Performance Evaluation of Mean Square Error of Butterworth and Chebyshev1 Filter with Matlab

Mamta Katiar
Associate professor
Mahararishi Markandeshwer University , Mullana
Haryana,India.

Anju
Lecturer, Kalpana Chawala Govt. Polytechnic for Women, Ambala city, Haryana, India

Abstract

A signal is any physical phenomenon which conveys information of any kind from one place or person to another. In communication system, during the processing of signal, some noise is added in the signal and signal becomes noisy. This is now mandatory to extract the signal buried under noise and periodic interference. In this paper, a signal is denoised by Butterworth filter and Chebyshev1 filter and calculating mean square error and signal to noise ratio from reconstructed signal at receiver and then compare the Butterworth and Chebyshev1 filter to find the best results. For this evaluation, all data is coded in the MATLAB.

Keywords
Butterworth filter, Chebyshev1 filter, Mean square error, Signal to noise ratio.

"1.Introduction"

The Digital Filtering is one of the most powerful tools of DSP. The digital filters consist of software and hardware. The input and output signals in the digital filter is digital or discrete time variant. The procedure for designing digital filters involves the determination of a set of filter coefficients to meet a set of design specifications. Digital filters come in two flavours: FIR and IIR. As the terminology suggest, these classifications refer to the filters impulse response. By varying the weight of the coefficients and number of filter taps, virtually any frequency response characteristics can be realised with an FIR filter. FIR filters have a very useful property: they can exhibit linear phase shift for all frequencies. IIR filters have infinite impulse response. IIR filters have much better frequency response than FIR filters of the same error. In IIR filters their phase characteristics is not linear, which can cause a problem to the systems which need phase linearity but in MATLAB software data processing is commonly performed "offline", i.e. the entire data sequence is available prior to filtering[1]. This allows for a non causal, zero phase filtering approach (via the filtfilt function), which eliminates the non linear phase distortion of an IIR filters.IIR filters can achieve the same level of attenuation as FIR filters but with far fewer coefficients. Therefore, an IIR filter can provide a significantly faster and most efficient filtering operation than an FIR filter. This paper considers two IIR filters: Butterworth and Chebyshev1.

A. Butterworth Filter

The butterworth filter has a maximally flat response, i.e., no passband ripple and roll-off of minus 20db per pole. Another name for it is "flat maximally magnitude" filters at the frequency of $\Omega = 0$, as the first 2N - 1 derivatives of the transfer function when $\Omega = 0$ are equal to zero. [2]. The Butterworth filters achieve its flatness at the expense of a relatively wide transition region from passband to stopband with average transient characteristics. This filter is completely defined mathematically by two parameters i.e. cut of frequency and number of poles. Compared to chebyshev filter, the phase linearity of buttorworth filter is better. In other words, the group delay (derivative of phase with respect to frequency) is more constant with respect to frequency. This means that the waveform distortion of the butterworth filter is lower. This Butterworth filters have the following characteristics [3].

- The magnitude response is nearly constant (equal to 1) at lower frequencies. That means pass band is maximally flat.
- The response is monotonically decreasing from the specified cut off frequencies.
- The maximum gain occurs at Ω= 0 and it is |H(0)|= 1.
- Half power frequency, or 3db down frequency, that corresponds to the specified cut off frequencies.

The magnitude squared response of low pass Butterworth filter is given by
H(Ω)|=1/1+(Ω/Ωc)2N (1)

This equation is also expressed as

|H(Ω)|2=1/1+ C2(Ω/Ωp)2N (2)

Here |H(Ω)|=Magnitude of analog low pass filter.
Ωc=Cut-off frequency (-3db frequency)
Ωp=Pass band edge frequency.
C=Parameter related to ripples in pass band.
N=Order of the filter.

The order of filter means the number of stages used in the design of filter. As the order of filter N increases, the response of filter is more close to the ideal response as shown in Fig.1.

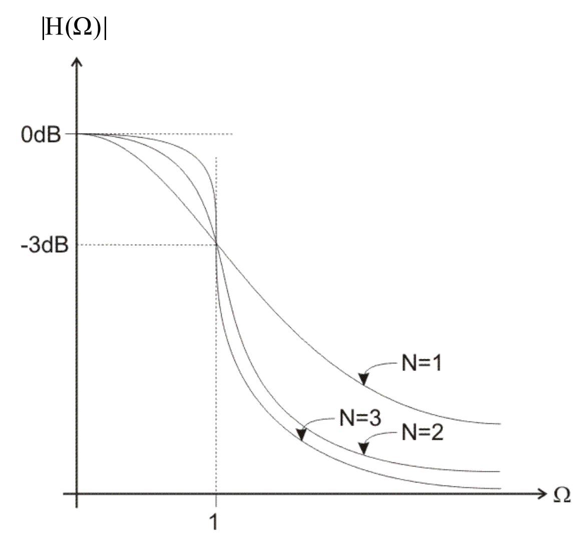


Fig.1.1- Effect of N on frequency response characteristics.

B. Chebyshev Type1 Filter

Chebyshev1 filters have a narrower transition region between the passband and the stopband. The sharp transition between the passband and the stopband of a chebyshev filter produces smaller absolute errors and faster execution speeds than a butterworth filter. The poles of chebyshev filter lies on an ellipse. ripple increase (band), the roll-off becomes sharper(good). The chebyshev filter is completely defined by three parameters-cut-off frequency, number of poles and passband ripples. The chebyshev response is a mathematical strategy for achieving a faster roll off by allowing ripple in the frequency response. The chebyshev response is an optimal trade-off between these two parameters. The magnitude squared frequency response is given by

|H(Ω)|2=1/1+ C2CN2(Ω/Ωp) (3)

Here |H(Ω)|=Magnitude of analog low pass filter.
C=Parameter related to ripples in pass band.
CN(x)=Chebyshev polynomial of order N

The chebyshev1 polynomials are determined by using the equations
CN+1(x)=2x CN(x)- CN-1(x) (4)
with C0(x)=1 and C1(x)=x

The following figure shows the frequency response of a lowpass Chebyshev1 filter.

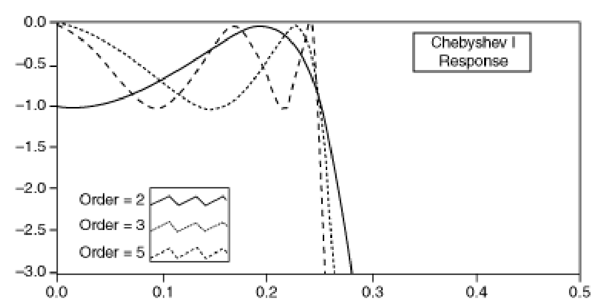


Fig.1.2- Effect of N on Chebyshev1 filter characteristics

Chebyshev

C. Mean Square Error

The Mean Square Error(MSE) has been the dominant quantitive performance matric in the field of signal processing. It is the standard criterion for the assessment of signal quality fidelity[4]. It is the method of choice for comparing competing signal processing methods of systems. It is one of the best choices of design engineers seeking to optimize signal processing algorithms.

The difference between the original signal & the reconstructed signal is Error signal which is denoted as "err". Mean squre error is calculated by taking the average of the "err". The value of MSE should be as low as possible. The formula for MSE is given by

$$MSE= [\Sigma \, err2]/M \qquad (5)$$

where M is the length of signal.
The MSE has many attractive features:

- MSE is simple.
- It is parameter free and inexpensive to compute, with a complexity of only one multiply and two additions per sample.
- It is also memory less—the squared error can be evaluated at each sample, independent of other samples.
- It has a clear physical meaning—it is the natural way the energy of the error signal.
- The MSE is an excellent metric in the context of optimization.

D. Signal to Noise Ratio

Signal to noise ratio (SNR) is a parameter use to quantify and compare the performance of algorithms and also determine the noise level in an reconstructed signal. The expression used to calculate signal to noise ratio is given by
SNR= 10log10[variance(So)/varience(So-Sf)]
Where So= original signal
and Sf = filtered signal.

2. METHOD

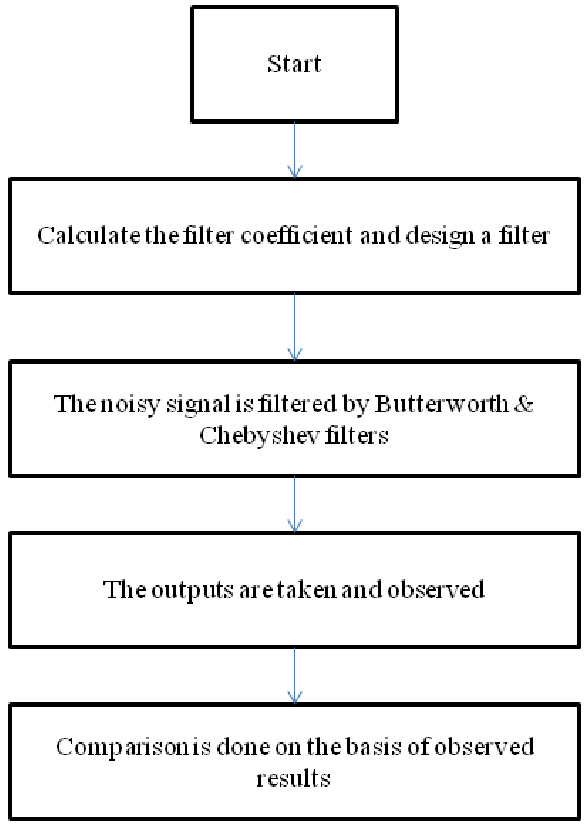
The transmitted signal is easily corrupted by noises such as Gaussian noise, Power line interference and so on. The process of adding noise to original noise is mathematically shown as

$$F(n)= X(n)+D(n), \quad n=1,2,3\ldots\ldots\ldots N \qquad (6)$$

X(n) is the original signal
D(n) is the Random Noise signal.
F(n) is the Signal+Noise

The F(n) signal is then filtered one by one at receiver by butterworth filter and chebyshev1 filter.

Flowchart for signal extraction buried in noise.



Steps for Calculating Mean Square Error:

1. Initially set the passband frequency (wp), stopband frequency (ws), passband ripples(rp) and stopband ripples(rs).
2. Determine the order and coefficients of filters .In MATLAB, use the command buttord() and cheb1ord() for butterworth filter and chebyshev1 filter respectively.
[n,wn] = buttord(wp,ws,rp,rs)
Where n is order of filter and wn is a cut off frequency.
3. Applying the command butter() to find the filter coefficients of butterworth filter .
[b,a] = butter (n, wn, 'ftype')
In case of chebyshev1 filter, use command cheby1().

[b,a] = cheby1(n,wn,rp,'ftype')
This function designs a highpass, lowpass or bandstop filter, where the string 'ftype' is 'high', 'low', or 'stop'. It returns the filter coefficients in length n+1 row vectors b and a, with coefficients in descending powers of z.

$$H(z)=[b(1)+b(2)z^{-1}+----+b(n+1)z^{-n}]/[1+a(2)z^{-1}+--------+a(n+1)z^{-n}] \quad (7)$$

4. Applying the same noisy signal as an input on the Butterworth filter and Chebyshev1 filter and plotting the graph.
5. Calculate the mean square error and signal to noise ratio.

3. RESULTS

Specifications taken for the design of Butterworth and Chebyshev1 filters are:
Sampling frequency=2000Hz.
Passband ripples=3db
Stopband ripples=43db

By giving different values of cut off frequency to Butterworth filter and chebyshev1 filter, we get the parameters as shown below in Table 3.1, 3.2, 3.3 and 3.4.

Table 3.1

Cut-off frequency 100Hz		
Parameters	Butterworth filter	Chebyshev1 filter
Wn	0.1322	0.1
Order	4	3
MSE	0.1985	0.2015
SNR	55.087	54.15

Table 3.2

Cut-off frequency 150Hz		
Parameters	Butterworth filter	Chebyshev1 filter
Wn	0.2527	0.15
Order	9	4
MSE	0.1985	0.2243
SNR	55.1523	54.6229

Table 3.3

Cut-off frequency 200Hz		
Parameters	Butterworth filter	Chebyshev1 filter
Wn	0.219	0.2
Order	7	4
MSE	0.1985	0.2259
SNR	55.1882	54.5916

Table 3.4

Cut-off frequency 250Hz		
Parameters	Butterworth filter	Chebyshev1 filter
Wn	0.2527	0.25
Order	9	5
MSE	0.1985	0.2502
SNR	55.1523	54.1481

The results showed in the tables states that as compare to chebyshev1 filter, the butterworth filters have better MSE and SNR values. The Order of butterworth filter is observed to be more than chebyshev1 filter at same cut off frequency. The following plots had been generated at a cut-off frequency of 200Hz.

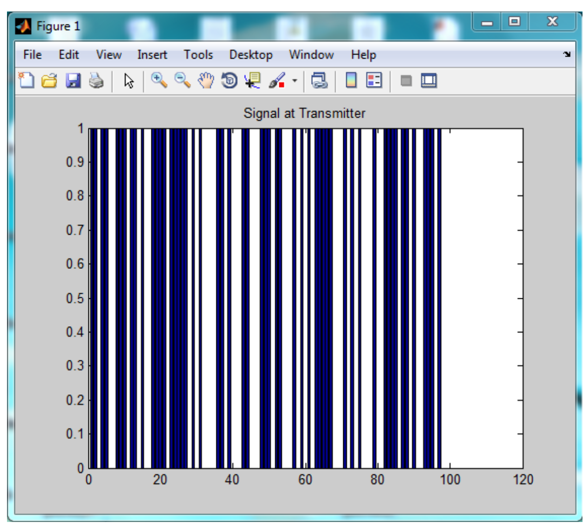


Fig 3.1- Original Signal at Transmitter

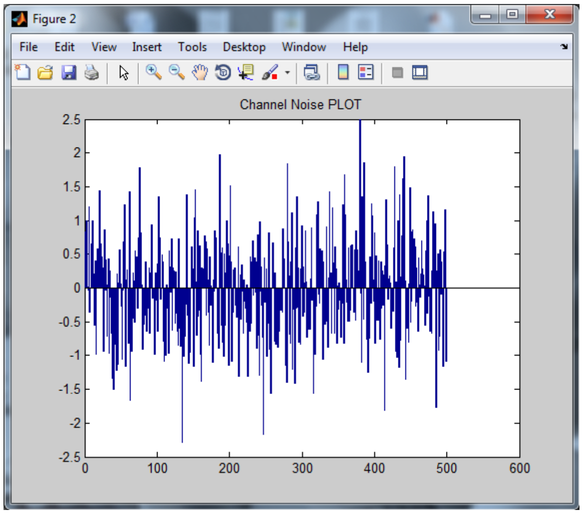


Fig 3.2- Graph of channel Noise

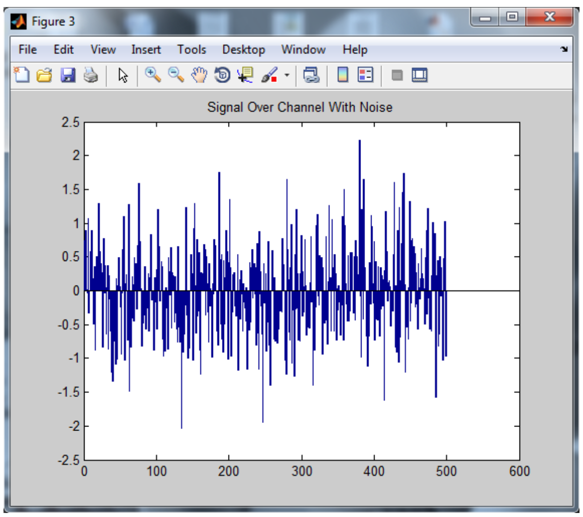


Fig 3.3- Signal over channel with noise

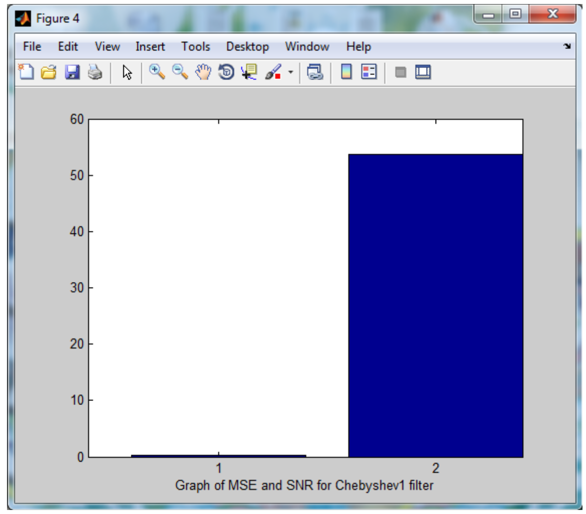


Fig 3.4- Graph of MSE and SNR for Chebyshev1 filter

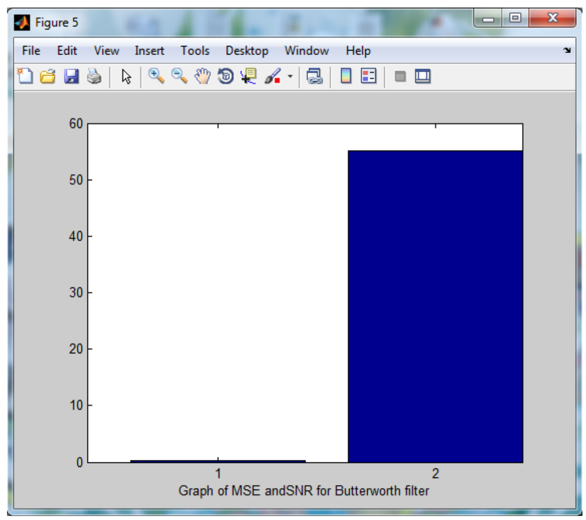


Fig 3.5- Graph of MSE and SNR for Butterworth filter

References

1. MATLAB (*The Language of Technical computing)*,the MathWorks Inc, Natick, mA.,1997

2. Mohit Bansal, Ritu Sharma and Parul Grover "*Performance evaluation of buttorworth filter for signal denoising*" IJECT Vol.1, Issue1,December 2010.

3. R.A.Barapate, J.S.Katre "*Digital Signal Processing*", Tech-Max January 2008 (Second revised edition).

ISSN: 2278-0181
Vol. 1 Issue 3, May - 2012

4. S.K. Mitra, *"Digital Signal Processing, A Computer based approach*", McGraw Hills, 2006. N.Y.(Third Edition)
5. Zhou Wang, "*Mean Squared Error: Love it or Leave it? A new look at signal fidelity Measures*" IEEE Signal Processing Magazine, Vol.26, Issue 1, Pages 98-117, January 2009.
6. Dolecek, G.J. "*Demo Programme for Teaching the Characteristics of Low Pass IIR Filters*", IEEE Conference Publication, Pg T4E1-T4E6,October 2011.
7. Samarjeet Singh, Uma Sharma "*MATLAB Based Digital IIR Filter Design*", IJECE, ISSN-2277-1956, 2012/01/PP-74-83.
8. Tmothy J. Schlichter "*Digital Filter Design Using MATLAB*"

6