

Performance Evaluation of Low cost INS/GPS System Integration

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Abstract— Nowadays, Navigation is becoming more common in different areas of industry and commercial sectors especially for the systems where position calculation is required. The main method depends on using a Global Positioning System (GPS) alone, however, high level of accuracy for specific applications is required, and it can be achieved by the integration between inertial measurement unit (IMU) and GPS. This paper discusses the overall inertial navigation system (INS) system structure using low cost IMU sensors, the navigator and error equation, enhancement of performance using GPS, integration sequence, and validation of the results after INS error correction of the internal navigation states. Finally, this paper presents the effectiveness of low cost INS system to calculate internal navigation states, verification of the error correction technique, and the performance of the overall system to obtain the navigation system requirements.

Keywords— IMU; INS; GPS; kalman filter; navigation states

I. INTRODUCTION

The idea of this paper is to combine the advantage of the short term precision of INS and the long-term stability of GPS to decrease the cost of the inertial navigation system. INS is a system of gyroscopes and accelerometers designed to measure specific force and angular rates with respect to an inertial frame to provide velocity, position and attitude [1]. INS cannot be jammed but the error of INS grows quickly over time. GPS is a satellite-based navigation system, which provides accurate position and velocity information but GPS receiver does not provide attitude data and can be jammed [2]. The major error sources in the inertial navigation system are due to gyro and accelerometer inertial sensor imperfections, incorrect navigation system initialization, and imperfections in the gravity model used in the computations. Whether the inertial sensor errors are caused by internal mechanical imperfections, electronics errors, or other sources, these error sources affect the indicated outputs of these devices. For the gyros, the major errors are in measuring angular rates. For the accelerometers, the major errors are in measuring specific force [3]. Integration of GPS with an inertial Navigation System improves the quality and integrity of each navigation system. Use of GPS permits calibration of inertial instrument biases, and the INS can be used to improve the tracking and reacquisition performance of the GPS receiver [4]. Kalman filtering is used when integrating an INS with aiding systems such as GPS for the estimation and compensation of the inertial sensors errors. Kalman filter is an algorithm for optimally estimating the error states of a system from measurements contaminated by noise. This is a sequential recursive algorithm that provides an optimal least mean variance estimation of the error states [5].

II. PRINCIPLES

A. INS mechanization process

INS mechanization is the process of determining the navigation states (position, velocity and attitude) from the raw inertial measurements (accelerations and angular rates) of IMU through solving the differential equations describing the system motion in the local level frame [6]. The rate of change of the position components is related to the velocity components as follow:

$$\dot{r}^l = \begin{bmatrix} \dot{\phi} \\ \dot{\lambda} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{R+h} & 0 \\ \frac{1}{R+h \cos \varphi} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V^N \\ V^e \\ V^u \end{bmatrix} = D^{-1} V^l \quad (1)$$

Where φ is the latitude angle, λ is the longitude angle, h is the altitude, R is the earth radius, V^N is the velocity component in the north direction, V^e is the velocity component in the east and V^u is the velocity component in the up direction. The acceleration components expressed in the local-level frame cannot directly provide the LLF velocity components of the moving platform due to three reasons:

The first reason is due to the Earth rotation rate ($\omega^e=15$ [deg/hr]), which is interpreted in the local-level frame as an angular velocity vector ω_{ie}^l :

$$\omega_{ie}^l = \begin{bmatrix} 0 \\ \omega^e \cos \varphi \\ \omega^e \sin \varphi \end{bmatrix} \quad (2)$$

The second reason is the change of orientation of the local-level frame with respect to the Earth. This change of orientation is due to the definition of the local North and vertical directions. The North direction (N) is tangential to the meridian all the time while the vertical direction is normal to the Earth's surface. This effect is expressed by the angular velocity vector ω_{el}^l :

$$\omega_{el}^l = \begin{bmatrix} -V^N \\ \frac{V^e}{R+h} \\ \frac{V^e \tan \varphi}{R+h} \end{bmatrix} \quad (3)$$

The third reason is due to the Earth's gravity field

$$g^l = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} \quad (4)$$

Taking into consideration all the previous factors, the rate of change of the velocity components of a moving body in LLF can be expressed as follows:

$$\dot{V}^l = R_b^l f^b - (2\Omega_{ie}^l + \Omega_{el}^l) V^l + g^l \quad (5)$$

Where Ω_{ie}^l is the skew symmetric matrix of ω_{ie}^l and Ω_{el}^l is the skew symmetric matrix of ω_{el}^l . Attitude mechanization equations are:

$$\dot{R}_b^l = R_b^l \Omega_{ib}^b = R_b^l (\Omega_{ib}^b - \Omega_{il}^l) \quad (6)$$

Where Ω_{ib}^b the skew symmetric matrix of is ω_{ib}^b , Ω_{il}^l is the skew symmetric matrix of ω_{il}^l and R_b^l is the rotation matrix. Then the overall mechanization equation is:

$$\begin{bmatrix} \dot{r}^l \\ \dot{V}^l \\ \dot{R}^l \end{bmatrix} = \begin{bmatrix} D^{-1} V^l \\ R_b^l f^b - (2\Omega_{ie}^l + \Omega_{el}^l) V^l + g^l \\ R_b^l \Omega_{ib}^b \end{bmatrix} \quad (7)$$

The structure of the algorithm is given by the scheme in Fig. 1.

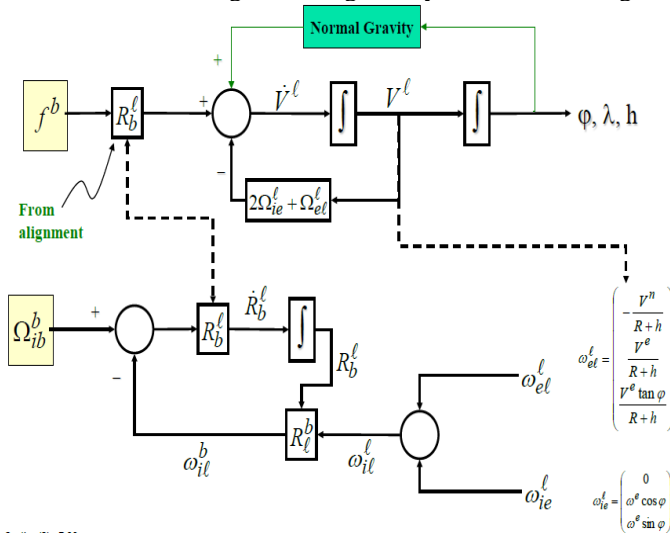


Fig 1. Scheme of INS mechanization defined for local Navigation frame.

B. GPS navigation equations

The GPS receiver calculates the positioning with the help of a group of satellites. Each satellite travels in its orbit around the earth. Using several known positions (of the satellites) and the measured distances between the receiver and the satellites, the position of the receiver can be estimated. The GPS receiver does this task by receiving the signals from at least four satellites organizes the navigation equations and by solving them can get the position of the user as follow [7]:

$$(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2 = (r_1 - c\Delta T_1)^2 \quad (8)$$

$$(x_2 - x)^2 + (y_2 - y)^2 + (z_2 - z)^2 = (r_2 - c\Delta T_2)^2 \quad (9)$$

$$(x_3 - x)^2 + (y_3 - y)^2 + (z_3 - z)^2 = (r_3 - c\Delta T_3)^2 \quad (10)$$

$$(x_4 - x)^2 + (y_4 - y)^2 + (z_4 - z)^2 = (r_4 - c\Delta T_4)^2 \quad (11)$$

Where ΔT_i is time difference between the receiver and i th satellite, r_i is the distance of i th satellite from the receiver, (x, y, z) is the receiver position, (x_i, y_i, z_i) is the i th satellite position and c is light velocity.

III. INS/GPS INTEGRATION SYSTEM

The integrated navigation system provides a full navigation parameters (position, velocity and attitude). Kalman filtering technique is employed to optimally fuse the INS and GPS positioning and navigation information to yield a reliable navigation solution. INS provides continuity in the navigation solution, attitude information and bridges GPS signal outages while, GPS prevents the inertial solution from drifting.

A. Dynamic error model of INS equations

The error state vector for the mechanization equations in the local-level frame consists of the errors along the curvilinear geodetic coordinates (latitude error $\delta\phi$, longitude error $\delta\lambda$, altitude error δh) as follow [8]:

$$\delta\dot{r}^l = \begin{bmatrix} \delta\dot{\phi} \\ \delta\dot{\lambda} \\ \delta\dot{h} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{M+h} & 0 \\ \frac{1}{(N+h)\cos\phi} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \delta V^e \\ \delta V^n \\ \delta V^u \end{bmatrix} \quad (12)$$

The errors along the earth-referenced velocities (east component error δV^e , north component error δV^n , up component error δV^u) as follow:

$$\delta\dot{V}^l = \begin{bmatrix} \delta\dot{V}^e \\ \delta\dot{V}^n \\ \delta\dot{V}^u \end{bmatrix} = \begin{bmatrix} 0 & f^u & -f^n \\ -f^u & 0 & f^e \\ f^n & -f^e & 0 \end{bmatrix} \begin{bmatrix} \delta p \\ \delta r \\ \delta A \end{bmatrix} + R_b^l \begin{bmatrix} \delta f^x \\ \delta f^y \\ \delta f^z \end{bmatrix} \quad (13)$$

Where p is the pitch angle, r is the roll angle and A is the azimuth angle. The errors along the three attitude angles (pitch error δp , roll error δr , azimuth error δA) as follow:

$$\Psi^l = \begin{bmatrix} \delta\dot{p} \\ \delta\dot{r} \\ \delta\dot{A} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{M+h} & 0 \\ \frac{-1}{N+h} & 0 & 0 \\ \frac{-\tan\phi}{N+h} & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta V^e \\ \delta V^n \\ \delta V^u \end{bmatrix} + R_b^l \begin{bmatrix} \delta\omega^x \\ \delta\omega^y \\ \delta\omega^z \end{bmatrix} \quad (14)$$

It also includes the accelerometers biases as follow [9]:

$$\delta f^b = \begin{bmatrix} \delta f^x \\ \delta f^y \\ \delta f^z \end{bmatrix} = \begin{bmatrix} -\beta_{fx} & 0 & 0 \\ 0 & -\beta_{fy} & 0 \\ 0 & 0 & -\beta_{fz} \end{bmatrix} \begin{bmatrix} \delta f^x \\ \delta f^y \\ \delta f^z \end{bmatrix} + \begin{bmatrix} \sqrt{2\beta_{fx}\sigma_{fx}^2} \\ \sqrt{2\beta_{fy}\sigma_{fy}^2} \\ \sqrt{2\beta_{fz}\sigma_{fz}^2} \end{bmatrix} w(t) \quad (15)$$

where β_{fx} , β_{fy} , β_{fz} are the reciprocals of the correlation times associated with the autocorrelation sequence of δf^x , δf^y , δf^z . σ_{fx}^2 , σ_{fy}^2 , σ_{fz}^2 are the variances associated with the accelerometer errors. $w(t)$ is white Gaussian noise with variance equal to one. The gyroscopes drift as follow:

$$\delta\dot{\omega}^b = \begin{bmatrix} \delta\dot{\omega}^x \\ \delta\dot{\omega}^y \\ \delta\dot{\omega}^z \end{bmatrix} = \begin{bmatrix} -\beta_{\omega x} & 0 & 0 \\ 0 & -\beta_{\omega y} & 0 \\ 0 & 0 & -\beta_{\omega z} \end{bmatrix} \begin{bmatrix} \delta\omega^x \\ \delta\omega^y \\ \delta\omega^z \end{bmatrix} + \begin{bmatrix} \sqrt{2\beta_{\omega x}\sigma_{\omega x}^2} \\ \sqrt{2\beta_{\omega y}\sigma_{\omega y}^2} \\ \sqrt{2\beta_{\omega z}\sigma_{\omega z}^2} \end{bmatrix} w(t) \quad (16)$$

Where $\beta_{\omega x}, \beta_{\omega y}, \beta_{\omega z}$ are the reciprocals of the correlation times associated with the autocorrelation sequence of $\delta\omega^x, \delta\omega^y, \delta\omega^z$. $\sigma_{\omega x}^2, \sigma_{\omega y}^2, \sigma_{\omega z}^2$ are the variances associated with the gyroscope errors. $w(t)$ is white Gaussian noise with variance equal to one. Finally the state equation for the INS errors is:

$$\dot{X}^l = \begin{bmatrix} \delta\dot{\gamma}^l \\ \delta\dot{V}^l \\ \dot{\psi}^l \\ \delta\dot{\omega}^b \\ \delta\dot{f}^b \end{bmatrix} \quad (17)$$

B. Models of INS/GPS integration and kalman filter

There are two models of the INS/GPS integration system: Firstly, the system model as follow [10]:

$$\dot{X}^l = F^l X^l + G^l w \quad (18)$$

Where F^l is the dynamic coefficient matrix, X^l is the state vector includes error components of position, velocity and attitude, G^l is the noise distribution vector and w is the unit-variance white Gaussian noise. Secondly, the measurement is modeled as follow [11]:

$$Z_k = H_K X_k + \eta_k \quad (19)$$

Where Z_k is consists of the differences between the position coordinates and velocities predicted by the INS and the corresponding values measured by the GPS, H_K is the measurement design matrix at time t_k , X_k is the state vector of INS, η_k is a vector of measurement noise which is zero-mean with covariance R_k . From the navigation error equations, a state model is settled which is required for the operation of Kalman filter as follow [12]:

$$X_k = \phi_{k-1} X_{k-1} + G_{k-1} w_{k-1} \quad (20)$$

Where X_k is the state vector, ϕ_{k-1} is the state transition matrix (STM), G_{k-1} is the noise distribution matrix, w_{k-1} is the process noise vector k is the measurement epoch. Kalman filter estimates the states of the system by operating as feedback loop and the operation of Kalman filter has two phases (prediction phase and correction phase) [13]. Prediction or time update phase as follow [14]:

$$\hat{X}_k(-) = \phi_{k-1} X_{k-1}(+) \quad (21)$$

$$P_k(-) = \phi_{k-1} P_{k-1}(+) \phi_{k-1}^T + G_{k-1} Q_{k-1} G_{k-1}^T \quad (22)$$

Where $\hat{X}_k(-)$ is Predicted or a priori value of the estimated state vector of a linear dynamic system, $X_{k-1}(+)$ is corrected or a posteriori value of the estimated state vector of a linear dynamic system, $P_k(-)$ is predicted or a priori matrix of the estimation covariance of state estimation uncertainty in matrix form and $P_{k-1}(+)$ corrected or a posteriori matrix of estimation covariance of state estimation uncertainty in matrix form. Correction or measurement update phase as follow:

$$K_k = P_k(-) H_k^T / [H_k P_k(-) H_k^T + R_k] \quad (23)$$

$$\hat{X}_k(+) = \hat{X}_k(-) + K_k [Z_k - H_k \hat{X}_k(-)] \quad (24)$$

$$P_k(+) = [I - K_k H_k] P_k(-) \quad (25)$$

Where K_k is the Kalman gain. The integration using Kalman filter is given by the scheme in fig.2.

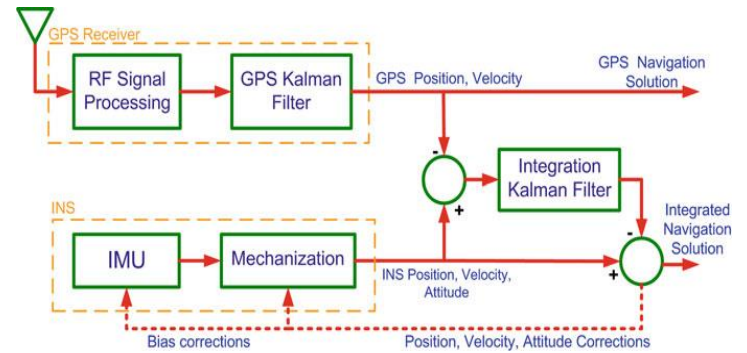


Fig 2. A block diagram of INS/GPS integration

IV. RESULTS AND SIMULATION

In this section, some simulation and performance results are shown. The input data of the simulation model comes from stationary IMU (MPU 6050). The simulation model is developed using SIMULINK in Matlab environment.

A. INS mechanization simulation results

Using the raw data (accelerations and angular rates) from stationary IMU can provide navigation states of INS by mechanization process.

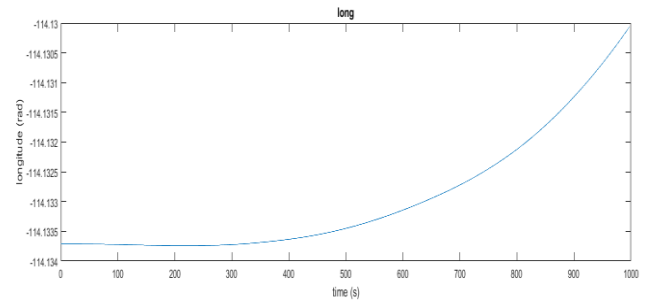


Fig 3. Longitude

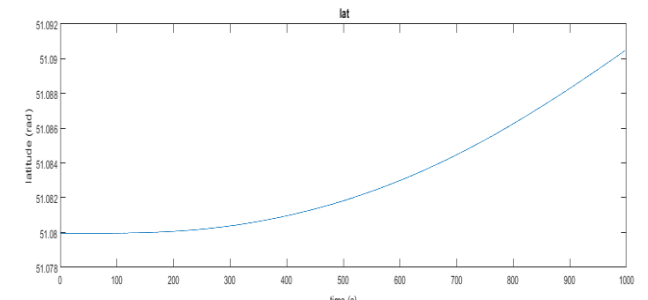


Fig 4. Latitude

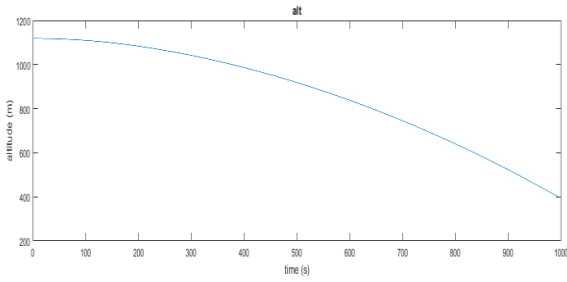


Fig 5. Altitude

Figures 3, 4 and 5 describe the position parameters (longitude, latitude and altitude).

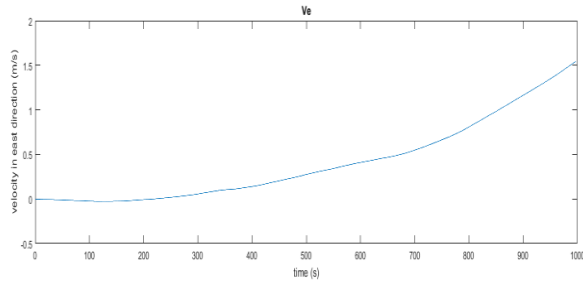


Fig 6. East velocity

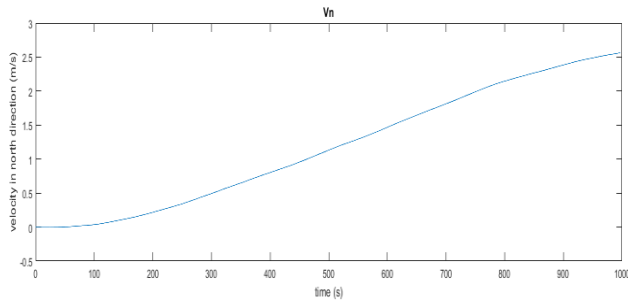


Fig 7. North velocity

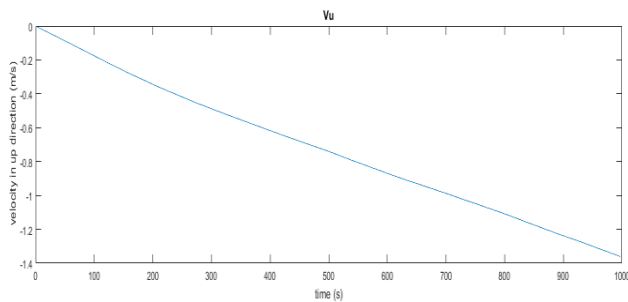


Fig 8. Up velocity

Figures 6, 7 and 8 describe the velocity parameters (east velocity, north velocity and up velocity).

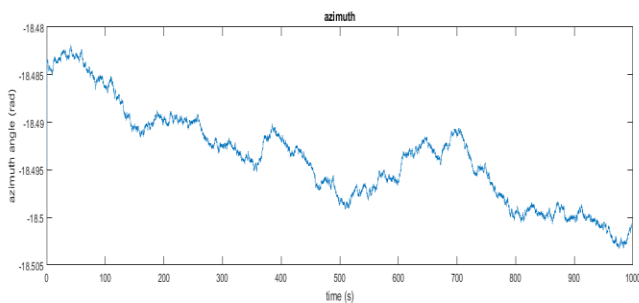


Fig 9. Azimuth angle

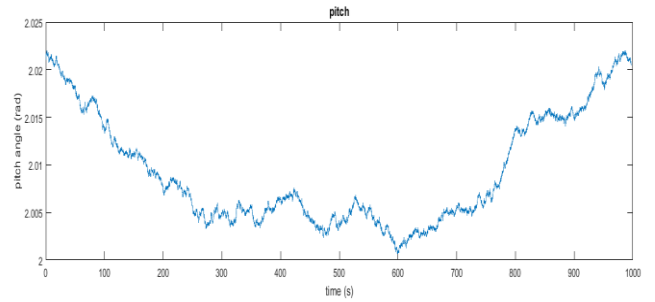


Fig 10. Pitch angle

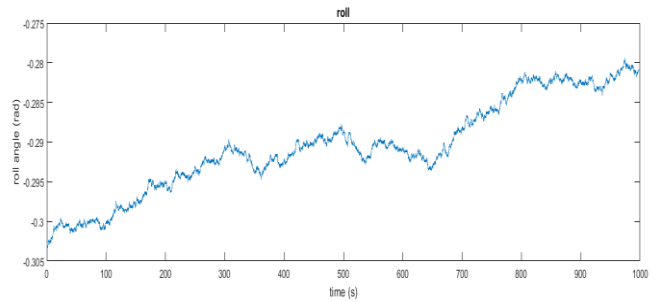


Fig 11. Roll angle

Figures 9, 10 and 11 describe the attitude parameters (azimuth angle, pitch angle and roll angle).

B. INS/GPS integration system simulation results

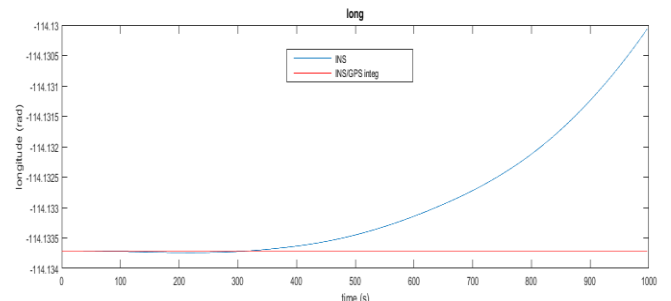


Fig 12. Comparison between longitude by INS and longitude by INS/GPS integrated system.

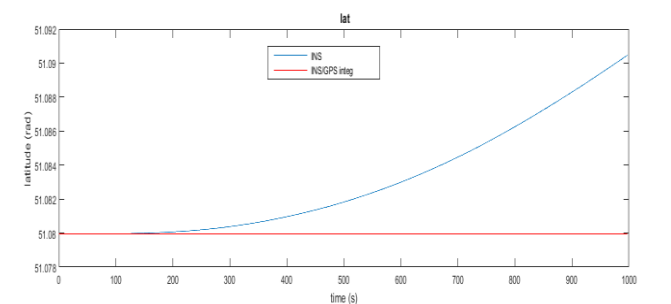


Fig 13. Comparison between latitude by INS and latitude by INS/GPS integrated system

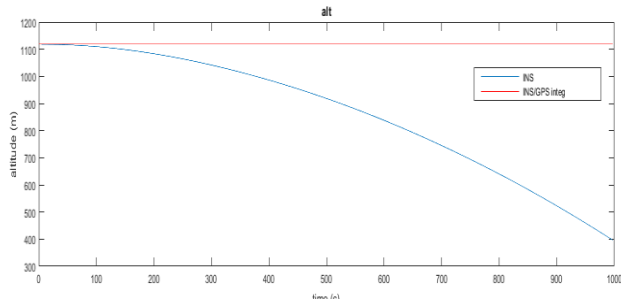


Fig 14. Comparison between altitude by INS and altitude by INS/GPS integrated system.

Figures 12, 13 and 14 describe the comparison between INS and INS/GPS integrated system in position parameters. It is clear that the integration process improves the position accuracy.

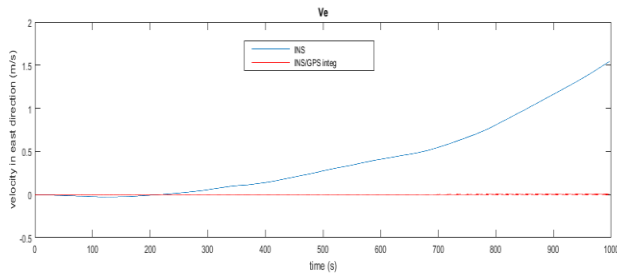


Fig 15. Comparison between east velocity by INS and east velocity by INS/GPS integrated system.

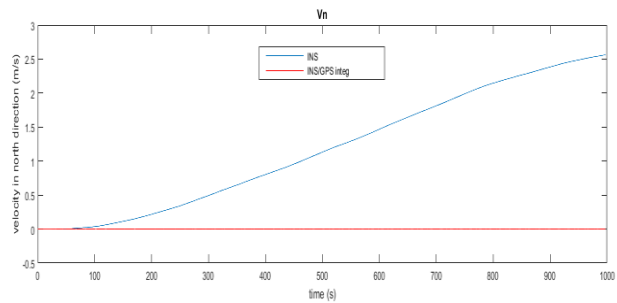


Fig 16. Comparison between north velocity by INS and north velocity by INS/GPS integrated system.

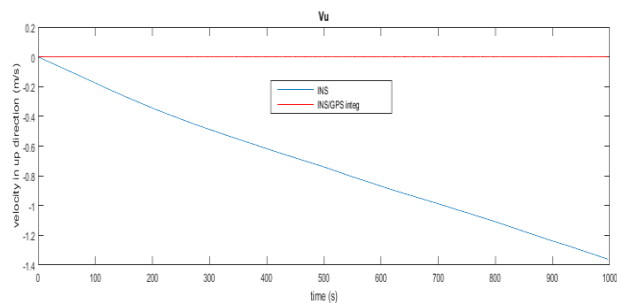


Fig 17. Comparison between up velocity by INS and up velocity by INS/GPS integrated system

Figures 15, 16 and 17 describe the comparison between INS and INS/GPS integrated system in velocity parameters. It is clear that the integration process improves the velocity accuracy.

CONCLUSION

In the present study, an INS/GPS integration using Kalman filter is carried out in order to gain the privilege of combination of the advantages of the short-term precision of INS and long-term stability of GPS. Also, an error analysis is carried out to

stand up on the possible sources of errors in this integrated system. The analysis of the present study leads to the following conclusions:

- 1- The combination used leads to a remarkable decrease in the inertial navigation system cost.
- 2- Integration of INS with GPS improves the quality of overall navigation system performance.
- 3- Usage GPS permits calibration of inertial instrument biases.
- 4- Usage INS improves the tracking and reacquisition performance of GPS receiver.
- 5- For both instruments, the two largest errors are usually a bias instability which is measured in deg/hr for gyro bias drift, or micro g for the accelerometer bias, and a scale-factor stability, which is usually measured in parts per million (ppm) of the sensed inertial quantity.
- 6- Kalman filter provides real time statistical data related to the estimation accuracy of the error states, which is very useful for quantitative error analysis.

Generally, it can be said that the integration of INS with GPS using Kalman filter helps improving remarkably the accuracy of navigation data (position and velocity) at low cost.

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