

Performance Estimation Of OFDM System Using Space Time Trellis Coded Modulation

Abstract

Space-Time Trellis Codes (STTCs) are based on well defined trellis structures and can be decoded using soft-decision decoding techniques at the receiver, such as Viterbi decoding. In this paper, an encoder and Viterbi decoder are implemented to simulate the performance of Orthogonal Frequency Division Multiplexing (OFDM) system using STTCs in a second order diversity i. e. two transmit and one receive antennas in terms of bit error rate (BER), symbol error rate (SER), frame error rate (FER) and channel capacity.

Keywords: STTCs, OFDM, BER, SER, FER.

I. Introduction

Space-Time Trellis Codes (STTCs) were introduced in 1998 [1] as a high- data rate, bandwidth and power-efficient method of communication over wireless Rayleigh and Rician fading channels. STTCs can achieve a diversity advantage by placing the diversity burden on the BS, and hence leaving the MS to maintain its mobility and practicality [1]. Tarokh et al. [2] derived the design criteria for STTCs over slow frequency non-selective fading channels. The design criteria were shown to be determined by the distance matrices constructed from pairs of distinct codewords. The minimum rank of the distance matrices was used to determine the diversity gain, and the minimum distance of the distance matrices was used to determine the coding gain [3]. The trellis codes used in the simulations are shown in Table 1.1 and Table 1.2 [3], [4], and [5].

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Table 1.1 Coefficient pairs for 4PSK, 4-, 8- and 16-state STTC codes

Table 1.2 Coefficient pairs for 8PSK, 8-state STTC code

V	(a_0^1, a_0^2)	(a_1^1, a_1^2)	(b_0^1, b_0^2)	(b_1^1, b_1^2)	(c_0^1, c_0^2)	(c_1^1, c_1^2)
3	(0,4)	(4,0)	(0,2)	(2,0)	(0,1)	(5,0)

Each G matrix has the dimensions of $(i + s) * n$,

V	(a_0^1, a_0^2)	(a_1^1, a_1^2)	(a_2^1, a_2^2)	(b_0^1, b_0^2)	(b_1^1, b_1^2)	(b_2^1, b_2^2)
2	(0,2)	(2,0)	-	(0,1)	(1,0)	-
3	(0,2)	(2,0)	-	(0,1)	(1,0)	(2,2)
4	(0,2)	(2,0)	(0,2)	(0,1)	(1,2)	(2,0)

where $i = \log_2 M$ represents the number of information bits transmitted, s represents the number of shift registers in the encoder and n represents the number of transmit antennas. The elements of this matrix define the coefficient pairs described earlier in the encoder structure. The matrix for any number of states (4, 8, 16, 32), for a two-transmit antenna space-time code is

$$G = \begin{bmatrix} a_0^1 & a_0^2 \\ b_0^1 & b_0^2 \\ a_1^1 & a_1^2 \\ b_1^1 & b_1^2 \\ a_2^1 & a_2^2 \\ b_2^1 & b_2^2 \\ \vdots & \vdots \\ a_{v_1}^1 & a_{v_1}^2 \\ b_{v_2}^1 & b_{v_2}^2 \end{bmatrix}$$

The codes presented here provide the best tradeoff between data rate, diversity advantage, and trellis complexity [3].

A. Trellis Coded Modulation

In [6] author explained trellis based error correcting codes and signal constellations such as 8-PSK modulation. The paths in the trellis specify the allowable sequences of transmitted symbols. Note that since pair of parallel transitions differs in only the most significant bit, the phases associated with this pair of parallel transitions differ from each other by 180° . For the decoding operation when one transmits one of these 8 phases over an AWGN channel, the noisy received signal is first mapped by the front end of the decoder into a point in the plane

say this point $R = (R_x, R_y)$. If there were no noise, the point would be the point corresponding to the transmitted phase. With noise, the point will be displaced from the transmitted phase by a noise vector. This is shown in Figure 1.1 (a) and (b). The proper branch matrix to be used by the Viterbi decoder is the squared Euclidean distance between the phase corresponding to that branch and the received point R.

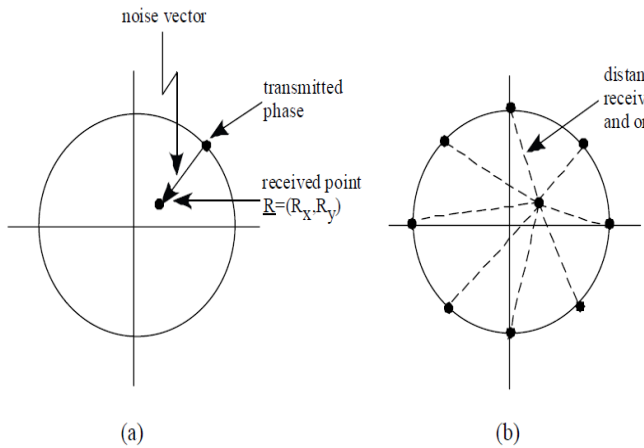


Fig. 1.1 Effects of noise on 8 PSK and Euclidean Distance

It should be noted that when Viterbi decoding is applied for every pair of parallel branches, only the branch with the smaller squared Euclidean is used. However, which of these two branches is used at every step in the winning path. This is how the trellis code protects the most significant bit. The system model for STTC modulation is shown in figure 1.2.

A. Performance Criteria

Consider the STTC codeword

$$c = c_1^1 c_1^2 c_1^3 \dots c_1^n c_2^1 c_2^2 \dots c_2^n \dots c_L^1 c_L^2 \dots c_L^n \quad (1.1)$$

that is transmitted in a frame of length L from the n transmit antennas, and the erroneous codeword

$$e = e_1^1 e_1^2 e_1^3 \dots e_1^n e_2^1 e_2^2 \dots e_2^n \dots e_L^1 e_L^2 \dots e_L^n \quad (1.2)$$

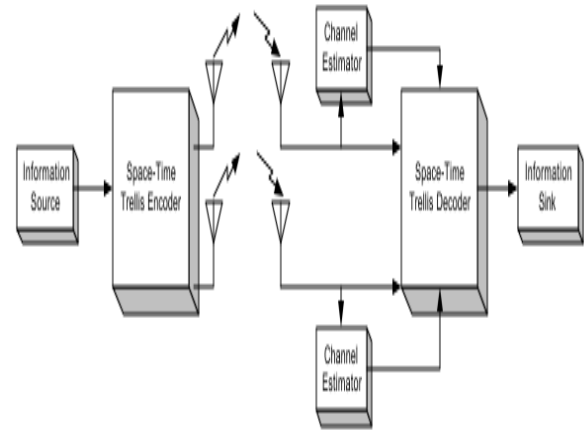


Fig 1.2 Space Time Trellis Code Model

In order to achieve the maximum diversity order $n \cdot m$ where m receive antennas and n transmit antennas), the $(n \times L)$ difference matrix has to be full rank for the codewords c and e .

$$B(c, e) = \begin{bmatrix} e_1^1 - c_1^1 & e_2^1 - c_2^1 & \dots & e_L^1 - c_L^1 \\ e_1^2 - c_1^2 & e_2^2 - c_2^2 & \dots & e_L^2 - c_L^2 \\ \vdots & \vdots & \ddots & \vdots \\ e_1^n - c_1^n & e_2^n - c_2^n & \dots & e_L^n - c_L^n \end{bmatrix}$$

If $B(c, e)$ has minimum rank r over the set of pairs of distinct codewords, then a diversity of $r \cdot m$ is achieved [7].

Let $A(c, e) = B(c, e)B^*(c, e)$ be the distance matrix, where $B^*(c, e)$ is the Hermitian of $B(c, e)$. The rank of A is r ; the kernel of A has a minimum dimension $n-r$, and exactly $n-r$ eigenvalues of A are zero. The non-zero eigenvalues of A can be denoted by $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n$. Assuming perfect channel state information (CSI), the probability of transmitting c and deciding in favour of e at the decoder is given by [4]

$$P(c \rightarrow e | h_{ij}, i = 1, 2, \dots, m) \leq \exp\left(-d^2(c, e) \frac{E_s}{4N_0}\right) \quad (1.3)$$

where $N_0/2$ is the noise variance per

dimension and the Euclidean distance is

$$d^2(c, e) = \sum_{j=1}^m \sum_{t=1}^L \left| \sum_{j=1}^n h_{ij} (c_t^j - e_t^j) \right|^2 \quad (1.4)$$

It follows from [3] that the pair-wise error bound is given by

$$P(c \rightarrow e) \leq \left(\prod_{l=1}^r \lambda_l \right)^{-m} \left(\frac{E_s}{4N_0} \right)^{-rm} \quad (1.5)$$

Thus to achieve the best performance for a given system satisfy the following rank and determinant criteria [4].

Maximize the diversity gain given by $r \cdot m$, or maximize over all possible codeword pairs c and e the minimum rank of matrix $A(c, e)$. The minimum rank of the matrix A taken over all codeword pairs is also called the rank of the code. Maximize the coding gain specified by $(\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n)^{\frac{1}{r}}$ where $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n$ is the absolute value of the sum of determinants of all the principal $(r \times r)$ cofactors of A . If full rank $r = n$ is achievable, it is equivalent to maximizing the determinant of $A(c, e)$ taken over all possible codeword pairs c and e [4]. The encoder structure for the 4PSK scheme with two transmits antennas and one receive antenna is shown in figure 1.4.

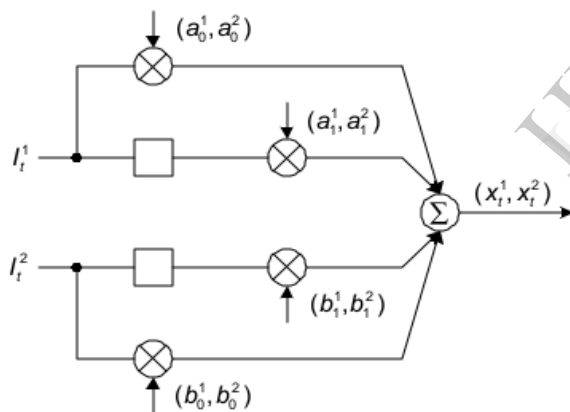


Fig. 1.4 4-State, PSK Encoder Structure

II. Literature Survey

Chang et al. proposed the original OFDM principles in 1966, and successfully achieved a patent in January of 1970. The concept of multicarrier transmission was first explicitly proposed by Chang [8] in 1966. A detailed description of multicarrier can also be found in [9] and [10]. In 1971, Weinstein and Ebert proposed a modified OFDM system [11] in which the discrete Fourier Transform (DFT) was applied to generate the orthogonal

subcarriers waveforms instead of the banks of sinusoidal generators. The implementation of MC systems with equalization was investigated by Hirosaki et al. [12, 13] and Peled and Ruiz [14]. Cyclic prefix (CP) or cyclic extension was first introduced by Peled and Ruiz in 1980 [14] for OFDM systems with the trade-off of the transmitting energy efficiency, this new scheme can result in a phenomenal ISI (Inter Symbol Interference) reduction. Hence it has been adopted by the current IEEE standards. In 1980, Hirosaki introduced an equalization algorithm to suppress both inter symbol interference (ISI) and ICI [12], which may have resulted from a channel distortion, synchronization error, or phase error. Zimmerman and Kirsch [15] discussed the applications of MC in HF radio in 1967. More materials on the HF application of MC can be found in [16]. In 1985, Cimini first applied OFDM in mobile wireless communications [17]. In [18], Casas and Leung discussed the applications of MC over mobile radio FM channels. Shannon's classical paper in 1948 suggested that the highest data rate can be achieved for frequency-selective channels by using an MC system with an infinitely dense set of sub-channels and adapting transmission powers and data rates according to the signal-to-noise ratio (SNR) at different sub-channels. Some of the earlier inventions on practical loading algorithms for OFDM or DMT systems were in [19, 20]. In the late 1970s, Ungerboeck [21] and Imai and Hirakawa [22] independently presented two of the most powerful applicable coded modulation techniques namely trellis coded modulation (TCM) and multilevel coded modulation (MLC), respectively. Ungerboeck's TCM is based on mapping by binary set partitioning, whereby the signal set, with an underlying signal constellation of $M = 2^m$ points, is successively binary partitioned in m or fewer steps to define a mapping of binary addresses to signal points. It maximizes the minimum intra-subset Euclidean distance. In the encoder, the binary addresses are usually divided into least significant binary symbols, which are convolutionally encoded, and most

significant binary symbols, which if present are left uncoded. An exhaustive computer search is usually used to find the corresponding code parameters, in order to maximize the minimum distance between coded sequences in Euclidean space. A simple analogy might be helpful in understanding the overall goals in TCM. Imagine that there is an all-knowing wizard at the transmitter. As the message bits enter the system, the wizard recognizes that some of the bits are most vulnerable to the degradation effects of channel impairments; hence, they are assigned modulation waveforms associated with the best distance properties. Similarly, other bits are judged to be very robust, and hence, they are assigned waveforms with poorer distance properties. Modulation and coding take place together. The wizard is assigning waveforms to bits (modulation), but, the assignment is being performed according to the criterion of better or worse distance properties (channel coding). Multi level coding (MLC) [22-24] and [25] splits the transmission channel into several logical sub-channels. The number of sub-channels depends on the size of the signal constellation of the underlying modulation scheme. As the sub-channels are separated, we can employ a multi stage decoder (MSD) which will decode the component codes sequentially starting with the most powerful component code first and use its output decisions (assumed to be correct) in the decoding of the subsequent and weaker code sequences. A MSD can potentially achieve the performance of a very large and complex code, but requires considerably lower decoding complexity [25]. The idea behind MLC, as originally described by Imai and Hirakawa [22], was to protect each bit in the label of a signal point with an independent binary code. This sort of protection implicitly assumes that some form of partitioning is being employed. Originally these codes were proposed for one-dimensional signaling combined with labeling by binary counting of the signal levels. The partitioning strategy was to maximize the minimum intra-subset Euclidean distance, in a similar manner to the TCM schemes developed by Ungerboeck.

Unlike TCM, however, the MLC approach provides flexible transmission rates, through the use of multiple component codes that may have different rates. Furthermore, any code can be used as a component code. Space time coded OFDM for high data rate wireless communication over wideband channels [26] gave us a design which combines the OFDM and modulated physical layer and combines coding and modulation. Space time codes were proposed for narrowband wireless channels [2]. These codes have high spectral density and operate at very low SNR.

III. Simulations Results and discussions

The proposed work reduces the Bit error rate (BER), Frame error rate (FER) and Symbol error rate (SER). Encoding of data is carried out by Trellis Encoding. M-PSK modulation is done with the help of 4/8/16/32 states considering an Additive White Gaussian Noise (AWGN) channel. Figure 3.1 presents the performance of the STTC with 4 PSK 4 states using 2x1 antennas.

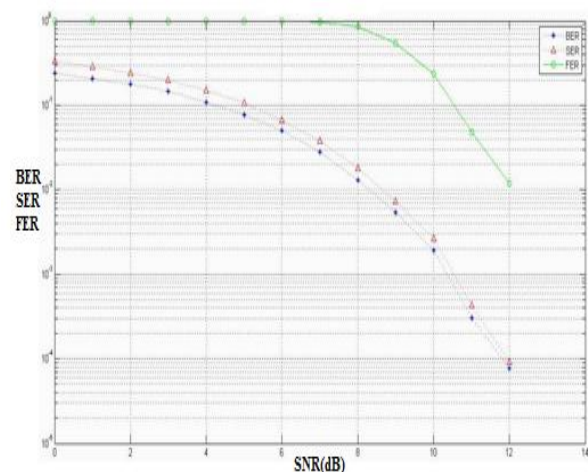


Fig 3.1 BER, SER & FER Graph of STTC 4 PSK 4 States Modulation with OFDM using 2x1 Antenna

Figure 3.2 and 3.3 presents the performance of the STTC with 4 PSK 4 states using 2x2 and 2x3 antennas. In figure 3.4, 3.5 and 3.6 presents the performance of the STTC with 8 PSK 32 states using 2x2 and 2x3 antennas. As the number of states and modulation increases, from 4 PSK 4 states to 8 PSK 32 states, the 2x1 antennas for 8 PSK 16 states

shows better performance, shown in figure 3.7, as compared to other antenna diversity system but the system complexity increases.

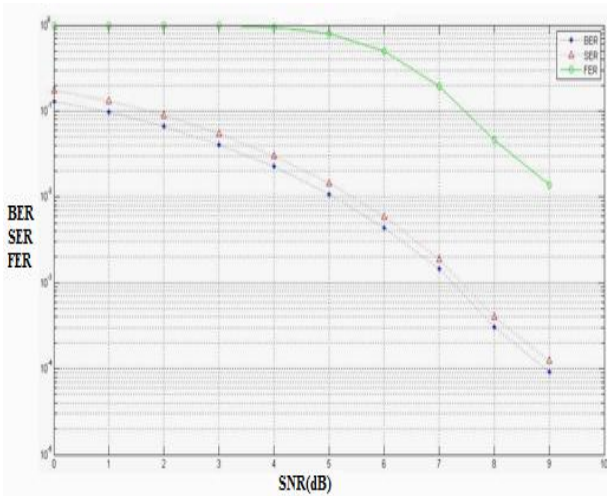


Fig 3.2 BER, SER & FER Graph of STTC 4 PSK 4 States Modulation with OFDM using 2x2 Antenna

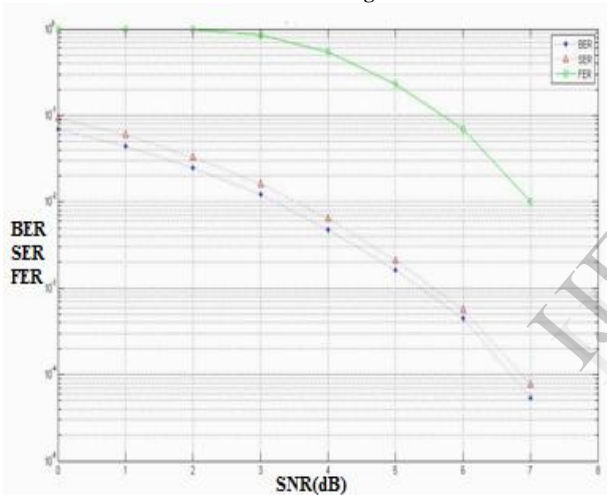


Fig 3.3 BER, SER & FER Graph of STTC 4 PSK 4 States Modulation with OFDM using 2x3 Antenna

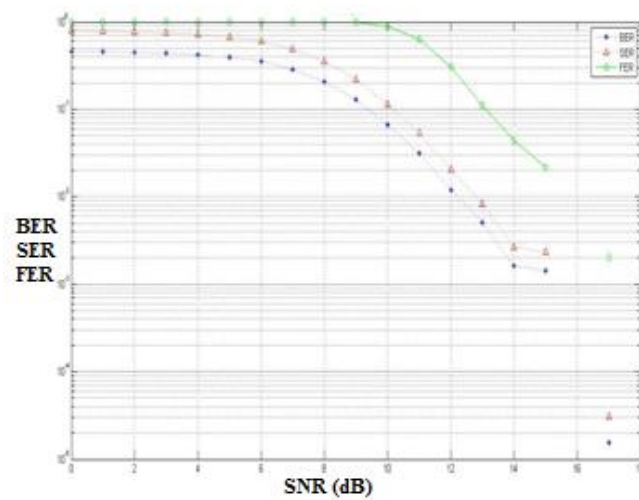


Fig 3.4 BER, SER & FER Graph of STTC 8 PSK 32 States Modulation with OFDM using 2x1 Antenna

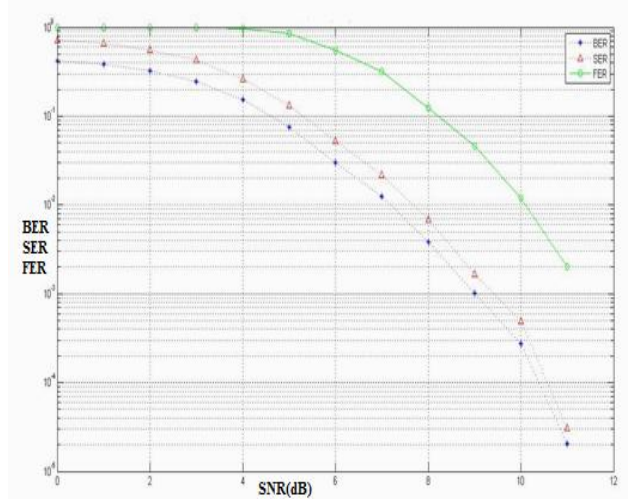


Fig 3.5 BER, SER & FER Graph of STTC 8 PSK 32 States Modulation with OFDM using 2x2 Antenna

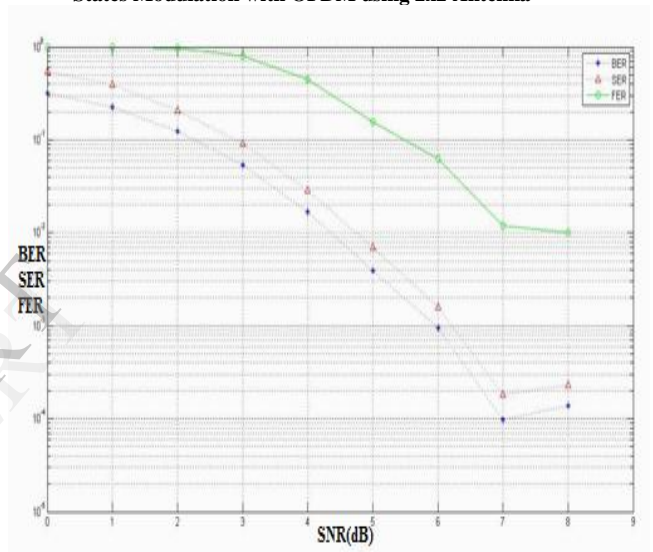


Fig 3.6 BER, SER & FER Graph of STTC 8 PSK 32 States Modulation with OFDM using 2x3 Antenna

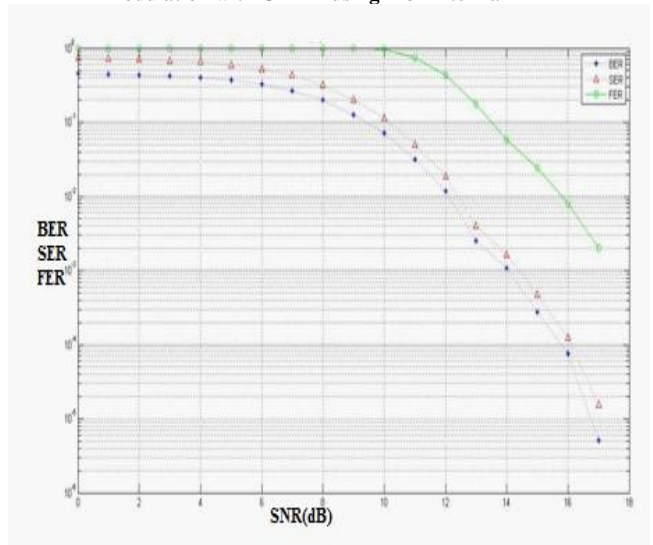


Fig 3.7 BER, SER & FER Graph of STTC 8 PSK 16 States Modulation with OFDM using 2x1 Antenna

Figure 3.8 shows the channel capacity using 4/8 PSK & QAM TCM OFDM with 2x3 Antenna.

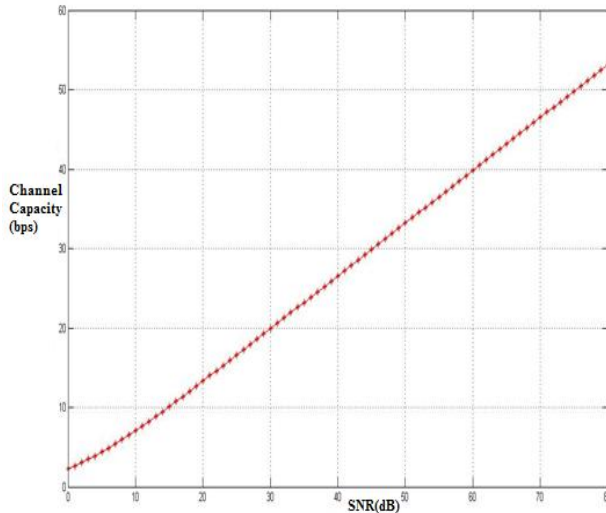


Fig 3.8 Channel Capacity using 4/8 PSK & QAM TCM OFDM with 2x3 Antenna

IV. Conclusions

This paper proposed the design of OFDM system with the help of QPSK/QAM using trellis code modulation and reduces Bit Error Rate (BER), Frame Error Rate and Symbol Error Rate and as Signal-to-Noise Ratio increases the BER, FER and SER decreases but at the expense of system complexity. This work can be implemented in VHDL and also with reduced complexity.

V. References

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