level domain, $(\lambda - mean)$ operation for image Im_{GL} is defined as:

$$\text{Im}_{GL}(i,j) = \frac{1}{w \times w}\sum_{m=i-w/2}^{i+w/2}\sum_{n=j-w/2}^{j+w/2}\text{Im}_{GL}^{\text{mod}}(m,n)$$

Where w is the size of the window, (m,n) is the location of the pixel centered the window. A $\lambda - mean$ operation for $P_{Ns}, \overline{P}_{Ns}^{\lambda}$ is defined as:

$$\overline{P}_{Ns}^{\lambda} = P\big(\overline{T}(\lambda), \overline{I}(\lambda), \overline{F}(\lambda)\big)$$

$$\overline{T}(\lambda) = \begin{cases} T & I < \lambda \\ \overline{T} & I \geq \lambda \end{cases}$$

$$\overline{T}^{\lambda}(i,j) = \frac{1}{w \times w} \sum_{m=i-w/2}^{i+w/2} \sum_{n=j-w/2}^{j+w/2} T(m,n)$$

$$\overline{F}(\lambda) = \begin{cases} F & I < \lambda \\ \overline{F} & I \geq \lambda \end{cases}$$

$$\overline{F}^{\lambda}(i,j) = \frac{1}{w \times w} \sum_{m=i-w/2}^{i+w/2} \sum_{n=j-w/2}^{j+w/2} F(m,n)$$
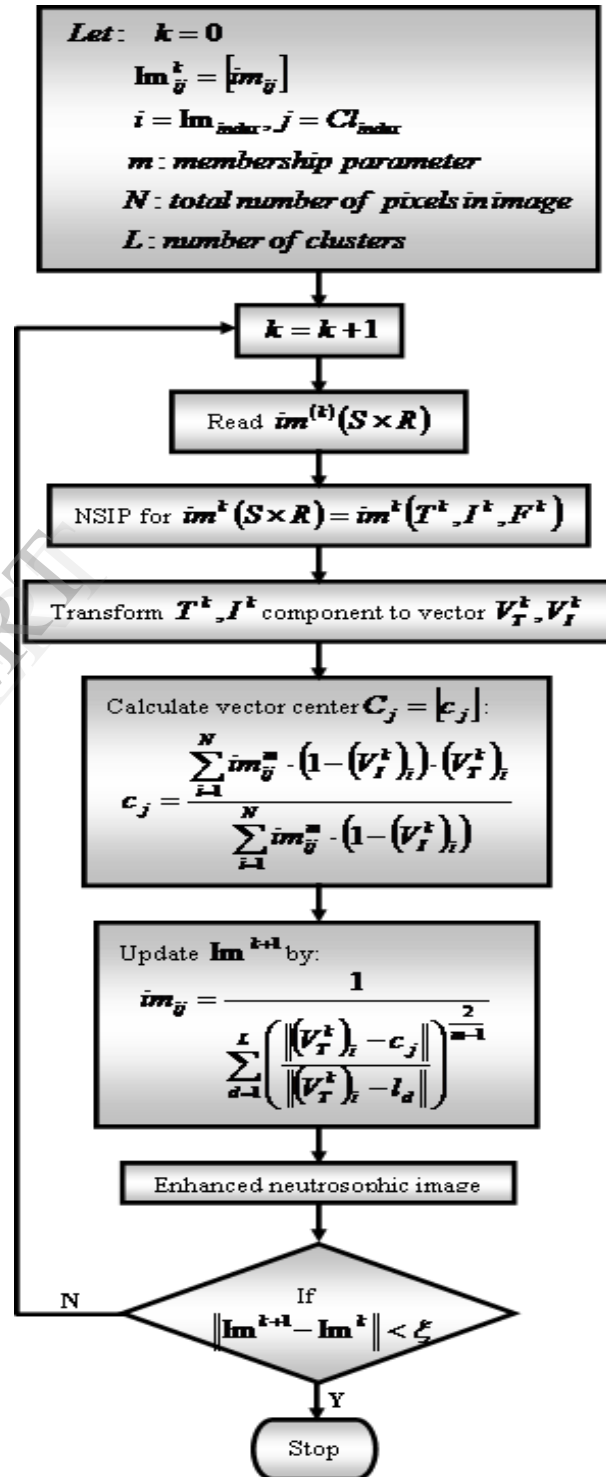

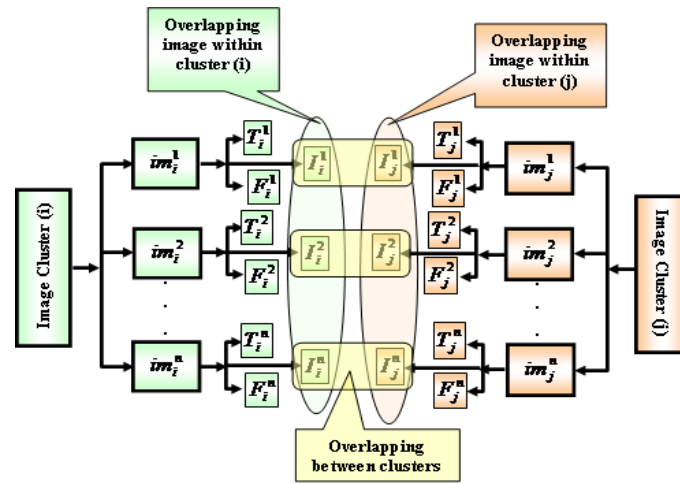Fig. 2: flowchart of neutrosophic image clustering.

Fig. 3: Overlapping within and between neutrosophic image cluster.

$$\bar{I}^{\lambda}(i,j) = 1 - \frac{\bar{H}(i,j) - \bar{H}_{\min}}{\bar{H}_{\max} - \bar{H}_{\min}}$$

Where $\bar{H}(i,j)$ is the homogeneity value of $\bar{T}^{\lambda}$ at (i, j).
w is the local window size.

After $\lambda - mean$ operation subset T became more homogeneous after removing the noise. By using a simple threshold method can be segnent the subset T accurately.

B.   Optimization neutrosophic image clustering:

Optimization $(N_S \, \text{Im})$Classification (ONsIC) method presents a new method to determine the best category to include the query images, where the characteristics of clusters are represented by neutrosophic sets. Suppose that a set of clusters $Cl = \{Cl_1, Cl_2, ..., Cl_m\}$ which consists

of *m* neutronsophic image clusters (NsIC) from which the most preferred cluster is to be selected to including the query image. Each NsIC is assessed on *n* different components as $\{im_1^{comp}, im_2^{comp}, ..., im_n^{comp}\}$. The evaluation of the cluster Cl_i with respect to the component of im_j^{comp} is a neutrosophic set. The neutrosophic index I_{ij} is such that the larger I_{ij} the higher a hesitation margin of NsIC Cl_i with respect to the components of im_j^{comp} whose intensity is given by T_{ij}. The NsIC matrix is given in the following form:

$$N_S IC = \begin{array}{c|cccc} & I_{comp}^1 & I_{comp}^2 & \cdots & I_{comp}^n \\ \hline Cl_1 & (T_{11}, I_{11}, F_{11}) & (T_{12}, I_{12}, F_{12}) & \cdots & (T_{1n}, I_{1n}, F_{1n}) \\ Cl_2 & (T_{21}, I_{21}, F_{21}) & (T_{22}, I_{22}, F_{22}) & \cdots & (T_{2n}, I_{2n}, F_{2n}) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ Cl_m & (T_{m1}, I_{m1}, F_{m1}) & (T_{m2}, I_{m2}, F_{m2}) & \cdots & (T_{mn}, I_{mn}, F_{mn}) \end{array}$$

Where the characteristics of NsIC (Cl_i) are given by:

$$Cl_i = \left\{ \frac{I_{comp}^1}{T_{i1}, I_{i1}, F_{i1}}, \frac{I_{comp}^2}{T_{i2}, I_{i2}, F_{i2}}, ..., \frac{I_{comp}^3}{T_{in}, I_{in}, F_{in}} \right\}, \quad where, 1 \le i \le m$$

$N_S IC(Cl_i)$ can present by another form as:

$$Cl_i = \left\langle \left\langle Cl_1, [K_{i1}^l, K_{i1}^u] \right\rangle, \left\langle Cl_2, [K_{i2}^l, K_{i2}^u] \right\rangle, ..., \left\langle Cl_n, [K_{in}^l, K_{in}^u] \right\rangle \right\rangle, \; where$$

$[K_{ij}^l, K_{ij}^u]$ is closed NsIC interval computed by:

$$[K_{ij}^l, K_{ij}^u] = \left[\min\left(\frac{T_{ij} + I_{ij}}{2}, \frac{1 - F_{ij} + I_{ij}}{2}\right), \max\left(\frac{T_{ij} + I_{ij}}{2}, \frac{1 - F_{ij} + I_{ij}}{2}\right) \right]$$

Obviously, $0 \le K_{ij}^l + K_{ij}^u \le 2$ for all $Cl_i \in Cl$ and $im_j^{comp} \in im$. The degrees to the alternative NsIC Cl_i satisfies and does not satisfy the can be measured by the evaluation function (*E*). The evaluation function $E(Cl_i)$ of alternative NsIC Cl_i can be expressed as:

$$E(Cl_i) = [K_{ij}^l, K_{ij}^u] \wedge [K_{ik}^l, K_{ik}^u] \wedge .... \wedge [K_{ip}^l, K_{ip}^u] \vee [K_{iQ}^l, K_{iQ}^u]$$
$$= [\min\{K_{ij}^l, K_{ik}^l, ..., K_{ip}^l\}, \min\{K_{ij}^u, K_{ik}^u, ..., K_{ip}^u\}] \vee [K_{iQ}^l, K_{iQ}^u]$$
$$= [\max\{\min\{K_{ij}^l, K_{ik}^l, ..., K_{ip}^l\}, K_{iQ}^l\}, \max\{\min\{K_{ij}^u, K_{ik}^u, ..., K_{ip}^u\}, K_{iQ}^u\}]$$
$$= [K_{Cl_i}^l, K_{Cl_i}^u]$$

$where, 1 \le i \le m$, and $(\wedge \, and \, \vee)$ denote the minimum and maximum operator of neutrosophic set respectively.

The score of alternative NsIC $S_c(Cl_i)$ can be evaluated by:

$$S_c(Cl_i) = 2(T_{Cl_i}^u - T_{Cl_i}^l)$$
$$= 2\left(\max\left(\left(\frac{T_{Cl_i} + I_{Cl_i}}{2}\right), \left(\frac{1 - F_{Cl_i} + I_{Cl_i}}{2}\right) \right) - \min\left(\left(\frac{T_{Cl_i} + I_{Cl_i}}{2}\right), \left(\frac{1 - F_{Cl_i} + I_{Cl_i}}{2}\right) \right) \right)$$

An accuracy function A_c is used to evaluate the degree of accuracy of neutrosophic elements as follows:

$$A_c(Cl_i) = \frac{1}{2}(T_{Cl_i}^l + T_{Cl_i}^u)$$
$$= \frac{1}{2}\left(\min\left(\left(\frac{T_{Cl_i} + I_{Cl_i}}{2}\right), \left(\frac{1 - F_{Cl_i} + I_{Cl_i}}{2}\right) \right) + \max\left(\left(\frac{T_{Cl_i} + I_{Cl_i}}{2}\right), \left(\frac{1 - F_{Cl_i} + I_{Cl_i}}{2}\right) \right) \right)$$

The larger value of $A_c(Cl_i)$ represents the more degree of accuracy of an element Cl_i in the neutrosophic set. Based on the score function S_c and the accuracy function A_c the degree of suitability that the corresponding cluster Cl_i satisfies the query images component can be measured by:

$$W(E(Cl_i)) = (S_c(E(Cl_i)))^2 - \left(\frac{1 - A_c(E(Cl_i))}{2}\right)$$

The coefficients in $W(E(Cl_i))$ have been chosen so that $W(E(Cl_i)) \in [0,1]$. The larger value of $W(E(Cl_i))$, the more suitability to which the alternative NsIC (Cl_i) satisfies to query image components, where $1 \le i \le m$.

Assume that there is a query images wants to choose an alternative NsIC which satisfies the components of QI_j, $where$, $1 \le j \le n$, each QI_j have a different degree of components $(\hat{T}, \hat{I}, \hat{F})$. Where,

$$0 \le \hat{T} \le 1,$$
$$0 \le \hat{I} \le 1,$$
$$0 \le \hat{F} \le 1,$$
$$and \quad 0 \le \hat{T} + \hat{I} + \hat{F} \le 3$$

$(\hat{T}, \hat{I}, and \, \hat{F})$ are the degrees of membership (object), indeterminacy (edge) and non-membership (background) of the images $im_j^{comp} \in im^{comp}$ to the vague concept importance of criterion respectively.

The weight of query image components lies in closed interval $[w_j^l, w_j^u]$ where,

$$[w_j^l, w_j^u] = \left[\min\left(\frac{\hat{T}_j + \hat{I}_j}{2}, \frac{1 - \hat{F}_j + \hat{I}_j}{2} \right), \max\left(\frac{\hat{T}_j + \hat{I}_j}{2}, \frac{1 - \hat{F}_j + \hat{I}_j}{2} \right) \right]$$

Where, for each query images are $0 \le w_j^l \le w_j^u \le 1$.

Then the suitability degree of alternative $NsIC(Cl_i)$ satisfies the query images components can be measured by:

$$R(Cl_i) = \max\left\{ (W(Cl_i))^2 - \left(\frac{1 - T(Cl_i)}{2}\right), (S_c([\hat{T}_{iq}^l, \hat{T}_{iq}^u])) - \frac{1 - A_c([\hat{T}_{iq}^l, \hat{T}_{iq}^u])}{2} \right\}$$

The optimal weights value can be computed via the following programming:

$$R(Cl_i) = \sum_{j=1}^{m} \left(2(K_{ij}^u - K_{ij}^l)^2 - \frac{1 - \frac{(K_{ij}^l + K_{ij}^u)}{2}}{2} \right) w_*^j$$

Subject to the conditions:

$$w_j^l \le w_*^j \le w_j^u$$
$$where, \quad j = 1,2,...,n$$

VI. Accuracy assessment:

A neutrosophic image is a combination of various components built on the intensity of pixels. The unsupervised classification applied to neutrosophic image sets result in clusters. The comparison between neutrosophic image components and clusters indicates the accuracy of the classification. Confusion matrices are used to assess this accuracy as they compare the relationship between the neutrosophic image components as the reference images and the corresponding results of the unsupervised classification technique.

A resulting cluster by unsupervised classification is not automatically labeled nor identified as corresponding to specific image. So, image is investigated with respect to all clusters, and the cluster containing most of image components closest to the mean of that image is considered as its corresponding to a specific cluster. Based on confusion matrix, the accuracy is then expressed in terms of the kappa statistic (k) where the difference between the clustering accuracy and the chance agreement between the images and the clusters is calculated [45].

It results in a value between 0 and 1 for each classification, where 0 indicates that the clustering is no better than grouping the image by chance:

$$k = \frac{N \sum_{i=1}^{c} y_{ij} - \sum_{i=1}^{c} (y_i \cdot y_j)}{N^2 - \sum_{i=1}^{c} (y_i \cdot y_j)}$$

Where:

$c:$ is the number of clusters $1 \le i \le c$

$N:$ is the total number of images in the image database classified

$i:$ refers to the images corresponding to cluster i

$y_{ij}:$ is the number of images in row i and column j in the confusion matrix

$y_i:$ is the number of images in row i in the confusion matrix

$y_j:$ is the number of images in column j in the confusion matrix

VII. EXPERIMENTAL AND RESULTS:

The WBIIS image database [46] is used to evaluate the ONsIC system. WBIIS image database consists of 10,000 generic images with variety size distributed on 10 categories of Africa, Beaches, Building, Buses, Dinosaurs, Elephants, Flowers, Foods, Horses and Natural. Several test images were used during experimentation and result as shown in figure (4).
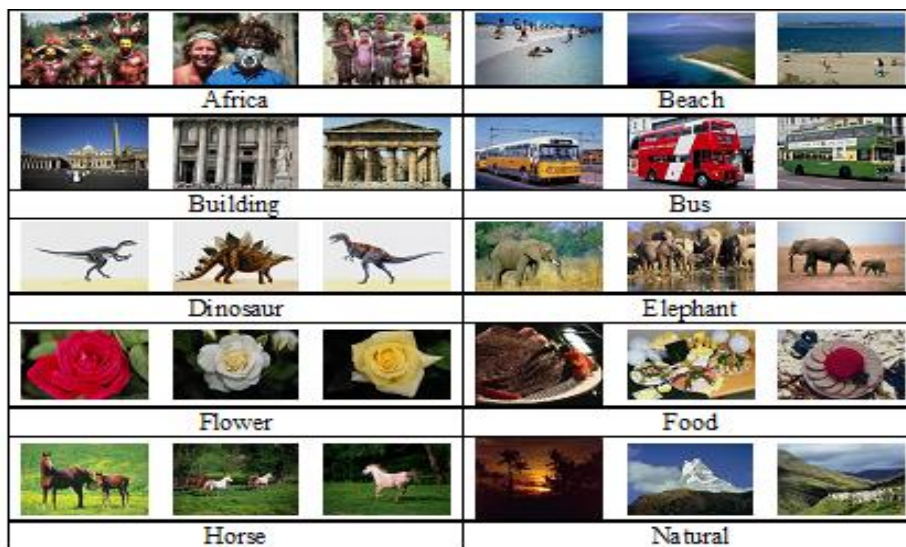

Fig. 4: Database categories

The experimental results are discussed under 4 heading:

A. Transform image to neutrosophic image.

Figure 5 illustrates three steps to transformation from RGB images to neutrosophic images. First step convert the RGB image to Gray image. A gray image is a simple kind of image that contains one domain, and each pixel in the image can be represented by an integer. Second step, an input image is mapped to T and F by the S-function [47] and indeterminacy domain I by homogeneity [48]. There are 12 features (4 parameter of GLCM $\times 3$ image component (T, I and F)) stored in features vectors database for the further processes.
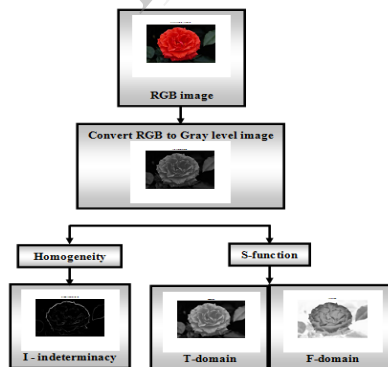

Fig. 5: Neutrosophic image transform steps.

B. Neutrosophic image clustering.

The major advantage of neutrosophic image clustering (NsIC) method based on fuzzy c-means can deal with indeterminate intensity regions effectively and accurately. Another advantage of NsIC is that it can smooth the complex background; therefore, it can prevent the object region from connecting with the false foregrounds. Besides the above two major advantages, NsIC finds more accurate object boundaries. Table 1 illustrate example of image clusters.

Table 1: Example of image clusters.

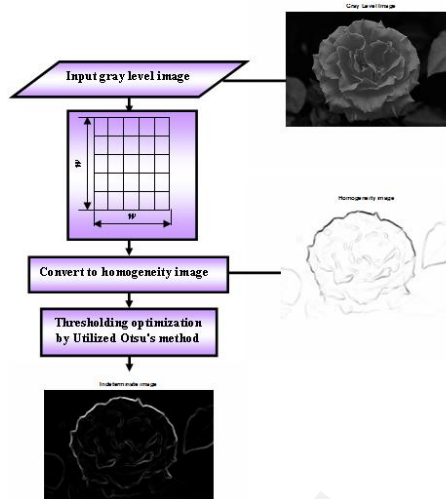| Clusters	im_1^{comp}			im_2^{comp}			im_3^{comp}			im_4^{comp}		
	T	F	I	T	F	I	T	F	I	T	F	I
Cl_{Africa}	0.5777	0.4223	0.0274	0.5503	0.4497	0.0209	0.5474	0.4526	0.0433	0.6357	0.3643	0.0280
Cl_{Beach}	0.4135	0.5865	0.0183	0.4839	0.5161	0.0212	0.5977	0.4023	0.0131	0.4668	0.5332	0.0149
Cl_{Build}	0.4806	0.5194	0.0474	0.4817	0.5183	0.0326	0.4333	0.5667	0.0353	0.5609	0.4391	0.0451

Fig. 6: Enhanced within neutrosophic image.

C. Neutrosophic image cluster enhanced.

The window size (5×5) is used for computing the standard deviation and discontinuity of pixels intensity. Figure 6 illustrates the homogeneity image in domain I. Otsu's method [49] is utilized a simple thresholding method. The global t optimum threshold is finds that minimizes the overlap variance of the background and objects within cluster and the overlap variance between images between clusters by the following equation:

$$\sigma(t) = c_1(t)\sigma_1(t) + c_2(t)\sigma_2(t)$$

Where, $\sigma(t)$ is the sum of variances of the two clusters as a t threshold function. $\sigma_i(t)$ and $c_i(t)$ are the variance and probability of class i , $i=1,2$ respectively. Threshold t that results in the minimization of σ (t) separates the two classes as the foreground and background, respectively.

D. Optimization neutrosophic image clustering.

ONsIC technique is applied on neutrosophic set of image clusters (NsIC) $\left\{ Cl_{(T_i, I_i, F_i)}^i , i = 1,2,...,m \right\}$ with evaluations as neutrosophic components. For example, there are two clusters each cluster including four images components (T, I, F) are represent as:

	I_{comp}^1	I_{comp}^2	I_{comp}^3	I_{comp}^4
$N_sC = Cl_1$	[0.8,0.6,015]	[0.68,0.46,0.2]	[0.45,0.1,0.5]	[0.5,0.4,0.8]
Cl_2	[0.4,0.8,0.45]	[0.75,0.9,0.05]	[1,0.5,1]	[0.5,0.6.0.9]

Evaluation of neutrosophic component for NsC $\left(E(NsC_{ij}) \right) = \left[K_{ij}^l, K_{ij}^u \right]$ is computed by from:

$$\left[K_{Cl_{ij}}^l, K_{Cl_{ij}}^u \right] = \left[\min\left(\frac{T_{ij}+I_{ij}}{2}, \frac{1-F_{ij}+I_{ij}}{2} \right), \max\left(\frac{T_{ij}+I_{ij}}{2}, \frac{1-F_{ij}+I_{ij}}{2} \right) \right]$$

The result is:

$$\left[\kappa_{Cl_{ij}}^L, \kappa_{Cl_{ij}}^U \right] = \begin{bmatrix} [0.7,0.725] & [0.57,0.63] & [0.275,0.3] & [0.3,0.6] \\ [0.6,0.675] & [0.825,0.925] & [0.25,0.75] & [0.35,0.75] \end{bmatrix}$$

Scoring and accuracy evaluation matrices S_c, A_c are computed by:

$$S_c = 2 \times \left[\kappa_{Cl_{ij}}^U - \kappa_{Cl_{ij}}^L \right] \quad , \quad A_c = 0.5 \times \left[\kappa_{Cl_{ij}}^L + \kappa_{Cl_{ij}}^U \right]$$

The result is:

$$S_c = \begin{bmatrix} 0.05 & 0.12 & 0.05 & 0.6 \\ 0.15 & 0.2 & 1 & 0.8 \end{bmatrix} , A_c = \begin{bmatrix} 0.7125 & 0.6 & 0.2875 & 0.45 \\ 0.6375 & 0.875 & 0.5 & 0.55 \end{bmatrix}$$

Weights of NsC $W(NsCl_{ij})$ are computed by:

$$W(NsCl_{ij}) = \left(S_c(NsC_{ij}) \right)^2 - \frac{1 - A_c(Nsc_{ij})}{2}$$

The result is:

$$W(N_sCl_{ij}) = \begin{bmatrix} -0.141 & -0.186 & -0.354 & 0.085 \\ -0.159 & -0.022 & 0.75 & 0.415 \end{bmatrix}$$

The coefficients of the linear programming problem are computed by sum each column in $W(NsC_{ij})$:

$$\sum W(NsCl_{ij}) = \begin{bmatrix} -0.3 & -0.208 & 0.396 & 0.5 \end{bmatrix}$$

Four neutrosophic query images $\left\{ QI_{(\hat{T}_j, \hat{i}_j, \hat{F}_j)}^j , j = 1,2,...,n \right\}$ are given as:

$$QI^4_{(\hat{T},\hat{I},\hat{F})} = \left\{ \frac{im^{comp}_{Q_1}}{0.25,0.3,0.25}, \frac{im^{comp}_{Q_2}}{0.35,0.6,0.41}, \frac{im^{comp}_{Q_3}}{0.32,0.55,0.67}, \frac{im^{comp}_{Q_4}}{0.64,0.98,0.57} \right\}$$

For simplifying computation, the neutrosophic set may be written as:

	$im^{comp}_{Q_1}$	$im^{comp}_{Q_2}$	$im^{comp}_{Q_{13}}$	$im^{comp}_{Q_{14}}$
T	0.25	0.35	0.32	0.64
I	0.3	0.6	0.55	0.98
F	0.25	0.41	0.67	0.57

Weights of query image are computed by formula:

$$\left[w^l_j, w^u_j \right] = \left[\min\left(\left(\frac{\hat{T}_j + \hat{I}_j}{2}\right), \left(\frac{1 - \hat{F}_j + \hat{I}_j}{2}\right) \right), \max\left(\left(\frac{\hat{T}_j + \hat{I}_j}{2}\right), \left(\frac{1 - \hat{F}_j + \hat{I}_j}{2}\right) \right) \right]$$

The result is:
$$\left[w^l_j, w^u_j \right] = \left[[0.275, 0.525] \quad [0.475, 0.595] \quad [0.435, 0.44] \quad [0.705, 0.81] \right]$$

Thus the linear programming now can be set as:

$$\max \sum_{j=1}^{n} \left(\sum_{i=1}^{n} w(A^{ij}) \right)$$

Subject to:

$$w^l_j \leq w^j \leq w^u_j$$

The result is:

Maximize : $\quad -0.3w_1 - 0.208w_2 + 0.396w_3 + 0.5w_4$

subject to :
$$0.275 \leq w_1 \leq 0.525$$
$$0.475 \leq w_2 \leq 0.595$$
$$0.435 \leq w_3 \leq 0.44$$
$$0.705 \leq w_4 \leq 0.81$$

The linear programming can easily solve to get w^j_* :

$$w^1_* = 0.275$$
$$w^2_* = 0.475$$
$$w^3_* = 0.44$$
$$w^4_* = 0.81$$

Then the degree of suitability that the corresponding cluster Cl_i satisfies the query images can be measured by the following function:

$$R(Cl_i) = \sum_{j=1}^{m} \left(2\left(K^u_{ij} - K^l_{ij}\right)^2 - \frac{1 - \left(K^l_{ij} + K^u_{ij}\right)}{2} \right) \times w^j_*$$

The result is:

$$R(Cl_1) = \left((0.05)^2 - \left(\frac{1 - 0.7125}{2}\right) \right) \times 0.275 + \left((0.12)^2 - \left(\frac{1 - 0.6}{2}\right) \right) \times 0.475$$
$$+ \left((0.05)^2 - \left(\frac{1 - 0.5}{2}\right) \right) \times 0.44 + \left((0.6)^2 - \left(\frac{1 - 0.45}{2}\right) \right) \times 0.81 = -0.21380375$$

$$R(Cl_2) = \left((0.15)^2 - \left(\frac{1 - 0.6375}{2}\right) \right) \times 0.275 + \left((0.2)^2 - \left(\frac{1 - 0.875}{2}\right) \right) \times 0.475$$
$$+ \left((1)^2 - \left(\frac{1 - 0.5}{2}\right) \right) \times 0.44 + \left((0.8)^2 - \left(\frac{1 - 0.55}{2}\right) \right) \times 0.81 = 0.61180625$$

Therefore we can see that the alternative cluster Cl_2 is the best choice. And the optimal ranking order of the alternatives is given by $Cl_2 \langle Cl_1$.

VIII. Performance results

Table 2 is show the confusion matrix for NsIC technique, the ten clusters found that represent a good approximation of the ten categories of database image.

Table 2: Confusion matrix for NsIC.

Categories	Clusters									
	Cl_1	Cl_2	Cl_3	Cl_4	Cl_5	Cl_6	Cl_7	Cl_8	Cl_9	Cl_{10}
Cat_{Afric}	89	0	5	0	1	0	3	0	2	0
Cat_{Beach}	0	98	1	0	0	0	0	0	0	1
Cat_{Build}	2	0	91	0	1	0	1	0	5	0
Cat_{Bus}	0	0	5	87	0	6	0	1	1	0
Cat_{Dins}	0	6	0	1	92	0	1	0	0	0
Cat_{Eleph}	6	2	0	0	0	88	2	1	1	0
Cat_{Flower}	0	0	0	2	0	3	95	0	0	0
Cat_{Food}	0	0	0	1	2	0	0	97	0	0
$Cat_{Texture}$	0	3	0	0	0	0	1	0	93	3
$Cat_{Natural}$	3	0	1	0	1	0	0	0	5	90

The experiment results have proved beyond any doubt that NsIcC technique can be of high-accuracy techniques.

Where, the accuracy of similar images was gathered in different clusters exceeded 90%, as shown in figure 7.
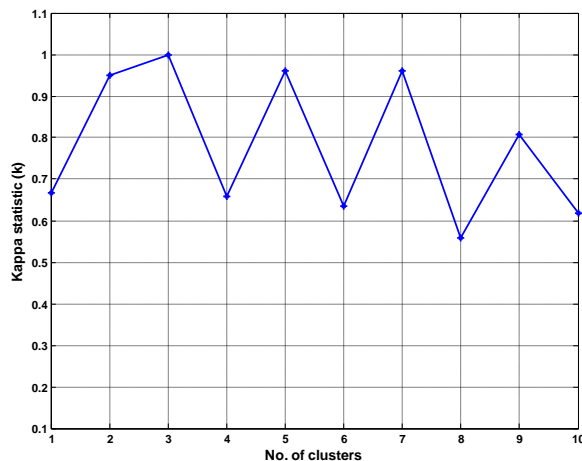
Fig. 7. The accuracy assessment (k) versus different clusters.
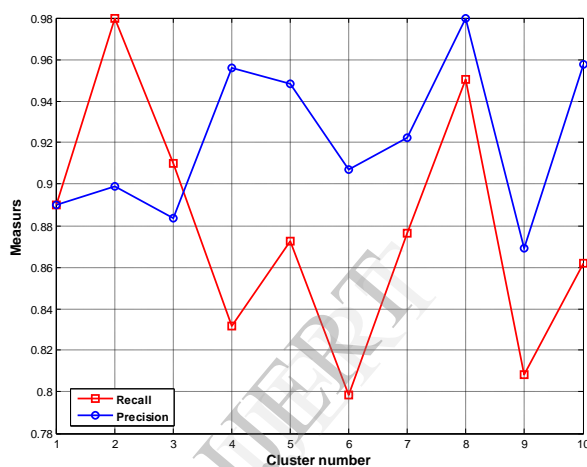

Fig. 8. The recall and precision measures versus different clusters.

The retrieval performance is defined by the precision and recall of the retrieved images [50]. As shown in figure 8 the beach category has been identified with high recall 98%. That is mean in this category the high number of similar images are gathered with very few errors. Whereas the low recall 80.8318% has been identified for house category. That is meaning there are   misclassified for some images in this category as a result of the weakness of I set that is working to determine the threshold used in image enhanced. The precision value ranging from 86.9195% to 97.9798% where the highest value belonging to the cluster 8 and lowest for the cluster 9. This means there is a cluster 8 has more than an image that dos not belong to texture category and vice versa in cluster 5.

The experiments were carried out on a large-scale over the WBIIS image database to measure the effectiveness of NSIC. The results focused on recall and precision measures and accuracy assessment as a criterion for comparison between NsIC technique and other technique namely G. Ciocca et al (2013).

G. Ciocca et al [51] proposed a new technique for unsupervised image classification based on purely semantic representation not on statistical method. This technique is composed of two parts namely supervised feature [52] and unsupervised classification. The idea of supervised features part is based on the hypothesis in the semantic realm regarding categories. That an image d belongs to category c_i with certain probability will not determine whether the same image belongs to a category c_j, but it will modify the probability that it be so. There are supervised feature descriptions are used to classify image regions into semantic concept classes such as Classemes [53], Prosemantic features [54], Object bank [55], CCA-based features [56], and Primitive features [57]. The second part unsupervised classification algorithm such as k-means based on the output of a limited number of classifiers can be embedded into a feature space by a clustering algorithm. G. Ciocca et al is used prosemantic features as supervised feature and k-means algorithm as unsupervised classifier and applied on image database. The results of recall, precision and accuracy assessment indicates that the NsIC technique achieved a high performance compared with the other technique as shown in figures 9, 10 and 11.
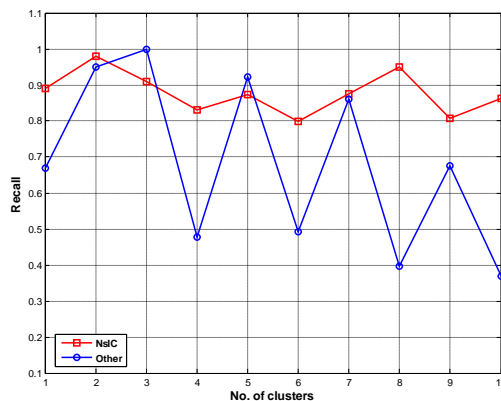
Fig. 9. The recall for compared two techniques.

In general, it is obvious that the NsIC technique has achieved a better precision, recall and accuracy assessment (k) at various numbers of clustered images than the other compared technique.

IX. CONCLUSION:

This paper presents a new technique to unsupervised classification for images based on neutrosophis sets and optimization linear programming. Neutrosophic theory was used to transform the gray image to neutrosophic image components (*O, E, B*). Indeterminacy set (*E*) was worked on determine the objects boundaries with high precision. Determining the boundaries of objects accurately blunted the effect of the overlapping problem within the cluster.
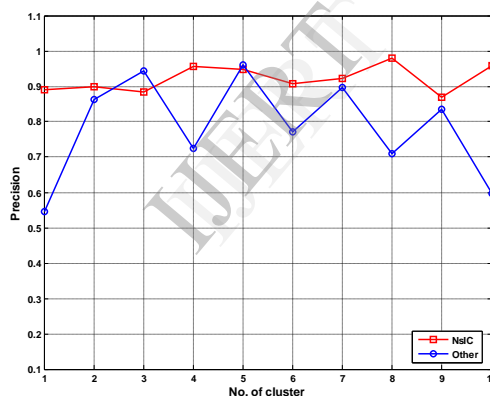

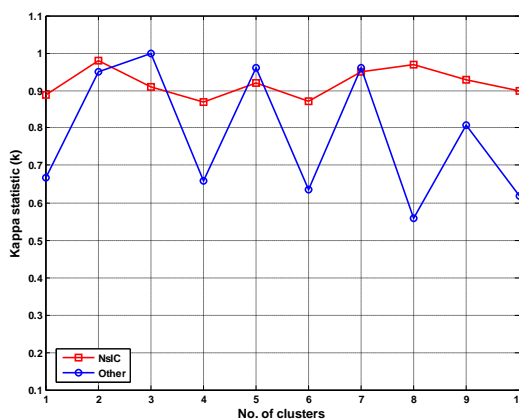Fig. 10. The precision for compared two techniques.

Fig. 11. The k statistic for compared two techniques.

Neutrosophic image clustering method based on fuzzy c-means is used. Neutrosophic image clustering has been enhanced by using the λ-mean operation which helped on solve the overlapping problem between clusters. Optimization neutrosophic image clustering is achieved by using the weight coefficient between image clusters and

images category as an object function in linear programming problem. Whereas, the constraints of linear programming problem are the weight limits for query images.

Practical results conducted on neutrosophic image clustering technique has proved its efficiency where it was to obtain the high performance rate in the accuracy of the resulting clusters as well as high values of recall and precision measures.

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