

Performance Analysis Of Block Diagonalization And Dirty Paper Coding Precoding Technique In Multi User Mimosystem

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Abstract-In the present papera we analyses performance of multi user Multiple-input multiple-output (MIMO) systems which has emerged recently as an important research topic. We check bit error rate(BER) vs. SNR performance for two algorithms which are Block-diagonalization and dirty paper coding precoding technique to cancel the interference cancellation broadcast channel.

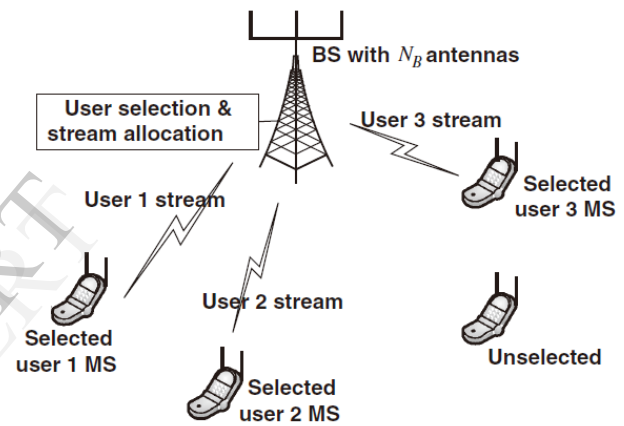
Key words-multi user mimo, broadcast channel, block-diagonalisation, dirty paper coding

I. INTRODUCTION

Multiple-input multiple-output (MIMO) Communication techniques have been an Important area of focus for next-generation Wireless systems because of their potential for high capacity, increased diversity, and interference Suppression. For applications such as Wireless LANs and cellular telephony, MIMO Systems will likely be deployed in environments Where a single base must communicate With many users simultaneously. As a result, the study of multi-user MIMO systems has emerged recently as an important research topic. Such systems have the potential to combine the high capacity achievable with MIMO Processing with the benefits of space-division Multipleaccesses. [1-4]we know that the channel capacity of the single user mimo with $N_r \times N_t$ mimo systems is proportional to $N_{min} = \min(N_t, N_r)$ [5]. In the single user mimo system ,a point to point high data rate transmission can be supported by spatial multiplexing while providing spatial diversity gain. However ,most communication systems deal with multiple users who are sharing the same radio resources.

Fig below illustrates a typical multi user mimo communication environment in whichthe multiple mobile stations are served by a single base station in the cellular

system. Suppose the base station and each mobile station is equipped with N_B and N_M antennas, respectively. As K independent users from a virtual set of $(K \cdot N_M)$ antennas which communicate with a single base station BS with N_B antennas, end to end Configuration can be considered as a $(K \cdot N_M) \times N_B$ MIMO system for downlink, $N_B \times (K \cdot N_M)$ MIMO system for uplink.



Multi user mimo communication system for $K=4$.

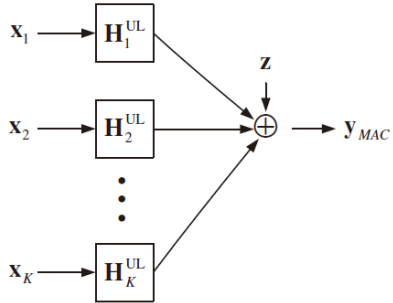
II.Mathematical model for Multi-user mimo system

A.Uplink channel model for multi user MIMO system; multiple access channel (MAC)

Consider K independent users in multi user MIMO system. We assume that the BS station and MS are equipped with N_B and N_M , respectively. Below fig .shows the Uplink channel known as a multiple access channel (MAC) for K independent users. Let $x_u \in \mathbb{C}^{N_M \times 1}$ and $y_{MAC} \in \mathbb{C}^{N_B \times 1}$ denote the transmit signal from the u th user , $u=1,2,\dots,K$, and the received signal at the received signal at the BS respectively. The channel gain between the u th user MS and BS is represented by $H_u^{UL} \in \mathbb{C}^{N_B \times N_M}$, $u=1,2,\dots,K$. The received signal is expressed as

$$y_{MAC} = \mathbf{H}_1^{UL}x_1 + \mathbf{H}_2^{UL}x_2 + \dots + \mathbf{H}_K^{UL}x_K + z$$

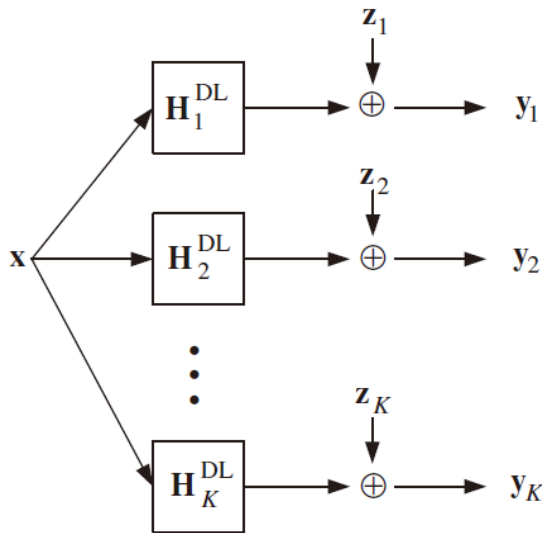
$$= \underbrace{\begin{bmatrix} \mathbf{H}_1^{UL} & \mathbf{H}_2^{UL} & \dots & \mathbf{H}_K^{UL} \end{bmatrix}}_{=\mathbf{H}^{UL}} \begin{bmatrix} x_1 \\ \vdots \\ x_K \end{bmatrix} + z = \mathbf{H}^{UL} \begin{bmatrix} x_1 \\ \vdots \\ x_K \end{bmatrix} + z$$



Uplink channel model for the multi user MIMO system: multiple access channel (MAC)

Now on the other hand, below fig. shows the downlink channel, known as Broadcast channel (BC) in which $\mathbf{X} \in \mathbb{C}^{NB \times 1}$ is the transmit signal from the BS and $\mathbf{y}_u \in \mathbb{C}^{NM,u \times 1}$ is the received signal at the u th user is expressed as

$$\mathbf{y}_u = \mathbf{H}_u^{DL} \mathbf{X} + \mathbf{Z}_u, u=1,2,\dots,K \text{ [6]}$$



Downlink channel model for multi user MIMO system: broadcastchannel (BC)

Where $\mathbf{Z}_u \in \mathbb{C}^{NM,u \times 1}$ is the additive white Gaussian noise at the u the user.

Representing all user signals by a single vector, the overall system can be represented as

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_K \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}_1^{DL} \\ \mathbf{H}_2^{DL} \\ \vdots \\ \mathbf{H}_K^{DL} \end{bmatrix}}_{\mathbf{H}_{DL}} \mathbf{x} + \underbrace{\begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_K \end{bmatrix}}_z$$

III. CONFIGURATION AND ANALYSIS OF BLOCK-DIAGONALIZATION AND DIRTY PAPER CODING PRECODING TECHNIQUES

In Block-diagonalization method is applicable to multiple users, each with multiple antennas to remove inter-antenna interference in its own signal as well as other user signal also cancelled noise enhancement of target user perspective. Let $N_{M,u}$ denote the number of antenna for the u th user , $u=1,2,\dots,k$, for the u the user signal $X_u \in \mathbb{C}^{NM,u \times 1}$, the received signal $y_u \in \mathbb{C}^{NM,u \times 1}$ given as

$$\mathbf{y}_u = \mathbf{H}_u^{DL} \sum_{k=1}^K \mathbf{W}_k \tilde{x}_k + \mathbf{z}_u$$

$$= \mathbf{H}_u^{DL} \mathbf{W}_u \tilde{x}_u + \sum_{k=1, k \neq u}^K \mathbf{H}_u^{DL} \mathbf{W}_k \tilde{x}_k + \mathbf{z}_u$$

Where $\mathbf{H}_u^{DL} \in \mathbb{C}^{NM,u \times NB}$ is the channel matrix between BS and the u th user, $\mathbf{W}_u \in \mathbb{C}^{NB \times NM,u}$ is the precoding matrix for the u th user and \mathbf{Z}_u is the noise vector. Consider the received signals for three user case (i.e. $K=3$),

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}_1^{DL} & \mathbf{H}_1^{DL} & \mathbf{H}_1^{DL} \\ \mathbf{H}_2^{DL} & \mathbf{H}_2^{DL} & \mathbf{H}_2^{DL} \\ \mathbf{H}_3^{DL} & \mathbf{H}_3^{DL} & \mathbf{H}_3^{DL} \end{bmatrix}}_{\mathbf{H}_{DL}} \underbrace{\begin{bmatrix} \mathbf{W}_1 \tilde{x}_1 \\ \mathbf{W}_2 \tilde{x}_2 \\ \mathbf{W}_3 \tilde{x}_3 \end{bmatrix}}_x + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{H}_1^{DL} \mathbf{W}_1 & \mathbf{H}_1^{DL} \mathbf{W}_2 & \mathbf{H}_1^{DL} \mathbf{W}_3 \\ \mathbf{H}_2^{DL} \mathbf{W}_1 & \mathbf{H}_2^{DL} \mathbf{W}_2 & \mathbf{H}_2^{DL} \mathbf{W}_3 \\ \mathbf{H}_3^{DL} \mathbf{W}_1 & \mathbf{H}_3^{DL} \mathbf{W}_2 & \mathbf{H}_3^{DL} \mathbf{W}_3 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

Where $\{\mathbf{H}_u^{DL} \mathbf{W}_k\}$ from the an effective channel matrix for the u th user receiver and the kth - user transmit signal($u,k=1,2,\dots,K$).

In channel matrix interference free transmission possible if above eq. can be Block- diagonalized , that is

$$\mathbf{H}_u^{DL} \mathbf{W}_k = \mathbf{0}_{NM,u \times NM,u}, \forall u \neq k$$

So , now we get

$$\mathbf{y}_u = \mathbf{H}_u^{DL} \mathbf{W}_u \tilde{x}_u + \mathbf{z}_u, u = 1, 2, \dots, K$$

Once we construct the interference- free signals in eq. various signal detection can be used to estimate \hat{X}_u .

So

$$\tilde{\mathbf{H}}_u^{DL} = \left[(\mathbf{H}_1^{DL})^H \cdots (\mathbf{H}_{u-1}^{DL})^H (\mathbf{H}_{u+1}^{DL})^H \cdots (\mathbf{H}_K^{DL})^H \right]^H$$

Which is equivalent to

$$\tilde{\mathbf{H}}_u^{DL} \mathbf{W}_u = \mathbf{0}_{(N_{M,total}-N_{M,u}) \times N_{M,u}}, \quad u = 1, 2, \dots, K$$

Received signal in case of K= 3 is expressed as

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1^{DL} \mathbf{W}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_2^{DL} \mathbf{W}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{H}_3^{DL} \mathbf{W}_3 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

Which is the appropriate dimension?

$$\tilde{\mathbf{H}}_u^{DL} = \tilde{\mathbf{U}}_u \tilde{\Lambda}_u \begin{bmatrix} \tilde{\mathbf{V}}_u^{\text{non-zero}} & \tilde{\mathbf{V}}_u^{\text{zero}} \end{bmatrix}^H$$

Now multiplying

$$\begin{aligned} \tilde{\mathbf{H}}_u^{DL} \tilde{\mathbf{V}}_u^{\text{zero}} &= \tilde{\mathbf{U}}_u \begin{bmatrix} \tilde{\Lambda}_u^{\text{non-zero}} & \mathbf{0} \end{bmatrix} \begin{bmatrix} (\tilde{\mathbf{V}}_u^{\text{non-zero}})^H \\ (\tilde{\mathbf{V}}_u^{\text{zero}})^H \end{bmatrix} \tilde{\mathbf{V}}_u^{\text{zero}} \\ &= \tilde{\mathbf{U}}_u \tilde{\Lambda}_u^{\text{non-zero}} (\tilde{\mathbf{V}}_u^{\text{non-zero}})^H \tilde{\mathbf{V}}_u^{\text{zero}} \\ &= \tilde{\mathbf{U}}_u \tilde{\Lambda}_u^{\text{non-zero}} \mathbf{0} \\ &= \mathbf{0} \end{aligned}$$

From above equation we $\overline{\mathbf{V}}_u^{\text{zero}}$ is the null space of $\overline{\mathbf{H}}_u^{DL}$

When the signal is transmitted in the $\overline{\mathbf{V}}_u^{\text{zero}}$, all but the u th user receives no signal at all. [9-10]. Thus $\mathbf{W}_u = \overline{\mathbf{V}}_u$ can be used for the precoding the u th user signal.

Dirty paper coding (DPC)

In this precoding technique an interference free transmission can be realized by subtracting the potential interference before transmission.

Dpc is implemented when the channel gains are completely known on the transmitter side.

To simply understand for K=3 user

Received signal is given as

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}_1^{DL} \\ \mathbf{H}_2^{DL} \\ \mathbf{H}_3^{DL} \end{bmatrix}}_{\mathbf{H}^{DL}} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

The channel matrix \mathbf{H}^{DL} can be LQ- decomposed as

$$\mathbf{H}^{DL} = \underbrace{\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}}_{\mathbf{L}} \underbrace{\begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \\ \mathbf{q}_3 \end{bmatrix}}_{\mathbf{Q}}$$

Where $\mathbf{H}_u^{DL} \in \mathbb{C}^{1 \times 3}$ is the channel matrix between BS and the u th user. [11-12]

By transmitting $\mathbf{Q}^H \mathbf{x}$ the effect of Q in eq. is eliminated through the channel. Leaving lower-triangular matrix after transmission, the received signal is given as [8]

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}_1^{DL} \\ \mathbf{H}_2^{DL} \\ \mathbf{H}_3^{DL} \end{bmatrix}}_{\mathbf{H}^{DL}} \mathbf{Q}^H \mathbf{x} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

So received signal of the first user is given as

$$\mathbf{Y}_1 = l_{11} \mathbf{x}_1 + z_1$$

so from the first – user perspective, therefore, the following condition needs to be met for the interference – free data transmission:

$$\mathbf{x}_1 = \bar{\mathbf{x}}_1$$

From eq it can be seen that the precoded signal x_1 is solely composed of the first user signal \bar{x}_1 .

Now received signal for the second user is given as

$$\begin{aligned} \mathbf{Y}_2 &= l_{21} x_1 + l_{22} x_2 + z_2 \\ &= l_{21} \bar{x}_1 + l_{22} x_2 + z_2 \end{aligned}$$

So, we find precoding cancels the interference component $l_{21} x_1$ on the transmitter side:

$$x_2 = \tilde{x}_2 - \frac{l_{21}}{l_{22}} x_1 = \tilde{x}_2 - \frac{l_{21}}{l_{22}} \bar{x}_1$$

Similarly we find received signal for K =3 user

$$\mathbf{Y}_3 = l_{31} x_1 + l_{32} x_2 + l_{33} x_3 + z_3$$

Here , the precoded signals x_1 and x_2 , are interference components in eq. , which can be canceled by the following precoding on the transmitter side:

$$x_3 = \tilde{x}_3 - \frac{l_{31}}{l_{33}} x_1 - \frac{l_{32}}{l_{33}} x_2$$

Now the precoding signals in equations are

$$\begin{bmatrix} x_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ \tilde{x}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{l_{21}}{l_{22}} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix},$$

and

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{l_{31}}{l_{33}} & -\frac{l_{32}}{l_{33}} & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \tilde{x}_3 \end{bmatrix}$$

Combining the above all eq. precoding matrices, we can express dpc in the following matrix form:

$$\begin{aligned} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{l_{31}}{l_{33}} & -\frac{l_{32}}{l_{33}} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -\frac{l_{21}}{l_{22}} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ -\frac{l_{21}}{l_{22}} & 1 & 0 \\ -\frac{l_{31}}{l_{33}} + \frac{l_{32} l_{21}}{l_{33} l_{22}} & -\frac{l_{32}}{l_{33}} & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix}. \end{aligned}$$

Using above precoding matrix, eq. can be re-written as

$$\begin{aligned} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} &= \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \\ &= \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -\frac{l_{21}}{l_{22}} & 1 & 0 \\ -\frac{l_{31}}{l_{33}} + \frac{l_{32} l_{21}}{l_{33} l_{22}} & -\frac{l_{32}}{l_{33}} & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \\ &= \begin{bmatrix} l_{11} & 0 & 0 \\ 0 & l_{22} & 0 \\ 0 & 0 & l_{33} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ x_2 \\ \tilde{x}_3 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \end{aligned}$$

Now interference free detection can be made for each user. We can see from equation that precoding matrix in dpc is a scaled inverse matrix of the lower triangular matrix which is obtained from the channel gain matrix, that is

$$\begin{bmatrix} 1 & 0 & 0 \\ -\frac{l_{21}}{l_{22}} & 1 & 0 \\ -\frac{l_{31}}{l_{33}} + \frac{l_{32} l_{21}}{l_{33} l_{22}} & -\frac{l_{32}}{l_{33}} & 1 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}^{-1} \begin{bmatrix} l_{11} & 0 & 0 \\ 0 & l_{22} & 0 \\ 0 & 0 & l_{33} \end{bmatrix}$$

IV.DESIGN AND SPECIFICATIONS

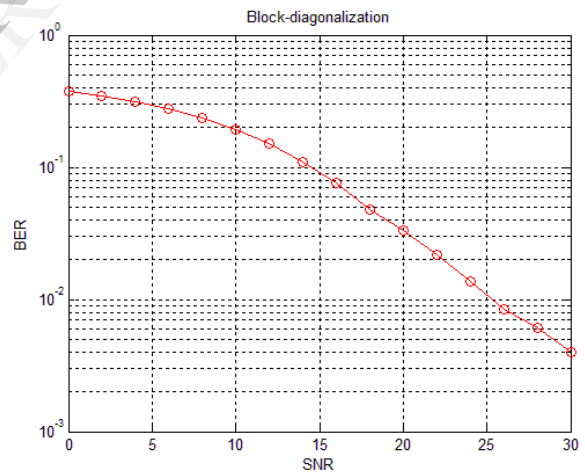
TABLE.1 Design Specifications for BD and DPC algorithm for multi user MIMO system.

NO. FRAMES	10
NO. PACKETS	100
NO.BITS PER QPSK	2
SYMBOL	
NO. BS. ANTENNA	4
NO. MS. ANTENNA	4
NO.ACT_USER	4/10

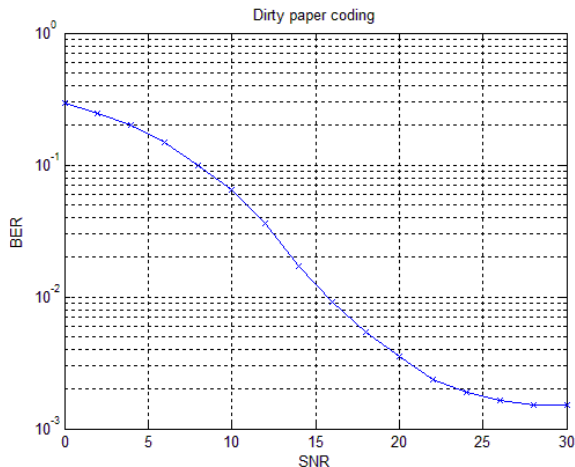
V. RESULTS AND DISCUSSION

We analysis bit error rate vs snr performance for two algorithm Block –diagonalization and dirty paper coding precoding techniques in using matlab tool.

We plot the below graph for the same.



And for dirty paper coding graph are given in below fig.



It is observed that the Block-diagonalization and dirty paper precoding technique gives bit error rate performance BER values **0.04 or 4%** bit error in Block-diagonalization and Values **0.01 or 1%** bit error in DPC at SNR of **20 db**.

VI. CONCLUSION

From the analysis it is concluded that we get better performance in terms of BER in dirty paper coding compares to Block-diagonalization precoding technique for multi user MIMO SYSTEM. Future problem we also measures BER using other algorithms like THP or also measures comparison of this precoding techniques for BER vs. SNR graph.

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