Performance Analysis of Various SNR Estimation Techniques under Different Wireless OFDM Channels

Ramchandar Rao P[1] ,Sampath Reddy Mula[2], T Anil Kumar [3], K Srinivas Rao [4]
[1,2] Department of ECE, SR Engineering College,Warangal,
Andhra Pradesh,India

[3] Department of ECE, Anurag Engineering College, Hyderabad,
Andhra Pradesh,India

[4] Department of ECE, TRR Engineering College, Hyderabad,
Andhra Pradesh,India

Abstract: *The Orthogonal Frequency Division Multiplexing (OFDM) is a promising approach to achieve higher data rates with sufficient performance in frequency selective channels. Hence it is the proposed multiplexing scheme for future Wireless communication and Wi-Max, and these schemes uses Adaptive Modulation & Coding (AMC) to select the appropriate modulation & coding scheme. The major input parameter for AMC is Signal to Noise ratio (SNR).So SNR estimation is required to operate the system with higher data rates. In this paper we compare various SNR estimators for different channel models based on preamble, and the suitable estimators are suggested for different channel models. The simulation results shows that the Boumard's estimator performs bad for lesser values of SNR in AWGN channel, for frequency selective channels the Periodic Sequence estimator(PS Estimator) performs well.*

Keywords: *OFDM, MMSE estimator, Boumard's estimator, Ren's estimator, Periodic Sequence estimator.*

1. INTRODUCTION

Orthogonal Frequency Divisional Multiplexing (OFDM) scheme has been adopted in many broadband systems. Compared to the conventional Single Carrier (SC) modulation schemes, OFDM enables the efficient use of the available channel bandwidth and easy control of the signal spectrum mask in the Transmitter and the Receiver side. OFDM support high data rates, allows simple equalization, and is robust to constant timing offset due to the adoption of Cyclic Prefix (CP) for which it is suggested as a multiple access scheme for future wireless communication. OFDM provides strong robustness against Inter symbol interference (ISI) by dividing the broadband channel into many narrowband sub channels in such a way that the attenuation across each sub channel stays flat. In OFDM the sub carrier frequencies are chosen as orthogonal to each other meaning that cross talk between the sub channels is eliminated and inter carrier guard bands are not required. Orthogonalization of sub channels is performed with low complexity by using FFT. A major advantage of OFDM systems is its ability to divide the input high rate data stream into many low-rate streams [11]-[12] that are transmitted in parallel, thereby increasing the symbol duration and reducing the inter symbol interference over frequency-selective fading channels.

To optimize the system performance it is required to estimate SNR at the receiver. In OFDM average SNR is estimated. There are two general categories of average SNR estimators. *Data-aided* (DA) estimators are based on either perfect or estimated knowledge of the transmitted data. However, a certain portion of data is needed for estimation purposes, which reduces bandwidth efficiency. *Blind* or *in-service* estimators derive SNR estimate from an unknown information-bearing portion of the received signal preserving efficiency at the cost of decreased performance. For packet based communications, block of information data is usually preceded by several training symbols (preambles) of known data used for synchronization and equalization purposes.

Therefore, DA SNR estimators can utilize preambles without additional throughput reduction. Most of the SNR estimators proposed in the literature so far are related to single carrier transmission. In [10], a detailed comparison of various algorithms is presented, together with the derivation of the Cramer-Rao bound (CRB)[9]. Most of these algorithms can be directly applied to OFDM systems in Additive White Gaussian Noise (AWGN) [2], while the SNR estimation in frequency selective channels additionally requires efficient estimation of Channel State information (CSI). In this paper, we compare various algorithms for the average SNR estimation in wireless OFDM systems. The SNR per subcarrier can be additionally estimated using channel estimates and the estimated average SNR. The proposed estimator utilizes preamble structure, proposed by Morelli and Mengali in [3]. Compared to Schmidl and Cox synchronization method [4], it allows synchronization over a wider frequency offset range with only one preamble, hence reducing the training symbol overhead. Since the proposed estimation algorithm relies on the signal samples at the output of the FFT, its performance depends strongly on the given preamble structure. The remainder of this paper is organized as follows. Section 2 provides the system model and specifies the SNR estimation problem. The Section 3 describes various channel models like AWGN, Rayleigh, and Rician, In Section 4 various SNR estimators are described such as Minimum Mean Square Error (MMSE), Boumard's, Ren's estimators and Periodic sequence estimator are briefly described and according SNR estimates are given. The performance is analyzed by computer simulations in Section 5. Finally, some concluding remarks are given in Section 6.

2. SYSTEM MODEL

The OFDM system design exploits the frequency selective channels by adaptive transmission. In this type of transmission particular range of frequencies suffer from the interference or attenuation, the carriers within that range can be disabled or made to run slower. Here the adaptable transmission parameters are bandwidth, data rate and power. In many wireless OFDM systems the transmission is normally organized in frames. The frame structure is given in Fig.1.

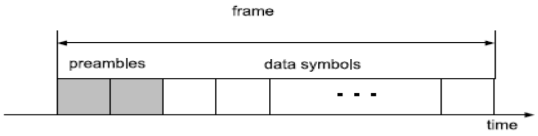

Fig. 1. Frame structure

From the Fig.1the sequence of data symbols is preceded by several preambles of known data for synchronization and channel estimation purposes. We consider general model of frame structure composed of *K* preambles where each preamble contains *N* modulated subcarriers. The SNR estimation is performed in frequency domain given model which contains only frequency domain characterization of received signal in frequency selective AWGN channel.

Let $C(k, n)$ denote the complex data symbol on *n*th subcarrier in k^{th} preamble, where $k = 0, \ldots, K - 1$ and $n = 0, \ldots, N - 1$. It is assumed that modulated subcarrier has unit magnitude, i.e.

$$|C(\mathrm{k},n)|^2 = 1 \tag{1}$$

Then at the receiver assuming, perfect synchronization after FFT. Then the received signal on n^{th} subcarrier in k^{th} preamble can be given as

$$Y(k,n) = \sqrt{S}C(k,n)H(k,n) + \sqrt{W}\,\eta(k,n)|C(\mathrm{k},n)|^2 = 1 \tag{2}$$

where $\eta(k, n)$ is sampled complex zero-mean AWGN of unit variance, *S* and *W* are transmitted signal power and noise power on each subcarrier, respectively, and

$H(k, n)$ is the channel frequency response given by

$$H(k,n) = \sum_{l=1}^{L} h_l\,(kT_s).e^{-j2\pi\frac{n\tau_l}{NT_s}} \tag{3}$$

Where $h_l(kT_s)$ and τ_l denote the channel l^{th} path gain and delay during the k^{th} preamble, respectively, T_s is the duration of the OFDM preamble and *L* is the length of the channel memory.

Our initial assumption is that channel is constant during the whole frame, since we consider SNR estimation algorithms for the purposes of adaptive transmission. Therefore, time index *k* is omitted during the estimation procedure, i.e. $H(k, n)$ is replaced by $H(n)$. Then the average SNR of the k^{th} received OFDM preamble can be expressed as

$$\rho_{av}(k) = \frac{E\{\frac{1}{n}\sum_{n=0}^{N-1}|\sqrt{S}C(k,n)H(n)|^2\}}{E\{\frac{1}{n}\sum_{n=0}^{N-1}|\sqrt{W}\eta(k,n)|^2\}} = \frac{S}{W} \tag{4}$$

Where

$$\sum_{n=0}^{N-1}|H(n)|^2 = N \tag{5}$$

is satisfied, while the SNR of the n^{th} subcarrier is given by

$$\rho(k,n) = \frac{E\{\left|\sqrt{S}C(k,n)H(n)\right|^2\}}{E\{\left|\sqrt{W}\eta(k,n)\right|^2\}} = \frac{S|H(n)|^2}{W} = \rho_{av}.|H(n)|^2 \tag{6}$$

3. VARIOUS WIRELESS CHANNEL MODELS

The performance of any wireless system is affected by the medium of propagation, namely the characteristics of the channel. The received signal consists of a combination of attenuated, reflected, refracted, and diffracted replicas of the transmitted signal. On top of all this, the channel adds noise to the signal and can cause a shift in the carrier frequency, if the transmitter or receiver is moving (Doppler Effect). Understanding these effects on the signal is important because the performance of a radio system is dependent on the radio channel characteristics.

3.1 AWGN channel

For the Additive White Gaussian Noise (AWGN) channel the received signal is equal to the transmitted signal with some portion of white Gaussian noise added [10]. This channel is particularly important for discrete models operating on a restricted number space, because this allows optimizing the circuits in terms of their noise performance.

$$\hat{s}(t) = s(t) + n(t) \tag{7}$$

Where $n(t)$ is a sample of a Gaussian random process. This represents white Gaussian noise.

3.2 Rayleigh Fading

Rayleigh fading is caused by multipath reception. The mobile antenna receives a large number, say *N* reflected and scattered waves. Because of wave cancellation effects, the instantaneous received power seen by a moving antenna becomes a random variable, dependent on the location of the antenna. The Rayleigh distribution is commonly used to describe the statistical time varying nature of the received signal power [15]. It describes the probability of the signal level being received due to fading. The relative phase of multiple reflected signals can cause constructive or destructive interference at the Receiver. This is experienced over very short distances (typically at half wavelength distances), thus is given the term fast fading. These variations can vary from 10-30dB over a short distance.

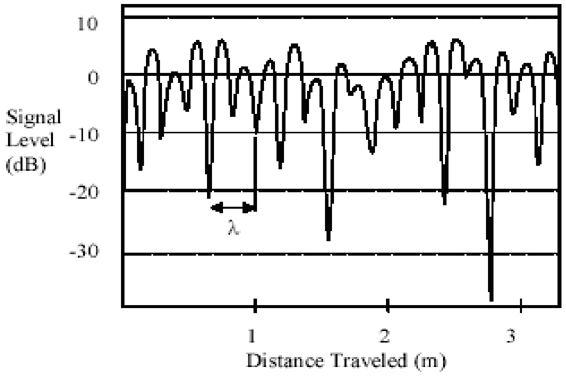

Fig. 2. Typical Rayleigh fading when the mobile unit is moving

3.3 Rician Fading

Rician fading is a stochastic model for radio propagation anomaly caused by partial cancellation of a radio signal by itself. The signal arrives at the receiver by several different paths, and at least one of the paths is changing (lengthening or shortening). Rician fading occurs when one of the paths, typically a line of sight signal, is much stronger than the others. In Rician fading, the amplitude gain is characterized by a Rician distribution.

4. SNR ESTIMATORS

4.1. MMSE Estimator

The term MMSE specifically refers to estimation in a Bayesian setting, i.e., it is a method of statistical interference in which some kinds of evidence or observations are used to calculate the probability that a hypothesis may be true, or else to update its previously calculated probability .The mean squared error of an estimator is one of many ways to quantify the difference between values implied by an estimator and the true values of the quantity being estimated.

The MMSE average SNR estimate is given by [2]

$$\hat{\rho}_{av,MMSE} = \frac{\hat{s}_{MMSE}}{\widehat{W}_{MMSE}} \tag{8}$$

Where

$$\hat{S}_{\ MMSE} = \left|\tfrac{1}{n}\textstyle\sum_{n=0}^{N-1} Y(n)C(n)^{*}\right|^{2} \tag{9}$$

And

$$\widehat{W}_{\ MMSE} = \frac{1}{n}\sum_{n=0}^{N-1} |Y(n)|^{\ 2} - \hat{S}_{\ MMSE} \tag{10}$$

are the MMSE estimates of *S* and *W*, respectively.

4.2. Boumard's Estimator

The main advantage of this estimator is that it exists both in time and frequency domain. It is the second-order moment-based SNR estimator for MIMO OFDM system in slow varying channel in both time and frequency domain.

Assume that the channel is time-invariant and that two identical preambles are used for SNR estimation, i.e. $k = 0, 1$ and $C(0, n) = C(1, n) = C(n)$, for $n = 1, \ldots, N$. Average SNR estimate can be expressed as

$$\hat{\rho}_{av,bou} = \frac{\hat{s}_{bou}}{\widehat{W}_{bou}} \tag{11}$$

Where

$$\hat{S}_{bou} = \frac{1}{N}\sum_{n=0}^{N-1}\left|\widehat{H}(n)\right|^{2} \tag{12}$$

$$\widehat{W}_{bou} = \frac{1}{4N}\sum_{n=0}^{N-1}\left|\begin{matrix} C(n-1)\big(Y(0,n)+Y(1,n)\big) \\ -C(n)Y(0,n-1)+Y(1,n-1)\end{matrix}\right|^{2} \tag{13}$$

These are the estimates of *S* and *W*, respectively, and

$$\widehat{H}(n) = \frac{C^{*}(n)}{2}\big(Y(0,n)+Y(1,n)\big) \tag{14}$$

Is the least square estimate of H(n) averaged over two preamble symbols, Using $\widehat{H}(n)$ SNR on n^{th} subcarrier is estimated as

$$\hat{\rho}(n) = \frac{\left|\widehat{H}(n)\right|^{2}}{\widehat{W}_{bou}} \tag{15}$$

4.3 Ren's Estimator

It is the estimator which is very robust against frequency selectivity [5].This also based on second-order moments and more accurate compared with Boumard's estimator, and it requires two identical preambles for the estimation of average SNR The average SNR is given as

$$\hat{\rho}_{av,Ren} = \frac{\hat{s}_{Ren}}{\widehat{W}_{Ren}} \tag{16}$$

Where

$$\widehat{W}_{Ren} = \frac{4}{N}\sum_{n=0}^{N-1}\Big\{Im\Big[Y(0,n)C^{*}(0,n)\widehat{H^{*}}(n) \\ /\left|\widehat{H^{*}}(n)\right|\Big]\Big\}^{2} \tag{17}$$

And

$$\hat{S}_{Ren} = \frac{1}{N}\sum_{n=0}^{N-1}|Y(0,n)|^{2} - \widehat{W}_{Ren} \tag{18}$$

The $\widehat{H}(n)$ defined in (14) will be used to estimate W and S. It is shown that the performance is independent of the channel frequency response estimation although the estimated channel states are used in average SNR estimation. Additionally, SNR on *n*th subcarrier is estimated as in (15) using the noise power estimate from (16).

4.4 Periodic Sequence Estimator

The periodic sequence estimator is based on the second order moments of received samples in the frequency domain [4]&[15]. For the coverage of total band width, in [3] a preamble of *Q* identical parts, each containing *N/Q* samples is proposed and it is depicted in time and frequency domain as shown in Fig. 3. The number of samples on loaded sub carriers can be given as $Np = N/Q$ samples. i.e ‘Q’ identical parts having ‘N’ samples and the remaining samples are the remainder then the remainder or nulled sub carriers can be given as $N_z = N - N_p = \frac{Q-1}{Q}N$ starting from the 0th, each Q^{th} subcarrier is modulated with a QPSK signal $Cp(m)$,where $m = 0, 1, \ldots, N_p - 1$ with $|Cp(m)| = 1$.For maintaining the total energy level constant over all symbols within the preamble, the power is scaled by a factor ‘*Q*’ then the total transmitted power in the loaded subcarriers is ‘*SQ*’. consider $n = mQ + q$, $m = 0, \ldots, Np - 1$, $q = 0, \ldots, Q - 1$. The transmitted signal on the n_{th} subcarrier is written as

$$C(n) = C(mQ+q) = \begin{cases} C_p(m), & q = 0 \\ 0, & q = 1, \ldots\ldots Q-1 \end{cases} \tag{19}$$

Where, ’Q’ is the identical part of the preamble, ’q’ is the remainder or nulled sub carriers, Cp(m) is the modulated signal. In PS estimator average SNR is obtained by the second order moment of received signal on loaded and nulled subcarriers [13]. The received signal for n^{th} sub carriers is given by

$$Y(n) = Y(mQ+q) = \begin{cases} Y_p(m), & q = 0 \\ Y_z(mQ+q), & q = 1, \ldots\ldots Q-1 \end{cases} \tag{20}$$

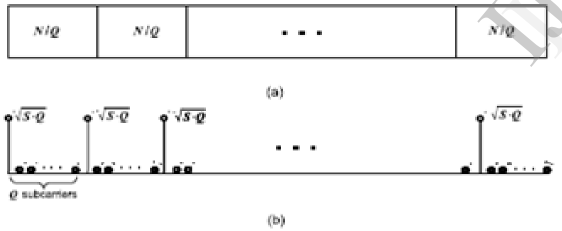

Fig. 3. Preamble structure a)Time domain b)frequency domain

Where

$$Y_p(m) = \sqrt{SQ}C_p(m)H(m) + \sqrt{W}\,\eta(m) \tag{21}$$

Is the received signal on loaded subcarriers, and the received signal on nulled sub carriers is given by

$$Y_z(mQ+q) = \sqrt{W}\,\eta(mQ+q) \tag{22}$$

Where the signal on the nulled subcarrier consists of only noise. Then the empirical second-order moment of loaded subcarriers is

$$\widehat{M}_{2,p} = \frac{1}{N_p}\sum_{m=0}^{N_p-1}\left|Y_p(m)\right|^2 \tag{23}$$

Its expected value is given as

$$E\{\widehat{M}_{2,p}\} = \frac{1}{N_p}\sum_{m=0}^{N_p-1}E\{\left|Y_p(m)\right|^2\} \tag{24}$$

Similarly, the empirical second moment of the received signal in nulled subcarriers

$$\widehat{M}_{2,z} = \frac{1}{N_p(Q-1)}\sum_{m=0}^{N_p-1}\sum_{q=0}^{Q-1}|Y_z(mQ+q)|^2 \tag{25}$$

Its expected value is given as

$$E\{\widehat{M}_{2,z}\} = \frac{1}{N_p(Q-1)}\sum_{m=0}^{N_p-1}\sum_{q=0}^{Q-1}E\{|Y_z(mQ+q)|^2\} \tag{26}$$

From (24)and(26) the average SNR ρ_{av} can be estimated as

$$\hat{\rho}_{av} = \frac{1}{Q}\frac{\widehat{M}_{2,p} - \widehat{M}_{2,z}}{\widehat{M}_{2,z}}$$

$$= \frac{1}{Q}\left((Q-1)\frac{\sum_{m=0}^{N_p-1}E\{\left|Y_p(m)\right|^2\}}{\sum_{m=0}^{N_p-1}\sum_{q=0}^{Q-1}|Y_z(mQ+q)|^2} - 1\right) \tag{27}$$

The channel estimates$\widehat{H}(n)$, $m = 0, 1, \ldots, Np - 1$, are available only for the loaded subcarriers. The channel estimation is more accurate since the transmitted power on each loaded subcarriers is increased by factor *Q*. Channel estimates for nulled subcarriers $\widehat{H}$ $(mQ+q)$, $m = 0, \ldots, Np-1$, $q = 1, \ldots, Q-1$, can be obtained by linear or DFT based interpolation [7]. Finally, increasing the number of identical parts on loaded sub carriers improves the accuracy of the noise power estimation and increases sensitivity of SNR per subcarrier estimates to frequency selectivity due to performed interpolation on nulled subcarriers during the channel estimation

5. SIMULATION RESULTS

The performance of various SNR estimators is compared under various channel conditions using Monte-Carlo simulations. The evaluation is measured in terms Normalized mean squared error (NMSE) of average SNR and SNR per subcarrier ,and these are given as

$$NMSE_{av} = \frac{1}{N_t}\sum_{i=i}^{N_t}\left(\frac{\hat{\rho}_{av,i} - \rho_{av}}{\rho_{av}}\right)^2 \tag{28}$$

Where $\hat{\rho}_{av,i}$ is the estimated average SNR in the i^{th} trial, and ρ_{av} is actual average value. Similarly

$$NMSE_{sc} = \frac{1}{NN_t}\sum_{i=i}^{N_t}\sum_{n=0}^{N}\left(\frac{\hat{\rho}(n)_i - \rho(n)}{\rho(n)}\right)^2 \tag{29}$$

The $\hat{\rho}(n)_i$is the i^{th} trial estimated value of$\rho(n)$.

These simulations are performed for OFDM systems parameters under Wi-Max specifications[8] which are given as

Name of the parameter	Values
No.of sub carriers	256
Cyclic prefix	32
Channel models	a)AWGN b)Rayleigh c) Rician
No.of channel Taps	3
Doppler shift	10
Modulation scheme	Qpsk
Estimators	Ren’s,Boumard’s,MMSE,Periodic sequence for Q=2,4,8

Fig.4. shows the $NMSE_{av}$ for AWGN channel and this is compared with Normalized Cramer Rao Bound (NCRB) to assess the performance where the NCRB is given as $NCRB = \frac{1}{N}\left(\frac{2}{\rho_{av}} + 1\right)$

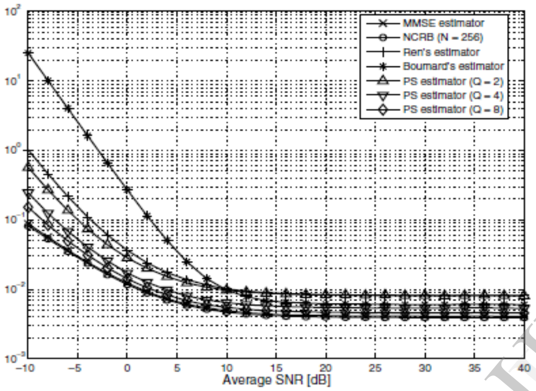

Fig. 4. NMSE vs Average SNR in AWGN channel

The Boumard’s estimator performs worse than compared to other estimators for smaller values of SNR. As the value of SNR increases the Boumard’s estimator performs similar to Ren’s and PS estimator. In PS estimator as the no.of identical parts (value of Q)are increasing to 2,4 and 8 then the curve approaches to NCRB.

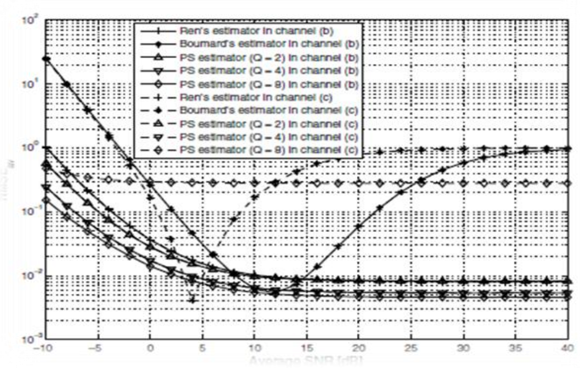

Fig. 5. NMSE vs Average SNR in Rayleigh and Rician channels

From Fig.5. frequency selective channels like Rayleigh and Rician the Boumard’s estimator is highly sensitive to frequency selectivity. The periodic sequence estimator performs well for these channels and the curves are similar to AWGN channel .For Channel b and c due to multipath fading more impact on Boumard’s estimator due to which it is very sensitive to frequency selective channels. the Fig.6. shows the average SNR per subcarrier for channel b and c. For lower SNR values all estimators performs bad and also it is observed that as the SNR values are increasing the $NMSE_{sc}$ is approaching to $NMSE_{av}$ are similar.

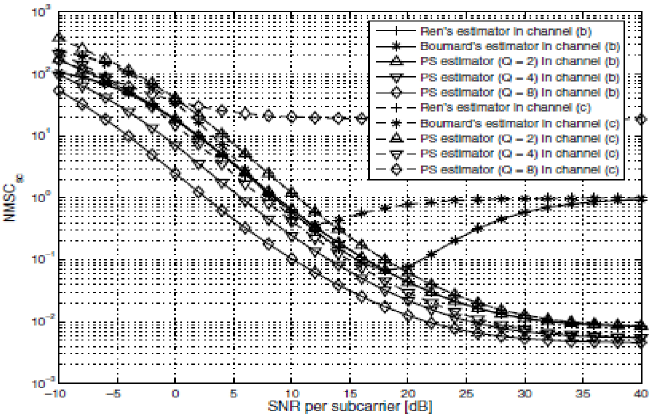

Fig. 6. NMSE vs Average SNR per sub carrier in Rayleigh and Rician channels

6. CONCLUSIONS

In this paper, we compare various SNR estimators for OFDM systems using various channel models. As the Boumard’s estimator is not performed well for low values of SNR, Similarly the Ren’s and Boumard’s estimator are very immune to frequency selectivity. The periodic sequence estimator performs well for frequency selective channels and as the number of identical parts is increasing the curve approaches to NCRB. From the derivation point of view the complexity of PS estimator is lower than others where it requires N multiplications. The performance of these estimators can be further improved by combining estimated average noise power with more sophisticated channel estimation algorithms using pilot subcarriers where the PS estimator uses only one preamble and also it does not require any knowledge of transmitted sub carrier.

References

1. Rana Shahid Manzoor and Wabo Majavu”Front-End Estimation of Noise Power and SNR in OFDM Systems” IEEE Trans. Intelligent and Advanced Systems,2007
2. D. Athanasios and G. Kalivas, “SNR estimation for low bit rate OFDM systems in AWGN channel,” in Proc. Of CN/ICONS/MCL 2006., pp. 198–198, April 2006.
3. M. Morelli and U. Mengali, “An improved frequency offset estimator for OFDM applications,” IEEE Commun. Lett., vol. 3, no. 3, pp. 75–77, Mar 1999 I
4. M. Morelli, C.-C. Kuo, and M.-O. Pun, “Synchronization techniques for orthogonal frequency division multiple access (OFDMA): A tutorial review,” Proc. IEEE, vol. 95, no. 7, pp. 1394–1427, July 2007.
5. G. Ren, Y. Chang, and H. Zhang, “SNR estimation algorithm based on the preamble for wireless

OFDM systems," Science in China Series F: Information Sciences, vol. 51, no. 7, pp. 965–974, July 2008.

6. S. Boumard, "Novel noise variance and SNR estimation algorithm for wireless MIMO OFDM systems," in Proc. of GLOBECOM '03., vol. 3, pp. 1330–1334 vol.3, Dec. 2003.
7. S.Coleri, M.Ergen, A. Puri, and A,Bahai, "Channel estimation techniques based on pilot arrangement in OFDM systems", IEEE Trans.Broadcast., vol.48,no.3,pp.223-229,Sep2002.
8. J. G. Andrews, A. Ghosh, and R. Muhamed, Fundamentals of WiMAX: Understanding Broadband Wireless Networking. Upper Saddle River, NJ, USA: Prentice Hall PTR, 2007.
9. N. Alagha, "Cramer-Rao bounds of SNR estimates for BPSK and QPSK modulated signals," IEEE Commun. Lett., vol. 5, no. 1, pp. 10–12, Jan 2001.
10. D. Pauluzzi and N. Beaulieu, "A comparison of SNR estimation techniques for the AWGN channel," IEEE Trans. Commun., vol. 48, no. 10, pp. 1681–1691, Oct 2000.
11. R. Nee and R. Prasad, OFDM for Wireless Multimedia Communication, Artech House, London, 2000.
12. A. Bahai, B. Saltzberg and M. Ergen, Multi carrier digital communications: Theory and applications of OFDM, Springer Science, New York, 2004
13. M.Zivkovic and R.Mathar "Preamble based SNR estimation in frequency selective channels for wireless OFDM systems", IEEE VTC 2009.
14. Yushi Shen and Ed Martinez , "Channel Estimation in OFDM Systems", IEEE VTC 2006
15. T.S.Gagandeep, and P.Darwin, "Performance analysis of extended SNR estimation for OFDM system under AWGN and Rayleigh channels",IJAEST-2011, Vol No. 9, Issue No. 1, 061 - 067