Performance Analysis of Non-Orthogonal Codes for CDMA Wireless Systems

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Abstract—The performance of a CDMA based wireless mobile systems is largely dependent on the characteristics of pseudo-random spreading codes. Thus, the choice of spreading codes is an important parameter to improve the performance of CDMA mobile systems. It is desirable to have a code dictionary consisting of spreading codes which possess both impulsive ACF and all zero CCF characteristics. Therefore, in this paper various non-orthogonal codes have been investigated with special emphasis on their peak as well as mean square correlation properties.

Keywords- CDMA; PN; Gold Codes; LFSR; ACF; CCF

I. INTRODUCTION

Nowadays mobile communication systems are penetrating at an exponential rate because these systems not only carry voice but also video and other high speed data information. Though the users can be identified on the basis of time and frequency but CDMA based mobile systems [1] prove to accommodate a large number of users without any overhead cost. Codes play an important role in the design of such mobile systems. These codes are not only used to differentiate the number of users but they also provide security to the system. In this paper we have evaluated the performance of the mobile system by using two non-orthogonal spreading codes i.e. PN and Gold codes.

A. Generation of Non-orthogonal Spreading Codes

Non-orthogonal spreading codes are those which give non-zero cross-correlation values for different time shifts. It results into multiple access interference and thus limits the maximum number of users supported by CDMA system. But these codes have ease of generation, randomness and good auto-correlation characteristics. In this section, the generation of non-orthogonal codes (PN codes and Gold Codes) using MATLAB programming is being discussed.

(a) Pseudo-Noise Codes

A pseudo-noise (PN) code is a periodic binary sequence generated using linear feedback shift register (LFSR) structure [2], [3] as shown in Figure 1. These codes are also known as Maximal length sequences (m-sequences).

The sequence a_i is generated according to the recursive formula [1]:

$$a_i = C_1 a_{i-1} + C_2 a_{i-2} + \dots\dots\dots\dots + C_n a_{i-n} = \sum_{k=1}^{n} C_k a_{i-k} \quad (1)$$

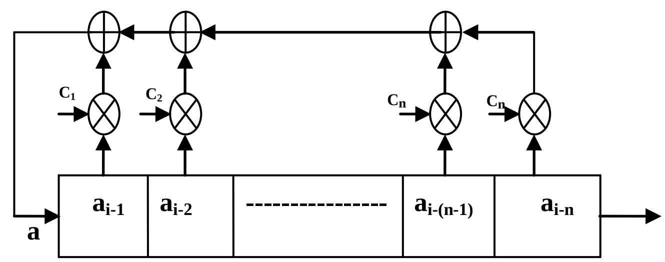

Fig1: PN-Sequence Generation

Here, all terms are binary (0 or 1) and addition and multiplications are modulo-2. The connection vector C_1, C_2,………C_n defines the characteristic polynomial of the linear feedback shift register (LFSR) sequence generator and determines the main characteristics of the generated sequence. The PN code so generated using LFSR with n number of flip-flops is periodic with period $N = 2^n-1$.

In this paper, the above mentioned generator structure was implemented through MATLAB programming. The PN codes having length N = 7 to 255 were generated and analyzed through programming. The feedback polynomials being used for the sequence generator structure to generate the above mentioned m-sequences are listed below:

7- Length PN sequence: X^3+X^2+1;
X^3+X+1 (Preferred)

15- Length PN sequence: X^4+X^3+1;
X^4+X+1 (Non-Preferred)

31- Length PN sequence: $X^5+ X^4+ X^3+X^2+1$;
$X^5+X^4+X^3+ X+1$(Preferred)

63- Length PN sequence: X^6+X+1;
$X^6+X^5+X^2+X+1$(Preferred)

127- Length PN sequence: X^7+X+1;
X^7+X^3+1 (Preferred Pair)

255- Length PN sequence: $X^8+X^6+X^5+X^3+1$;
$X^8+X^4+X^3+ X^2+1$
(Non-Preferred Pair)

Here, preferred pair of polynomials will generate PN codes having 3-valued cross-correlation function. The schematic used to generate one of the above PN sequences (N=127) is shown below in Figure 2. All the array elements *a[i]'s* are initialized to binary '1'. Then, with each clock cycle the feedback calculations (modulo-2 additions) are done, values in the array elements are shifted towards right and one element of the code sequence is obtained.

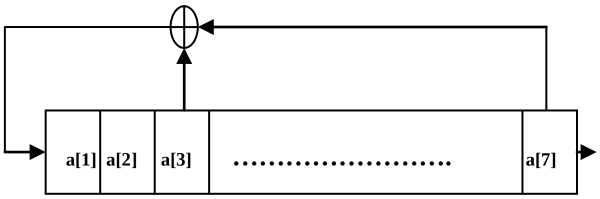

Fig 2: 127 Length PN(X^7+X^3+1)

It is thus observed that the generation structure of PN codes is very simple but family size of PN codes of different lengths is very small. As already mentioned, PN codes of each length are defined by their corresponding characteristic feedback polynomials. The PN code family size for lengths N = 7 to 1023 are tabulated in Table 1. Besides this, computer simulation was done to identify and verify all the characteristic polynomials for each of these lengths. Thus, the biggest disadvantage of PN codes is the small code family size and as a result lesser number of mobile users is supported.

TABLE 1: PN CODE FAMILY

PN Code Length	Code Family Size
7	02
15	02
31	06
63	06
127	18
255	16
511	48
1023	60

(b) Gold Code

Gold codes assume significance because of their large code family size as compared to their PN counterpart. In fact, these Gold codes are constructed using a pair of PN sequences (usually preferred pair) [2], [3], [4]. The PN codes designed in the previous subsection are used to construct the Gold codes of desired length. Let a and a^1 represent a preferred pair of PN sequences having period N= 2^n-1. In order to generate a set of all possible Gold codes for a given length; one of the above two PN codes is delayed by one chip at a time to generate a new Gold code. Thus, the family of Gold codes is defined by $\{a, a^1, a+a^1, a+Da^1, a+D^2a^1,\ldots, a+D^{N-1}a^1\}$, where D is the delay element. With the exception of sequences a and a^1, the set of Gold sequences are not maximal sequences. Thus, N number of Gold codes can be generated from a preferred pair of PN sequences of length N. Hence, by including this pair of generating PN codes in the Gold code family, the total number of Gold codes becomes N+2 for each length.

An illustration of how a Gold code (2^7-1 length) is being constructed is shown in Figure 3. Here, Gold code of length = 127 is generated using preferred pair of PN codes having characteristic polynomials X^7+X+1 and X^7+X^3+1 respectively.

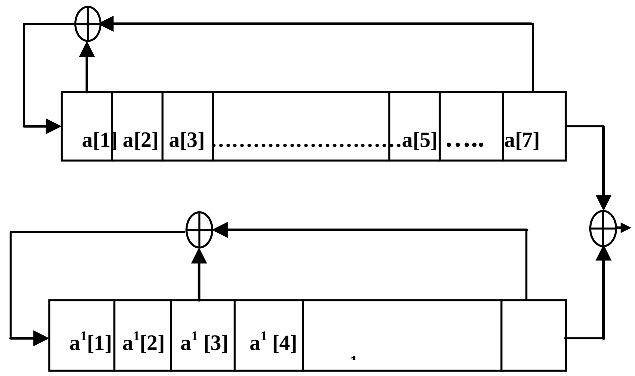

Fig. 3: 2^7-1 Length Gold Code

Thus, the entire gold code family having code length = 7 to 255 were generated through MATLAB programming by simulating the above structure. It has thus been observed that Gold codes have overcome the disadvantage of limited code family size as in PN codes. Also, finding preferred pairs of PN codes is necessary in defining sets of Gold codes.

II. EVALUATION OF PEAK RMS (RAC, RCC) AND AVERAGE MS (RAC, RCC) CHARACTERISTICS OF NON-ORTHOGONAL SPREADING CODES

In this section, the two types of Non-orthogonal codes i.e. PN codes and Gold codes are being considered for the evaluation of peak RMS correlation characteristics. Through exhaustive computer simulations using MATLAB, the sidelobe ACFs $C_{ii}(m)$ and CCFs $C_{ij}(m)$ for all possible combinations of entire code family of each length have been evaluated. Then, the peak or the highest RMS value of r_{ac} and r_{cc} for each code-set of particular length is being searched, tabulated and plotted. These peak RMS values of ACF and CCF will serve as a measure of maximum average interference [5] that may be produced by a particular code-set. Then, these $C_{ii}(m)$ and $C_{ij}(m)$ are used to evaluate average and normalized mean square correlation parameters R_{ac} and R_{cc} respectively. These parameters are also being tabulated and plotted for each code type.

(a) PN Codes:-

The PN codes were generated with length varying from L=7 to 1023 using the LFSR structure mentioned in the previous section. And the detailed results regarding the peak RMS values r_{ac}, r_{cc} and average MS values R_{ac} and R_{cc} for each PN code-set of particular length were tabulated in Table 2. Each of these Peak RMS and Average MS values for every PN code length is also plotted in figures 4 and 5 respectively.

TABLE 2: MEAN SQUARE CORRELATION PARAMETERS OF PN CODES

PN Code Length	Peak (acrms) rac	Peak (ccrms) rcc	Avg. MS (ACF) Rac	Avg. MS (CCF) Rcc
7	1	2.7873	0.2449	2.0612
15	1	3.8462	0.1244	1.9067
31	1	5.6987	0.0624	2.0187
63	1	8.0304	0.0312	2.0064
127	1	11.3355	0.0156	2.0043
255	1	16.0155	0.0078	1.9984
511	1	22.6384	0.0039	2.0013
1023	1	32.008	0.0020	2.0006

It has been observed from Figure 4 that peak RMS value of ACF sidelobes for PN codes is constant (i.e. '1') for all lengths. This is because of the two valued impulsive ACF characteristics possessed by each PN code. On the other hand, the peak RMS value for all CCFs of PN codes is monotonically increasing with code length. Further, in Figure 5, the averaged and normalized MS values are almost constant for both ACF sidelobes and CCFs.

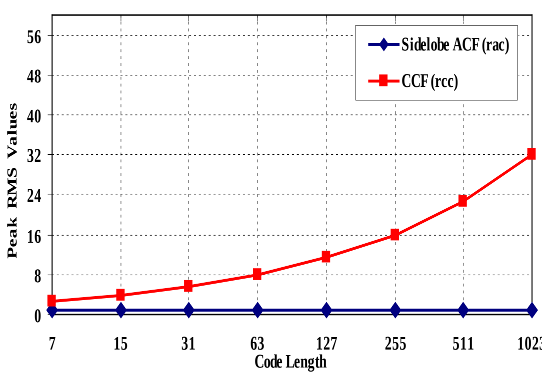

FIG 4: PEAK RMS VALUES OF ACF AND CCF FOR PN CODES

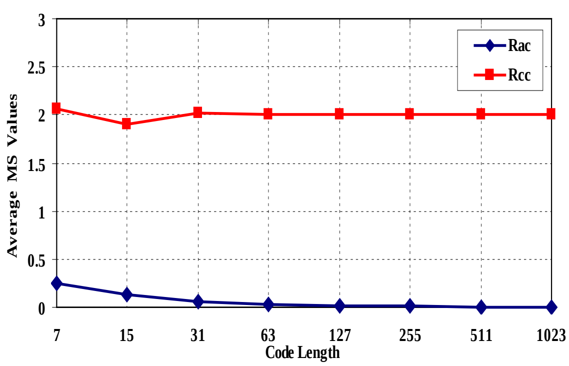

FIG 5 : AVERAGE RMS VALUES OF ACF AND CCF FOR PN CODES

This observation suggests that the average interference level introduced by PN codes is almost constant and independent of code length. Thus, in case of PN codes the main contribution towards interference is made by CCFs and not by ACF sidelobes.

(b) Gold Codes:-

The Gold codes were also generated with length varying from L=7 to 1023 by using the generator structure and pair of feedback polynomials. The entire set of Gold code family of each length is generated for exhaustive evaluation of mean square ACF and CCF characteristics. And the detailed results regarding the peak RMS values r_{ac}, r_{cc} and average MS values R_{ac} and R_{cc} for each Gold code-set of particular length are tabulated in Table 3. Like PN codes, each of these Peak RMS and Average MS values for every Gold code length is also plotted in Figures 6 and 7 respectively.

TABLE 3: MEAN SQUARE CORRELATION PARAMETERS OF GOLD CODES

Gold Code Length	Peak (acrms) rac	Peak (ccrms) rcc	Avg. MS (ACF) Rac	Avg. MS (CCF) Rcc
7	3.4157	3.4752	1.5510	1.7619
15	5.3318	4.6275	1.7642	1.8784
31	6.6081	6.9057	1.8787	1.9384
63	11.5382	9.8618	1.9385	1.9690
127	12.4665	12.6136	1.9690	1.9844
255	20.9985	20.1775	1.9844	1.9922
511	23.737	24.39	1.9922	1.9961
1023	39.041	36.2343	1.9961	1.9981

It has been observed from Figure 6 that unlike PN codes, the peak RMS value of ACF sidelobes for Gold codes is not constant and continuously increasing for all lengths.

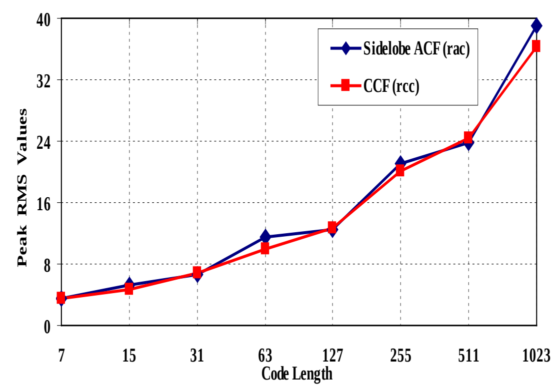

FIG 6: PEAK RMS VALUE OF ACF AND CCF GOLD CODES

This is so because no Gold code (except seed PN sequences) possesses the desirable impulsive ACF characteristics. Also, the peak RMS CCF values follow the same trend with varying code length. Further the averaged and normalized MS values in Figure 7 are almost constant with varying length for both ACF sidelobes and CCFs. But the striking observation is that the average interference level introduced by both ACF sidelobes as well as CCFs of Gold codes is almost same in direct contrast to PN codes. Thus, in case of Gold codes interference is caused due to ACF sidelobes as well as CCF.

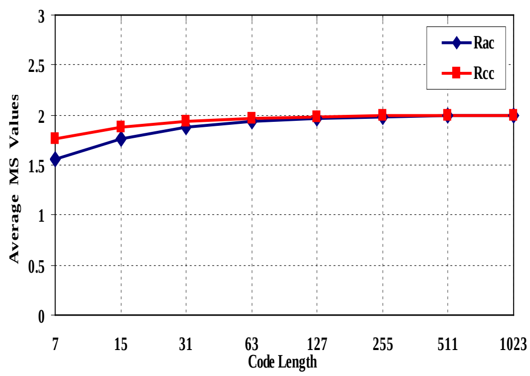

FIG 7: AVERAGE RMS VALUE OF ACF AND CCF GOLD CODES

III. CONCLUSION

Codes play an important role in determining the performance of a CDMA based wireless mobile system. And the choice of codes is mainly governed by the correlation characteristics. The simulation results proved that in case of PN codes the main contribution towards interference is made by CCFs and not by ACF sidelobes. On the other hand, in case of Gold codes interference is caused due to ACF sidelobes as well as CCF. The average interference level introduced by both ACF sidelobes as well as CCFs of Gold codes is almost same in direct contrast to PN codes. However, Gold codes possess the advantage of larger code family size than PN code family.

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