

# Performance Analysis of Multisplit Time Varying LMS Algorithm

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**Abstract**—In this paper multisplit time varying LMS algorithm (MS TVLMS) is proposed and its performance is compared with multisplit LMS (MS LMS) and affine combination of two LMS adaptive filters. Two performance criteria are utilized: minimum mean square error (MSE) and convergence rate. MS TVLMS utilizes the idea of split filtering with linearly constrained optimization scheme. Then multi split adaptive filter is obtained by introducing continuous split procedure that results into a Hadamard domain adaptive filter. Simulation results with proposed algorithm are presented and compared with MS LMS and affine combination. Moreover proposed algorithm has minimum mean square error and better convergence rate as compared with MS LMS and affine combination.

**Keywords**—TV LMS, MS LMS, affine combination

## I. INTRODUCTION

LMS algorithm is one of the most widely used algorithms for adaptive signal processing because of its simplicity and robustness. But its performance in terms of convergence rate and tracking ability depends on eigen value of input signal correlation matrix. Split adaptive filter has evolved as a better solution to improve convergence rate and reduce computational burden. Fundamental principles were proposed in [1] for real Toeplitz matrices. Subsequently the same technique was extended to classical algorithms in linear prediction theory [2]. K.C. Ho and P.C. Ching proposed split LMS adaptive filter for AR modeling [3] and then P.C. Ching and K.F. Wan generalized it to a so called unified approach [4], [5] by introduction of continuous splitting and corresponding application to a general transversal filtering problem.

This paper is organized as follows: Section II recalls principles of transversal filter and its connection with hadamard transform. Multisplit adaptive filter is considered and is updated with TV LMS. Simulation results are presented in Section IV. Conclusions are reported in Section V.

## II. MULTISPLIT AND HADAMARD TRANSFORM

Consider the classical scheme of an adaptive transversal filter as shown in Figure 1, in which the  $N$ -by-1 tap-weight vector of the filter  $\mathbf{w}(n) = [w_0(n), \dots, w_{N-1}(n)]$  has been split into its symmetric and antisymmetric parts:

$$\mathbf{w}(n) = \mathbf{w}_s(n) + \mathbf{w}_a(n) \quad (1)$$

where  $\mathbf{w}_s(n) = \frac{1}{2} [\mathbf{w}(n) + \mathbf{J}\mathbf{w}(n)]$ ,  $\mathbf{w}_a(n) = \frac{1}{2} [\mathbf{w}(n) - \mathbf{J}\mathbf{w}(n)]$  and  $\mathbf{J}$  is the reflection matrix.

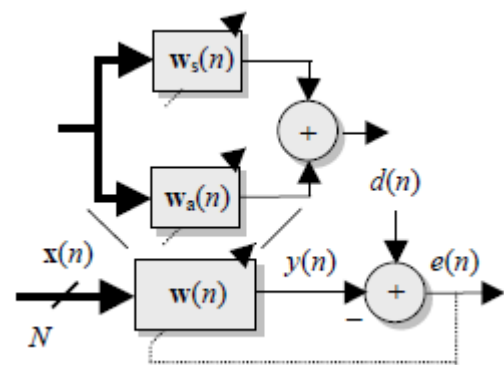


Fig.1. Split Adaptive transversal filtering

GSC structure [6] with symmetry and antisymmetry constraints the split transversal filtering scheme in Figure turns into the form represented by Figure 2.

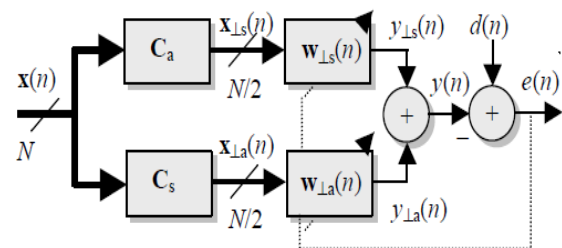


Figure 2. GSC implementation of split filter

Let  $N=2^M$ , where  $M$  is an integer number greater than one. Now, if each branch in Figure 1 is considered separately, the transversal filters  $\mathbf{w}_s(n)$  and  $\mathbf{w}_a(n)$  can also be split into their symmetric and antisymmetric parts [6].

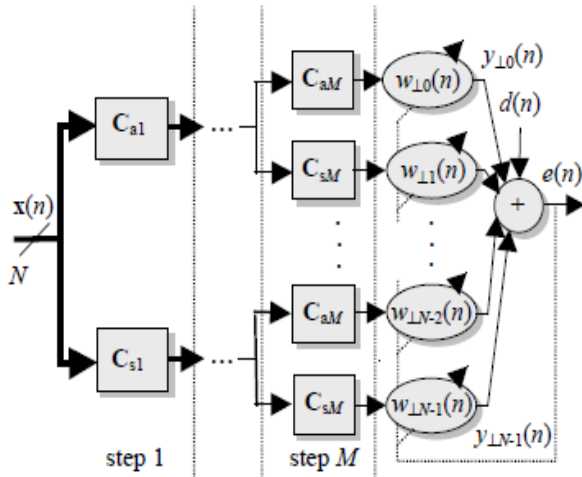


Figure 3. Multisplit adaptive filtering

The above multi-split scheme can be viewed as a linear transformation of  $x(n)$  denoted by

$$x_L(n) = T x(n) \tag{2}$$

where

$$T = \begin{bmatrix} C_{aM}^t & C_{aM-1}^t & \dots & C_{a1}^t \\ C_{sM}^t & C_{sM-1}^t & \dots & C_{s1}^t \\ \vdots & \vdots & \ddots & \vdots \\ C_{sM}^t & C_{sM-1}^t & \dots & C_{s1}^t \end{bmatrix}^t \tag{3}$$

Above transform results into Hadamard transform[6] as shown below

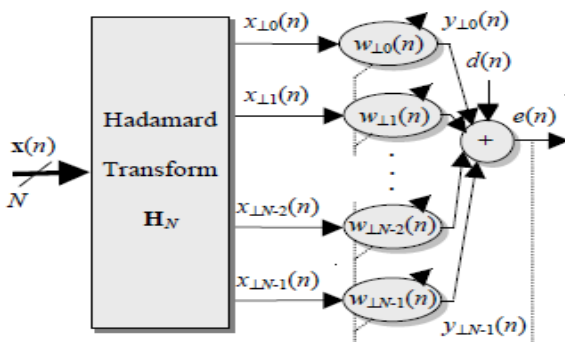


Figure 4. Hadamard transform of input  $x(n)$

Time varying LMS can be applied for updating the parameter With no increase in computational complexity.

Hadamard matrix of order  $2M$  is constructed as follows:

$$H_{2M} = \begin{bmatrix} H_M & H_M \\ H_M & -H_M \end{bmatrix} \tag{4}$$

MS TV LMS algorithm:

Initialization:

For  $i=0,1,\dots,N-1,\dots$  set  $w_i(0)=0$

Updating:

1) T transform on input  $x(n)$

2) TV LMS algorithm:

$$y(n) = W(n-1) x^T(n) \tag{5}$$

$$e(n) = d(n) - y(n) \tag{6}$$

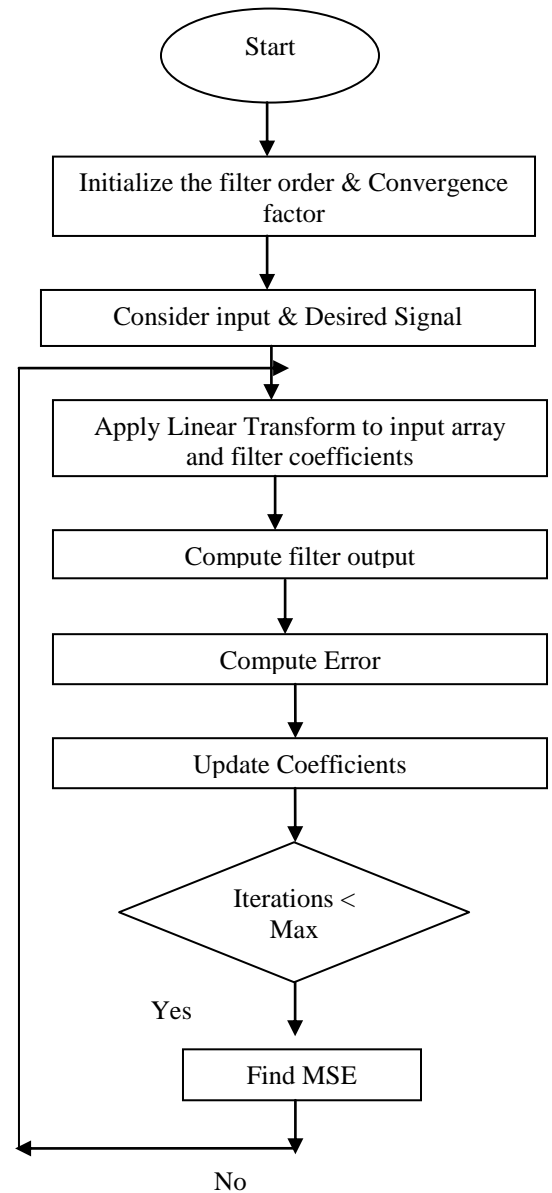
$$W(n) = W(n-1) + \mu_n e(n) x(n) \tag{7}$$

$$\alpha_n = C \frac{1}{1 + an^b} \tag{8}$$

$$\mu_n = \mu_0 * \alpha_n \tag{9}$$

### III. SIMULATION RESULTS

#### Simulation flowchart



#### Simulation Parameters

Input signal parameters:

Amplitude: 1

Frequency: 500Hz  
 Sampling Frequency: 10000Hz  
 Initial Phase: 0

Noise Parameters:

Amplitude: 0.15  
 Type: Gaussian  
 Mean: 0  
 Variance: 1  
 Initial Seed: 10

Filter Parameters

Filter Type: FIR  
 Order: 32  
 Structure: Direct form-I  
 Window: Rectangular  
 No. Of Iterations: variable  
 Convergence Factor: time varying

Desired signal parameters:

Sinusoidal signal of 500Hz frequency with amplitude 1.

Table 1: MSE for different algorithms

Iterations	MS LMS $\mu=0.05$	MS TVLMS $\mu=0.05$ $C=4, a=1,$ $b=0.7$	Affine Stochastic	Affine Error
100	0.0352	0.0256	0.0324	0.0289
200	0.0178	0.0133	0.0165	0.0148
300	0.0120	0.0092	0.0112	0.0100
400	0.0091	0.0071	0.0085	0.0077
500	0.0074	0.0058	0.0069	0.0063
600	0.0062	0.0050	0.0058	0.0053
700	0.0054	0.0044	0.0051	0.0047
800	0.0048	0.0039	0.0045	0.0042
900	0.0043	0.0035	0.0041	0.0038
1000	0.0039	0.0032	0.0037	0.0034

From above table it can be observed that MSE of MS TVLMS is less than MS LMS and affine combination.

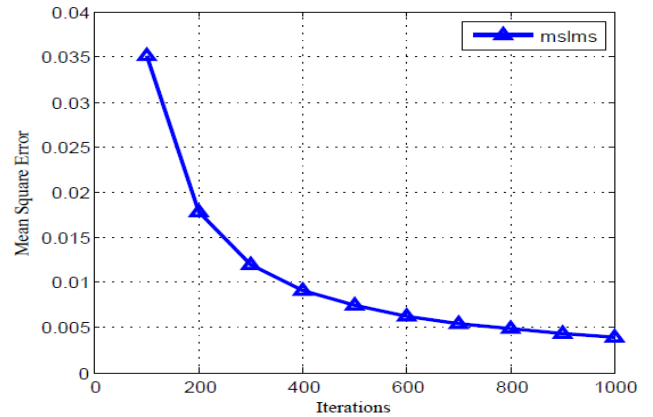


Figure 5. Convergence behavior of MS LMS

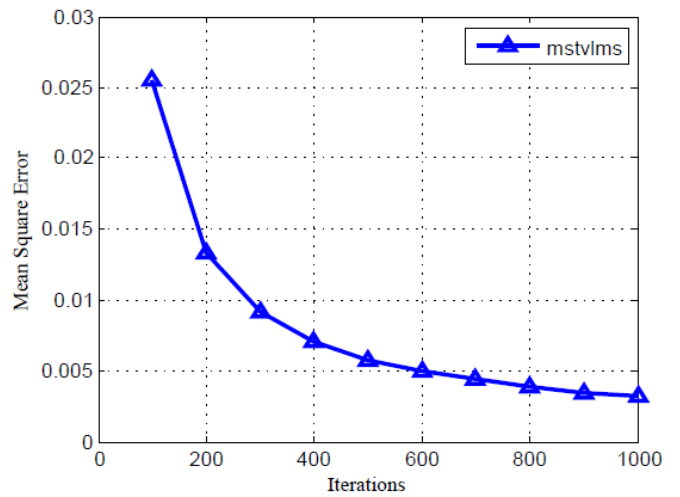


Figure 6. Convergence behavior of MSTV LMS

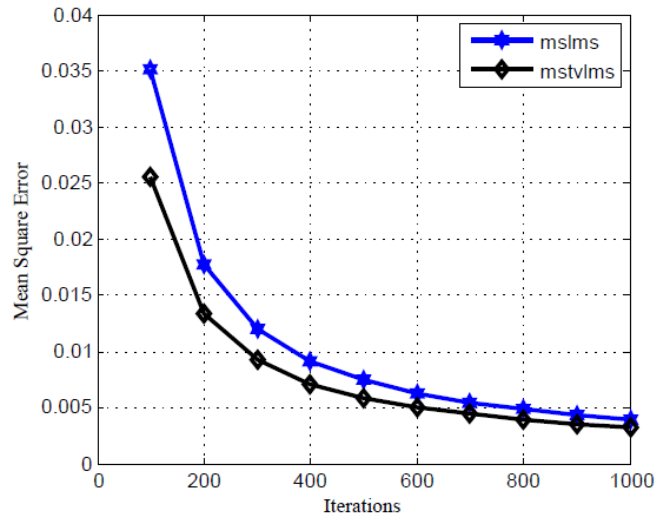


Figure 7. Convergence behavior of MSTV LMS and MS LMS

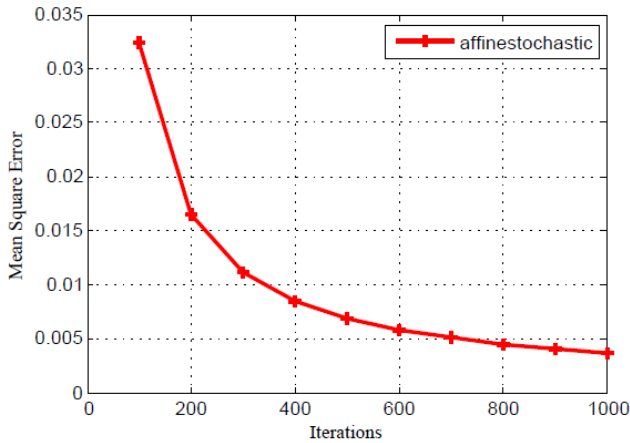


Figure 8. Convergence behavior of Affine Stochastic

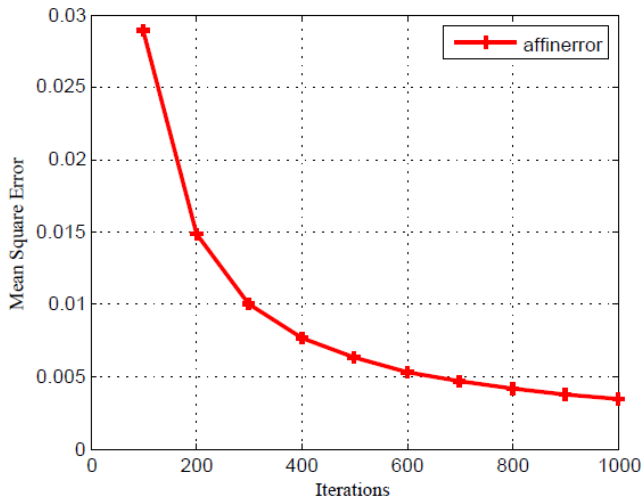


Figure 9. Convergence behavior of Affine Error

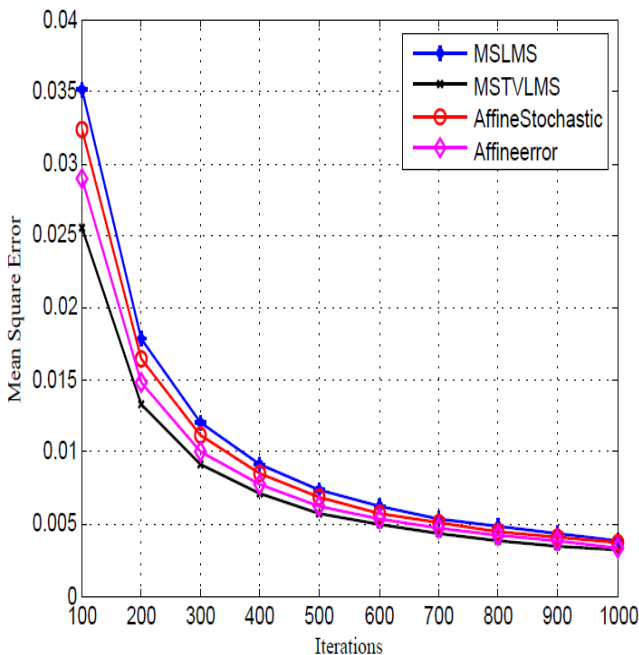


Figure 10. Comparison of MS LMS, MS TVLMS and affine combination

From above graphs it can be observed that MS TV LMS has minimum MSE and better convergence rate as compared to MS LMS and affine combination of two LMS adaptive filters.

#### IV CONCLUSIONS AND FUTURE SCOPE

This paper studied the performance of MS TV LMS algorithm. Here, input vectors as well as filter coefficients are split as symmetric and asymmetric parts using Hadamard transform. Hadamard transform is a linear transform, which operates on time domain samples of input and impulse response of filter. Simulation results show the better performance of proposed algorithm over MS LMS and affine combination in terms of MSE and convergence rate.

The same procedure can also be repeated in frequency domain. The input vector is split as low frequency part and high frequency part, each part is separately applied adaptive filtering algorithm, which leads to sub band adaptive algorithm. As no transformation is required and only requires filter banks, it is less computationally burden.

#### V. REFERENCES

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