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Orbital Maneuvering using Electrodynamic Tethers

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***Abstract*— Orbital debris has influence in future space missions considerably. A number of catastrophic collisions are expected in the following years. But to remove these threats from low earth orbit, the conventional techniques using propellant are not economically efficient. This innovative topic shows the method to remove debris using electrodynamic tethers, which is a non-conventional technique. The main objective is to use electrodynamic tethers for effective transfer of a debris removal satellite to different orbits for the collection of debris or attaching some deorbiting mechanism on the debris in the most optimal way. In brief, the paper defines optimal path for the debris removing vehicle to visit a predetermined number of debris. Matlab software is used as an optimization tool in this work. For this purpose various kinds of maneuvers that a satellite can perform are analyzed and the coding in Matlab for each maneuvers are performed. This coding is done in order to derive the delta-v required for various types of maneuvers. The Lambert's problem was used to determine the velocity vectors of the vehicle in orbit on defining the position vectors. Also the optimal path, which the vehicle can travel using electrodynamic tethers is solved using Matlab code. The method is chosen to reduce the payload required and thereby reduce the cost of maneuvering.**

Keywords— Electrodynamic Tethers (EDT), Travelling Salesman Problem (TSP), Lambert's Problem, Lorentz force, Orbital debris, Low Earth Orbit (LEO), Oblate Earth effect, RAAN rate

I.INTRODUCTION

Space explorations started several centuries back. In the beginning, these explorations helped in formulation of assumptions and theories regarding the evolution of earth. The Low Earth Orbit (LEO) extending from 160km to 2000km is the main operational area for active satellite, but due to the presence of space debris it is highly unstable. There are 300000 debris of the size around 1cm and 19000 debris of size more than 5cm including the inactive satellites and junks existing in the Low Earth Orbit. A number of collisions between artificial satellites have occurred in the recent past. One of the major events is the anti-satellite test by Chinese researchers in 2007 which completely destroyed the satellite 'Feng Yun' and created 2900 pieces of debris with size greater than 10cm. In 2009, the biggest hypervelocity collision ever happened when the satellite Iridium 33 collided with another satellite named Kosmos 2251 [6]. Iridum 33 was active during the collision event, but the inactive Russian satellite Kosmos 2251 (Inactivated in 1995) was left unnoticed which resulted in the collision. The collision created 1600 fragments which added threat to other active satellites. EDT can play a key role in changing the potential risks in the removal of debris. The method is cost effective because it doesn't need any fuel, fuel tank or additional propulsion accessories. All it requires is an electron emitter, balancing mass and long tether. Some experiments were already carried out by some agencies and organizations regarding the use of EDT in space.

Kessler Syndrome is the phenomena in which, the collision of one debris with another debris or an active satellite causes a series of collision process. Even if no launches are done from now on it can't be avoided. Only possibility is to remove the potentially risky debris.

The Lambert's problem is solved to find the velocity vectors. Gravity field variation for oblate earth effect also determined. The computation of velocity increment required for various orbital maneuvers are also coded in matlab. In order to determine the orbital elements and define the path of the vehicle, through different orbits Ode45 is used. The transfer from one debris orbit to another is found for a given current, tether length and initial conditions the orbit at any time can be defined.

As a future work travelling sales man problem is preferred to solve the logical concept involved in path optimization. However the problem solving using Branch and Bound algorithm is under progress.

II.ORBITAL MANEUVERS

Orbital maneuvers are performed in space for various purposes such as altitude raising, path changing, collision avoidance, etc. The Δv determinations for the following orbital maneuvers are completed [5].

A. Hohmann Transfer

In hohmann transfer, ΔV_a is provided at point A. This allows the satellite to enter an elliptical trajectory. After it reaches the point B in final orbit another firing is done in opposite direction as ΔV_b.

The total ΔV required is the sum of above velocity increments.

$$\Delta V_{total} = \Delta V_a + \Delta V_b \qquad (1)$$

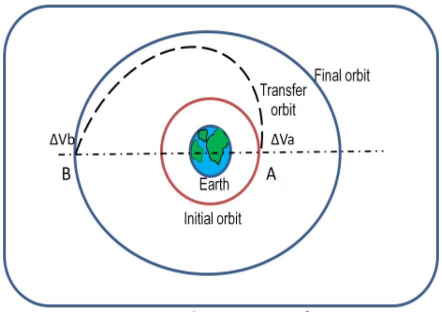

Fig 1: Hohmann Transfer

B. Bi-elliptic Hohmann Transfer

In the case of bi-elliptic hohmann transfer, two semielliptical transfer paths used for maneuvering as shown in the diagram from A to B and B to C. The transfer orbits must be coaxial.

$$\Delta V_{total} = \Delta V_a + \Delta V_b + \Delta V_c \qquad (2)$$

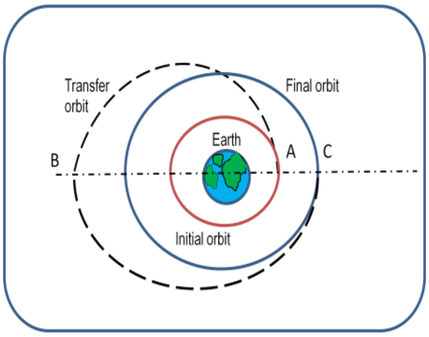

Fig 2: Bi-elliptic Hohmann transfer

C. Apse Line Rotation

Apse line can be said as an imaginary line which is defined by an orbit's eccentricity vector. The satellite can be shifted from one orbit to another at the point at which both the orbits intersect each other (A and B).

$$\Delta V = \sqrt{V_1^2 + V_2^2 - 2 V_1 V_2 \cos \Delta\gamma} \qquad (3)$$

V_1, V_2 – Velocity of satellite at orbits 1 and 2.
θ_1, θ_2 – True anomalies of satellite at orbits 1 and 2.
η – Apse line rotation angle.

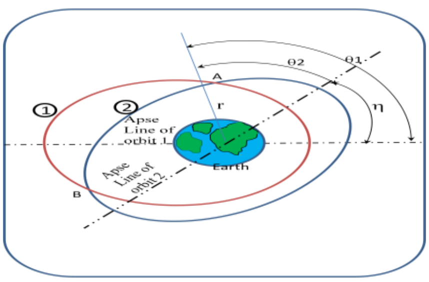

Fig 3: Apse line rotation

D. Phasing Maneuver

Phasing maneuver is used to change the position of a satellite in an orbit in order to rendezvous with another space object in the same orbit. From figure 5, it can be seen that the new orbit formed will be very much close to the initial orbit and dock with target, which allows proceeding the maintenance work, refueling, etc.

$$\text{Time Period, } T = \frac{2\pi}{\sqrt{\mu}} a^{3/2} \qquad (4)$$

Where, Gravitational parameter, $\mu \approx GM \approx 398600$ km/s^4
G-Gravitational constant of earth
M- Mass of earth; a- Semimajor axis

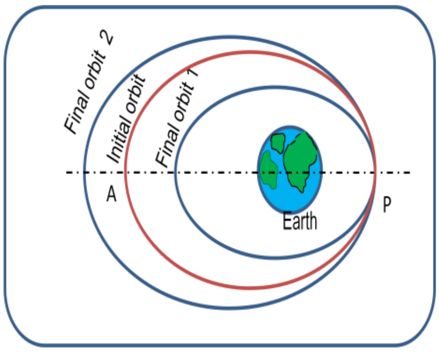

Fig 4: Phasing Maneuver

E. Non-Hohmann transfer with common Apse Line

This transfer strategy is used to transfer between orbits in same plane and sharing a common apse line. Since the non-hohmann transfer is preferred, the transfer trajectory is not necessarily a tangent to either orbit. Δv is same as apse line rotation.

r_a, r_d – Radius at initial and final orbit.
θ_a, θ_d – True anomaly at initial and final orbit.

$$\text{Eccentricity, } e = -\frac{r_a - r_d}{r_a \cos\theta_a - r_d \cos\theta_d} \qquad (5)$$

$$\text{Angular Momentum, } h = \sqrt{\mu r_a r_d}\sqrt{\frac{\cos\theta_a - \cos\theta_d}{r_a \cos\theta_a - r_d \cos\theta_d}} \qquad (6)$$

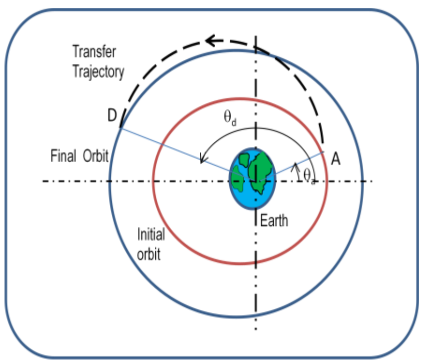

Fig 5: Non-Hohmann transfer with common apse line

F. Chase Maneuver

Chase maneuver is used to chase down a target space object. The chase maneuver is performed to catch up with an object and dock with it. Docking mechanism is used for maintenance, refueling, deorbiting, etc.

$$\text{Time Period, } T = \frac{2\pi}{\mu^2}\left(\frac{h}{\sqrt{1-e^2}}\right)^3 \quad (7)$$

$$\text{Velocity, } V_B = \frac{\mu}{h}\left(-\sin\theta_b\,\hat{i} + (e + \cos\theta_b)\hat{j}\right) \quad (8)$$

θ_a, θ_b – True anomaly at initial and final orbit.
h- Specific Angular momentum of the orbit
μ- Gravitational parameter

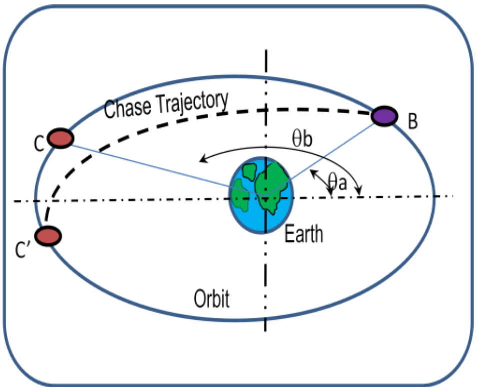

Fig 6: Chase Maneuver

G. Plane Changing Maneuver

The plane changing maneuver is a complex one because the velocity increment should be given at an angle so that the satellite moves into an orbit in another plane. In this case, both the orbits will share a common focus even though they don't have a common plane.

$$\Delta V = \sqrt{\left(V_{r_1} - V_{r_2}\right)^2 + V_{t_1}^2 + V_{t_2}^2 - 2V_{t_1}V_{t_2}\cos\alpha} \quad (9)$$

V_r, V_t – Radial and Tangential velocity at an orbit

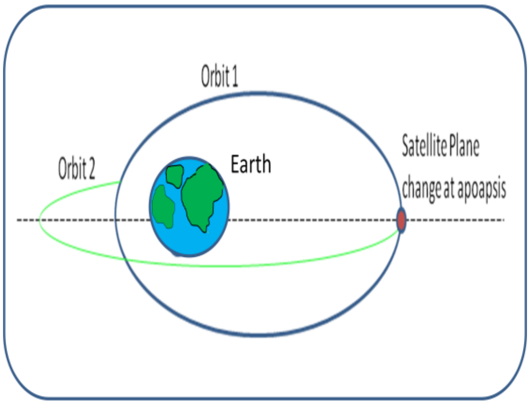

Fig 7: Plane changing maneuver

III. Optimization Techniques

Optimization techniques usually include an iterative process which determines the solution for a given problem.

A. Lambert's Problem

Lambert's problem is aimed at determining the velocity vectors from the given position vectors (r_1, r_2) of a vehicle in any orbit [5].

Through Newton's iterative process, the value of z can be accurately determined. This z value can be used for further calculations where 'A' is a constant determined by the true anomaly and the magnitude of position vectors. The functions used are:

$$F(z) = \left[\frac{y(z)}{C(z)}\right]^{3/2} S(z) + A\sqrt{y(z)} - \sqrt{\mu}\,\Delta t \quad (10)$$

Where,

$$y(z) = r_1 + r_2 + A\frac{zS(z) - 1}{\sqrt{C(z)}}$$

$$S(z) = \frac{1}{6} - \frac{z}{120} \qquad C(z) = \frac{1}{2} - \frac{z}{24}$$

The Newton's iterative process is,

$$z_{i+1} = z_i - \frac{F(z_i)}{F'(z_i)} \quad (11)$$

After iterative process, the z value is introduced in the Lagrange variables given by,

$$f = 1 - \frac{y(z)}{r_1}$$

$$\dot{f} = \frac{\sqrt{\mu}}{r_1 r_2}\sqrt{\frac{y(z)}{C(z)}}\,[z\,S(z) - 1] \quad (12)$$

$$g = A\sqrt{\frac{y(z)}{\mu}}$$

$$\dot{g} = 1 - \frac{y(z)}{r_2}$$

Now these Lagrange variables can be directly introduced to determine the velocity vector.

$$v_1 = \frac{1}{g}\left(\vec{r}_2 - f\,\vec{r}_1\right) \quad (13)$$

$$v_2 = \frac{1}{g}\left(\dot{g}\,\vec{r}_2 - \vec{r}_1\right)$$

After determining the velocity vector the orbital elements can be easily determined. The orbital elements are Semimajor Axis (a), Eccentricity (e), RAAN (Ω), Argument of Perigee (ω), Inclination (i) and True anomaly (θ) as indicated in the figure 8.

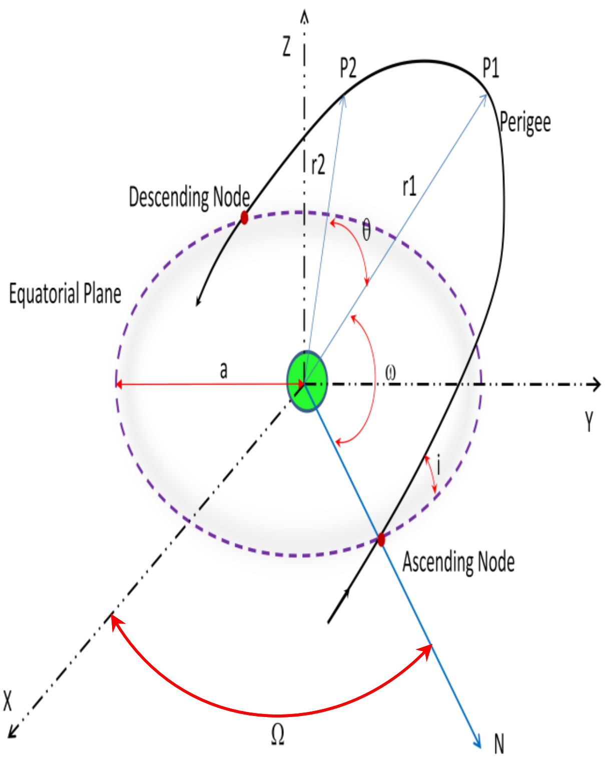

Fig 8: Orbital Elements

B. Travelling Salesman Problem

TSP is a logical approach towards the complex problem. The minimal path can be found using this technique. For example, to visit 5 cities in the most optimal way as illustrated in figure 9, we need to choose the most efficient path from the combination of all the paths. Branch and bound algorithm is the method used for this purpose.

The branch and bound algorithm consists of a number of constraints. It eliminates the chance of sub tours between the cities which creates more complexity to the problem. The following diagram shows one of the possible paths to travel between the cities.

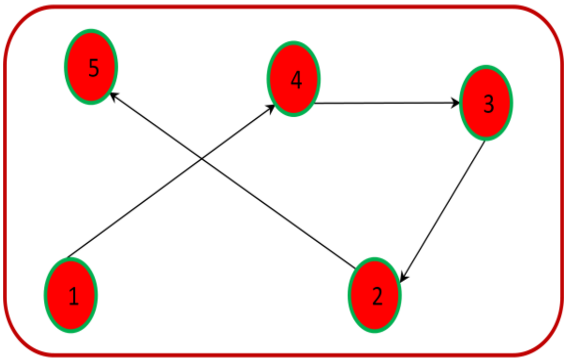

Fig 9: Travelling Salesman Problem

In the case of space object, the condition becomes more complex, because the distance between space objects vary with time increment.

TABLE I. Velocity increment for various maneuvers

Maneuver	Conditions	Velocity Increment Required (km/s)
Hohmann transfer	First orbit apogee radius = 11000 km First orbit perigee radius = 9000 km Second orbit radius = 20000 km	1.7832
Bi-elliptic hohmann transfer	First orbit radius = 10000 km Second orbit radius = 20000 km First elliptical transfer orbit apogee radius = 15000 km	1.834
Apse line rotation	First orbit apogee radius =15000 km First orbit perigee radius = 9000 km Second orbit apogee radius =12000 km Second orbit perigee radius =8000 km Apse line rotation angle=45 degree	0.6256
Phasing Maneuver	First orbit apogee radius =11000 km First orbit perigee radius = 9000 km Second orbit true anomaly= 90 degree	1.0587
Chasing maneuver	Apogee radius =20000 km Perigee radius =10000 km True anomaly of chaser = 45 degree True anomaly of target = 150degree Time required for the chase = 1 hr	12.6817
Non-hohmann transfer with common apse line	Enter the apogee of initial orbit 20000 Enter the perigee of initial orbit 10000 True anomaly of initial orbit with respect to apse line 45	20.8669

IV. Electrodynamic Tethers

Electrodynamic tethers are long conducting wires which can be used in satellites for an economic maneuvering process as shown in figure 10.

The basic principle of tethered satellite is that of a motor. That is the conversion of electric potential into kinetic energy in the presence of a magnetic field. In real case the electric potential across the tether in the presence of earth's magnetic field generates a Lorentz force.

The magnetic field around the earth is affected by so many factors. Among them the main factors are,

- Magnetic and rotational tilt of earth
- Solar wind effect

To determine the magnetic field action Euler-Hill reference frame is considered and the magnetic field components are noted [4].

In order to set the orbital elements, the basic parameters required are, the semi lactus rectum, specific angular momentum and the radius of the orbit. The parameters are given by,

$$p = a\left(1-e^2\right) \tag{14}$$

$$h = \sqrt{\mu p} \tag{15}$$

$$r = \frac{p}{\left(1+e\cos(\nu)\right)} \tag{16}$$

The components of magnetic field vector can be taken as,

$$B_i = -2\left(\frac{\mu_m}{r^3}\right)\sin(\omega+\theta)\sin(i)$$

$$B_j = \left(\frac{\mu_m}{r^3}\right)\cos(\omega+\theta)\sin(i) \tag{17}$$

$$B_k = \left(\frac{\mu_m}{r^3}\right)\cos(i)$$

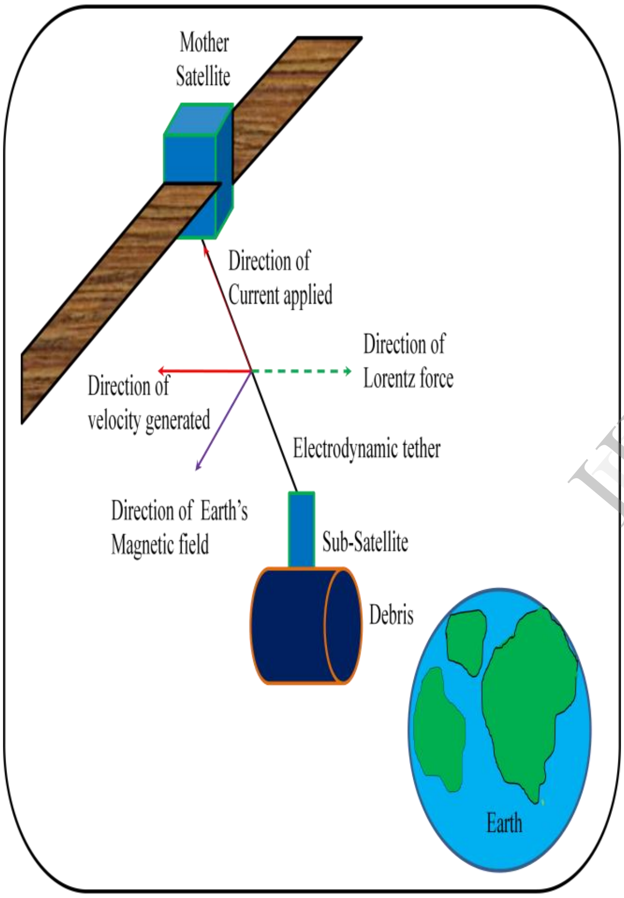

Fig 10: Electrodynamic Tethers

Components of perturbative acceleration act along the radial, tangential and normal directions of the orbit.

Thus for a non-liberating EDT, the perturbative accelerations are,

$$f_r = \left(\frac{IL}{m}\right)\left(B_k\sin(\theta)\cos(\varphi) - B_j\sin(\varphi)\right)$$

$$f_\theta = \left(\frac{IL}{m}\right)\left(B_i\sin(\varphi) - B_k\cos(\theta)\cos(\varphi)\right) \tag{18}$$

$$f_h = \left(\frac{IL}{m}\right)\left(B_j\cos(\theta)\cos(\varphi) - B_i\sin(\theta)\cos(\varphi)\right)$$

In order to find the trajectory of satellite after the tether based perturbative acceleration, the rate at which the orbital elements change over a period of time is found.

By using the Gauss form of variational equation,

$$\frac{da}{dt} = \left(\frac{2a^2}{h}\right)\left[e\sin(\theta)f_r + \frac{p}{r}f_t\right]$$

$$\frac{de}{dt} = \left(\frac{1}{h}\right)\left\{p\sin(\theta)f_r + \left[(p+r)\cos(\theta) + re\right]f_t\right\}$$

$$\frac{d\omega}{dt} = \left(\frac{1}{he}\right)\left[-p\cos(\theta)f_r + (p+r)\sin(\theta)f_t\right] - \left(\frac{r\sin(\omega+\theta)\cos(i)}{h\sin(i)}\right)f_h \tag{19}$$

$$\frac{d\Omega}{dt} = \left(\frac{r\sin(\omega+\theta)}{h\sin(i)}\right)f_h$$

$$\frac{di}{dt} = \left(\frac{r\cos(\omega+\theta)}{h}\right)f_h$$

$$\frac{d\theta}{dt} = \left(\frac{h}{r^2}\right) + \left(\frac{1}{he}\right)\left[p\cos(\theta)f_r - (p+r)\sin(\theta)f_t\right]$$

The rate of change in orbital parameters can be found out using variational method. All the six orbital elements of initial orbit are essential in further calculation.

The rate of orbital element change with time can be used over a period of time to get the exact position of the spacecraft.

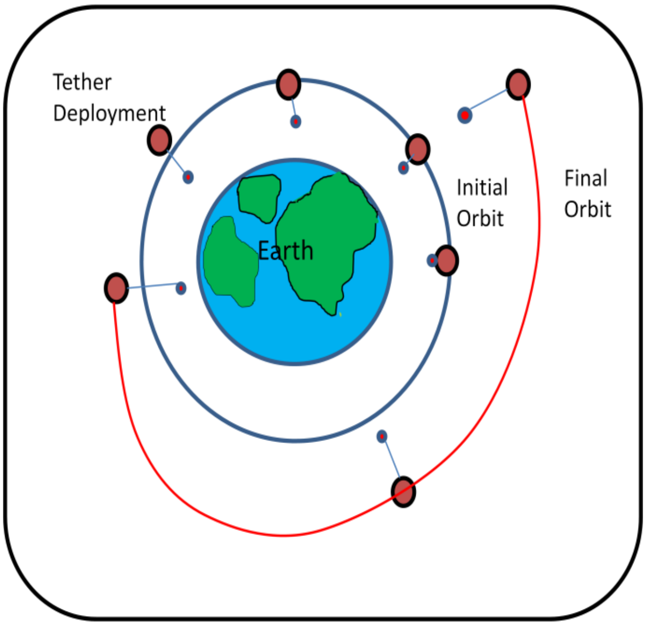

Fig 11: Tether Deployment

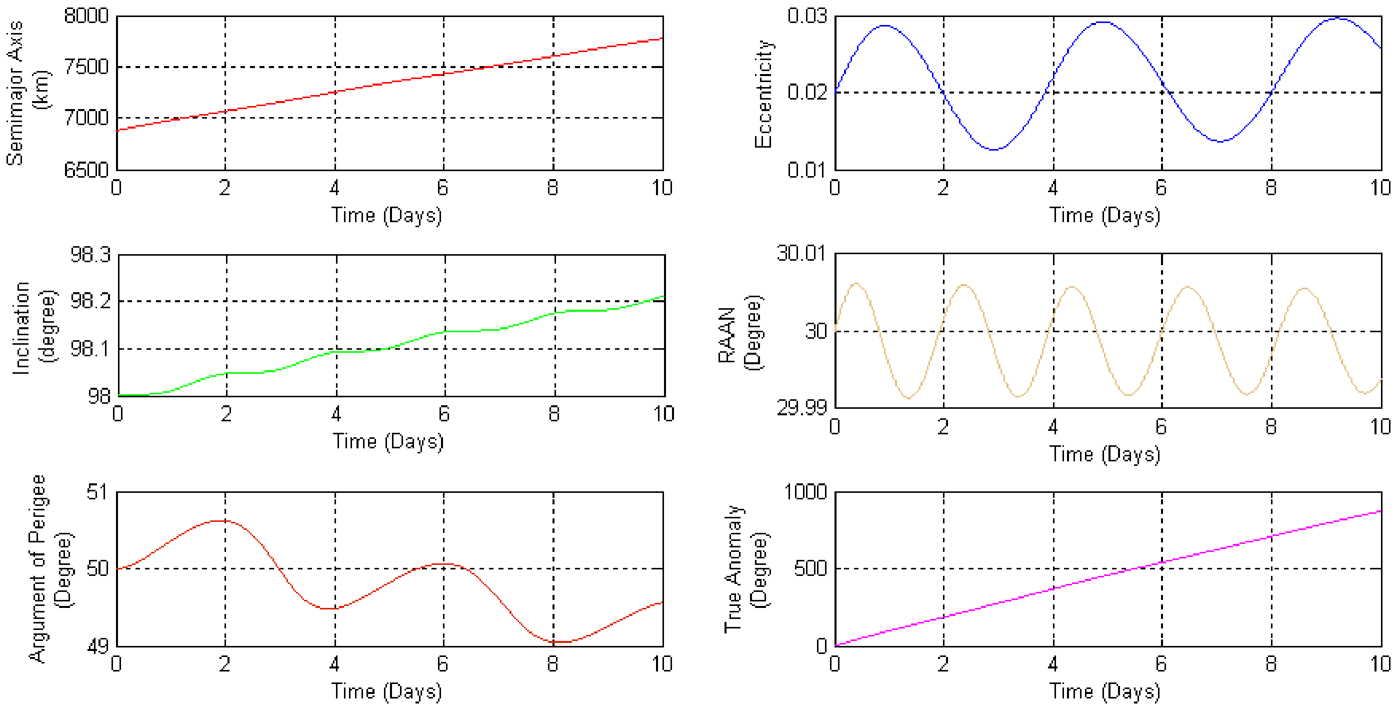

Fig 12: Orbital element variation generated in Matlab

For a given, mass of satellite = 420kg ; Current applied = 5 A ; Liberation angle, θ = ϕ = 0 deg ;Length of tether = 15km; Semimajor axis = 6878km ; Eccentricity = 0.02;RAAN= 30 deg ; Inclination= 98 deg ; Argument of perigee = 50 deg; True anomaly = 0 deg; Time = 10 days

The result shown in the figure 12 was generated using ode45 in Matlab. Ode45 is used to solve the ordinary equations with given initial values. Orbital element variation over the orbits with time increment is determined from the graph obtained.

The Semimajor axis shows huge variation considering other orbital elements. Over a period of 10 days, the semimajor axis varies by a distance of above 1000km.

The eccentricity shows slight variation, which shows that the satellite will still be in an almost circular orbit.

The true anomaly shows much higher variation because of the position change over time.

RAAN rate has based on natural perturbations are not considered here. Only effect of the EDT is considered. The inclination is the angle created between the equatorial plane and the satellite orbital plane which also shows slight change during the process.

V. FUTURE APPLICATION

- Orbital debris removal in the most cost effective way.
- Refueling of active satellites in the earth orbit.
- For maintenance of active satellites in space.
- Kessler Syndrome can be avoided.
- To remove the potentially risk debris from the path of active satellites to avoid collision.

VI. CONCLUSION

The space debris has become a huge threat to the future space missions. Removing the potentially risk debris will avoid the risk of collision with active satellites. NASA is now researching the scope of electrodynamic tethers in this field. In this work, the travel between one orbit to another using electrodynamic tethers is determined. The result shows that within a short span of time tethers can maneuver the satellite. Huge velocity increment required in conventional method can be avoided. As a future work, TSP inclusion into the work is under process. By including TSP any number of debris can be visited by the spacecraft in optimal way.

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