

# Performance Analysis of PID Controller and Its significance for Closed Loop System

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**Abstract:** The Proportional Integral Derivative (PID) Controller is the most widely used control technique in industry. The popularity of PID Controller because of their robust performance in a wide range of operating condition & its simple functionality. Industrial processes are subjected to variation in parameters and parameters perturbations. In this paper presents an analysis of P-I-D parameters in mathematical simulator software for second order closed loop system and discusses them. Simulation results are demonstrated the performance analysis and its significance in closed loop system.

**Key words:** PID Controller, Proportional gain ( $K_p$ ), Integral gain ( $K_I$ ), Derivative gain ( $K_D$ ).

## I. INTRODUCTION:

The basic characteristics of Proportional, Integral and Derivative controller & how to use them in closed loop application with their desired response. In this paper, consider a unity feedback system as seen below [5]:

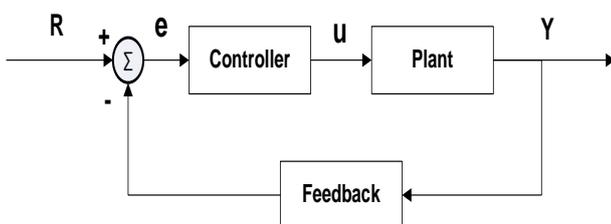


Fig. 1: Block Diagram of closed Loop System

Here, a Plant is stands for a controlled system and Controller stands for provides the excitation of the plant and designed to control the overall system behaviour.

The transfer function of the PID Controller in S-domain is shown below [7]:

$$K_p + \frac{K_I}{s} + K_D s = \frac{K_D s^2 + K_p s + K_I}{s} \quad (1)$$

Where,  $K_p$  is the proportional gain,  $K_i$  is the integral gain, and  $K_d$  is the derivative gain. First, take an effect of PIC controller on closed loop system from above bock diagram. In that, error ( $e$ ) is the difference of desired reference value ( $R$ ) and the actual output ( $Y$ ).The controller takes this error signal and computes both its derivative and its integral [5].

The signal which is sent to the actuator ( $u$ ) is as following [7].

$$u = K_p \cdot e + K_I \int e dt + K_D \frac{de}{dt} \quad (2)$$

This output signal ( $u$ ) will be sent to the plant and new output ( $Y$ ) will be getting. This new output ( $Y$ ) will be sent back with feedback and again new error ( $e$ ) will be obtained [8]. The PID Controller takes new error signal and calculates its derivative and it's integral again. This process goes on and on [2].

Generally, for an open-loop transfer function which have the canonical second-order form [7]:

$$\frac{1}{s^2 + 2\delta\omega s + \omega^2} \quad (3)$$

This paper begins with a brief introduction of the closed loop PID controller and its basics parameter with equation. Section 2 gives the information about the characteristic of the PID controller and effect of each parameter on closed loop system. Section 3 provides a basic example problem for understanding each parameters effect on real time problem and simulate the results in mathematical simulation software. Section 4 describes the application of PID controller in real world. Section 5 provides the results of this paper and discusses technical term regarding this. Section 6 provides the conclusion about this paper and section 7 shows the reference which is used for this paper.

## II. CHARACTERISTICS OF PID CONTROLLERS:

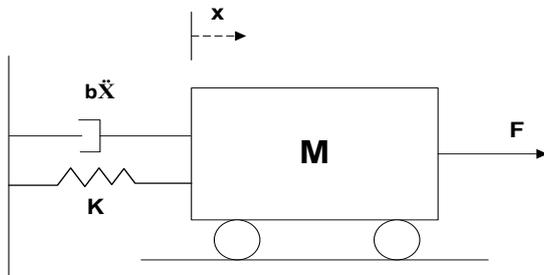
The effect of proportional gain ( $K_p$ ) will reducing the rise time but never eliminates the steady- state error [1]. Integral gain will eliminate the steady state error but it makes transient response very poor. The effect of a derivative gain ( $K_d$ ) will increase the stability of the system, reduce the overshoot, and improve the transient response. Effects of all gain  $K_p$ ,  $K_i$ , and  $K_d$  in closed loop feedback system will be conclude in below given table [4]:

Gain	Rise time	Overshoot	Settling time	S-S error
$K_p$	Decrease	Increase	Not change	Decrease
$K_i$	Decrease	Increase	Increase	Eliminate
$K_d$	Not change	Decrease	Decrease	Not change

This table should be used for only reference because this correlation may not be exactly accurate and  $K_p$ ,  $K_i$  &  $K_d$  are dependent on each other. In fact, if any change in one variable from above table the effect on other variable [2].

### III. PROBLEM STATEMENT:

In this paper, taken a simple mass, spring, and damper example problem [6].



**Fig. 2: Example Problem of mass, spring and Damper**

The modelling equation for this system is [6]

$$M\ddot{x} + b\dot{x} + kx = F \quad (4)$$

Taking the Laplace transform of the modelling equation (4)

$$MS^2X(s) + bsX(s) + kX(s) = F(s) \quad (5)$$

The transfer function between the displacement  $X(s)$  and the input  $F(s)$  is as given below [6]:

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + bs + k} \quad (6)$$

Let's assume,  $M = 1\text{kg}$ ,  $b = 20\text{ N.s/m}$ ,  $k = 30\text{ N/m}$ ,  $F(s) = 1$ . Put all these values into the above transfer function.

$$\frac{X(s)}{F(s)} = \frac{1}{s^2 + 20s + 30} \quad (7)$$

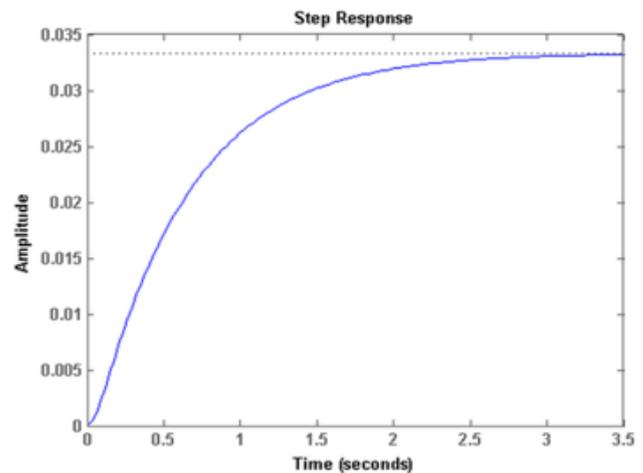
The aim of this paper is to how each of  $K_p$ ,  $K_i$  and  $K_d$  contributes to obtain a fast rise time, minimum overshoot and no steady-state error [5].

#### A. OPEN - LOOP RESPONSE:

Many PID controllers are designed by the trial & error selection of the variables  $K_p$ ,  $K_i$ , and  $K_d$ . There are some rules of thumb to determine good values for start-up [1].

In this paper consider a second-order plant transfer function:

$$\frac{X(s)}{F(s)} = \frac{1}{s^2 + 20s + 30} \quad (8)$$



**Fig. 3: Output Step Response for Open Loop System**

The DC gain of the plant transfer function is  $1/30$ , so  $0.033$  is the final value of the output for a unit step input. From above figure the steady state error is  $0.95$  which is quite large. Furthermore, the rise time is about  $0.95$  second, and the settling time is about  $2.5$  seconds. Most likely, this response will not be adequate. Therefore, need to add some control [5].

#### B. PROPORTIONAL (P) CONTROL:

The effect of Proportional gain ( $K_p$ ) will reduce the rise time, increase the overshoot and reduces the steady state error. The closed-loop transfer function of the above system with a proportional controller is [1]:

$$\frac{X(s)}{F(s)} = \frac{K_p}{s^2 + 20s + (30 + K_p)} \quad (9)$$

Now take proportional gain  $K_p = 20$ .

The plant and controller transfer functions have to be multiplied together before the loop is closed [4]. It should also be noted that it is not a good idea to use proportional control to reduce the steady-state error, because it's not able to eliminate the error completely [8]. This fact will become evident below. For this control the plot as shown in figure 4(a).

Now, the rise time has been reduced and the steady-state error is smaller, if use a greater  $K_p$  [5], the rise time and steady-state error will become even smaller. Change the  $K_p=600$  value in the simulator which shown in figure 4(b).

The rise time is 0.095 seconds and the steady state error is very small. But the overshoot has gotten very large. From this example show a large proportional gain will reduce the steady-state error but at the same time, worsen the transient response. If needed a small overshoot and a small steady-state error, a proportional gain alone is not enough [6].

C. PROPORTIONAL – DERIVATIVE (PD)

CONTROL:

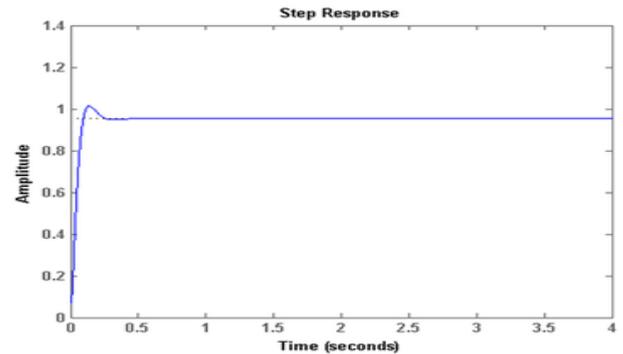
The effect of derivative gain (Kd) reduce a overshoot and settling time. The closed-loop transfer function of the given system with a PD controller is [7]:

$$\frac{X(s)}{F(s)} = \frac{K_D s + K_P}{s^2 + (20+K_D)s + (30 + K_P)} \quad (10)$$

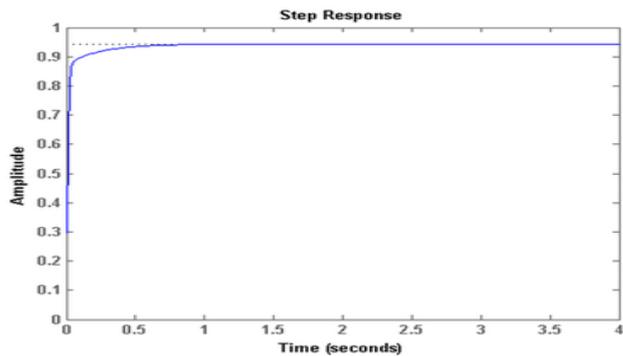
The rise time is now probably satisfactory (rise time is about 0.095 second). Now add a derivative controller to the system to see if the overshoot can be reduced [7]. Add another variable, Kd, set it equal to 15 simulate again and the plot which shown in figure 5(a).

The overshoot is much less than before. It is now only 20% instead of almost 45%. Now try to improve that even more. Try increasing Kd to 110 [4], it will shown in figure 5(b), the overshoot eliminated completely.

Now system with a fast rise time and no overshoot [5]. Unfortunately, there is still about a 5% steady-state error. It would seem that a PD controller is not satisfactory for this system. Let's try a PI controller instead.



[a]



[b]

Fig. 5: Output Response for Proportional –Derivative (PD) Controller [a] For Kp = 600 & Kd = 15, [b] For Kp = 600 & Kd = 110.

This plot shows that the derivative controller reduced both the overshoot and the settling time, and had small effect on the rise time and the steady-state error [3].

D. PROPORTIONAL – INTEGRAL (PI)

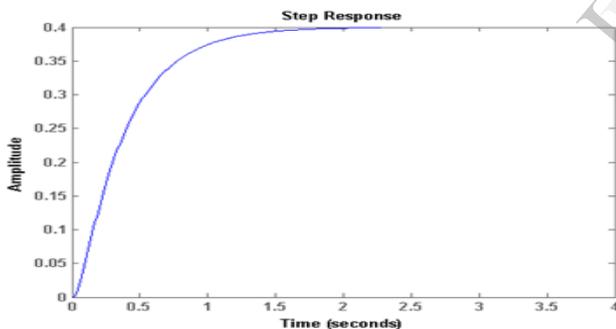
CONTROL:

Before going into a PID control, let's take a look at a PI control. The effect of integral gain (Ki) will decrease the rise time, increase the overshoot and settling time, eliminate the steady state error. For the given system, the closed-loop transfer function with a PI control is:

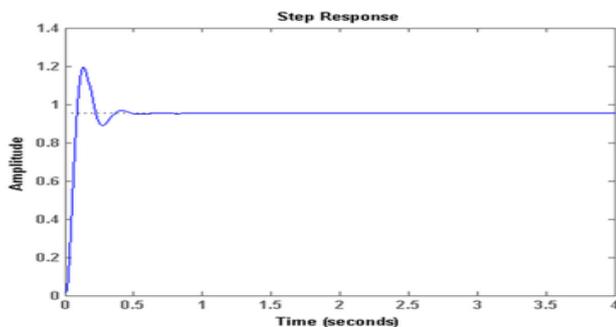
$$\frac{X(s)}{F(s)} = \frac{K_P s + K_I}{s^3 + 20s^2 + (30 + K_P)s + K_I} \quad (11)$$

Proportional control will reduce the steady-state error, but at the cost of a larger overshoot [2]. Furthermore, proportional gain will never completely eliminate the steady-state error. For that purpose try to integral control [1]. Now implement a PI controller and start with a small Ki. its plot as shown in figure 6(a).

The Ki controller really slows down the response [6]. The settling time becomes more than 180 seconds. To reduce the settling time, try to increase Ki, but by doing this, the transient response will get worse (e.g. large overshoot) [4]. Try Ki=15, by changing the Ki variable. The plot as shown in figure 6(b).



[a]



[b]

Fig. 4: Output Response for Proportional Controller [a] For Kp = 20, [b] For Kp = 600.

Now having a large overshoot again, while the settling time is still long. To reduce settling time and overshoot, a PI controller by itself is not enough.

#### E. PROPORTIONAL – INTEGRAL – DERIVATIVE (PID) CONTROL:

Now, let's take a look at a PID controller. The transfer function of a PID controller in given closed loop system is as shown below:

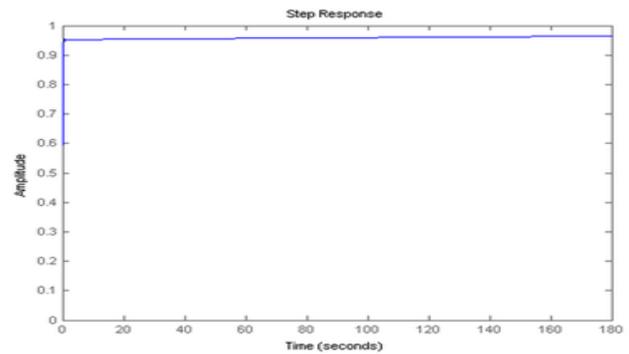
$$\frac{X(s)}{F(s)} = \frac{K_D s^2 + K_P s + K_I}{s^3 + (20 + K_D)s^2 + (30 + K_P)s + K_I} \quad (12)$$

From the two controllers above, we see that if we want a fast response, small overshoot, and no steady-state error [8], neither a PI nor a PD controller will suffice. Let's implement both controllers and design a PID controller to see if combining the two controllers will yield the desired response [3]. Recalling that PD controller gave us a pretty good response, except for a little steady-state error. Let's start from there, and add a small  $K_i = 1$ . Change the following to implement the PID controller and plot the closed loop response as shown in figure 7(a).

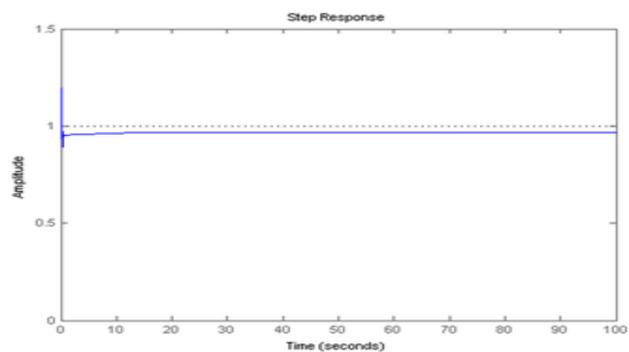
The settling time is still very long. Increase  $K_i$  to 105 and re-run the program and plot as shown in figure 7(b).

The settling time is much shorter, but still not small enough. Increase  $K_i$  to 505 and plot the response as shown in figure 7(c).

Now the settling time reduces to only 0.95 second. This is probably an acceptable response for this system [4]. To design a PID controller, the general rule is to add proportional control to get the desired rise time [2], add derivative control to get the desired overshoot, and then add integral control (if needed) to eliminate the steady-state error [3].



[a]



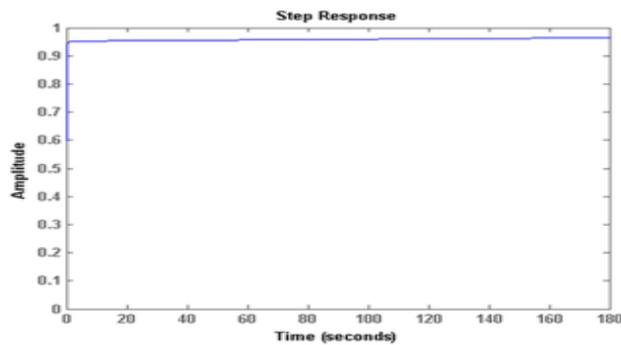
[b]

Fig. 6: Output Response for Proportional - Integral (PI) Controller [a] For  $K_p = 600$  &  $K_i = 1$ , [b] For  $K_p = 600$  &  $K_i = 15$

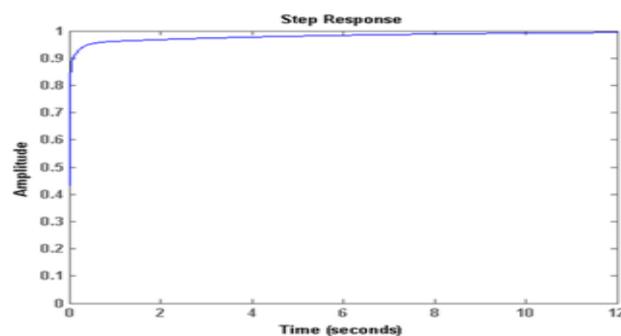
#### IV. APPLICATION OF PID CONTROLLER:

PID temperature control is required in any industrial process where it is critical for temperature to be maintained within strict limits for any reason. The number of industrial applications is vast, but here are a few examples [1]:

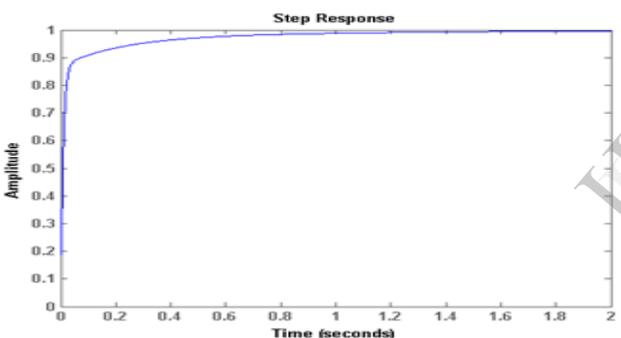
- Laboratory incubators
- Industrial ovens
- Environmental chambers
- Autoclaves
- Smoke machines
- Catering water baths
- Packaging machinery



[a]



[b]



[c]

Fig. 7: Output Response for Proportional - Integral - Derivative (PID) Controller [a] For  $K_p=600$ ,  $K_i=1$  &  $K_d=105$ , [b] For  $K_p=600$ ,  $K_i=1$  &  $K_d=105$ , [c] For  $K_p=600$ ,  $K_i=1$  &  $K_d=105$ .

## V. RESULTS AND DISCUSSION:

In above, Figure 3 shows output step response of open loop controller. Figure 4 indicates the results of Proportional controller [7], Figure 5 describes the performance of Proportional-Derivative controller, and Figure 6 shows output result of Proportional- Integral controller. Finally Figure 7 indicates the output response for PID controller as shown in results.

## VI. CONCLUSION:

This paper reports Performance Analysis of PID Controller Parameters, its significance and method to do the same for Closed Loop System using mathematical simulation

tool. It is concluded that for designing of a PID controller for any system, require some steps to obtain a desired response few things are to be added like proportional gain ( $K_p$ ) to improve the rise time, a derivative gain ( $K_d$ ) to improve the overshoot, internal gain ( $K_i$ ) for eliminate the steady state error.  $K_p$ ,  $K_i$  and  $K_d$  are being analyzed and optimized until an desired overall response is obtain. At last also conclude that do not need to implement all three controller (proportional, derivative and integral) into a single system, if not necessary.

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