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Performance of Functionally Graded Rotating Disk with Variable Thickness

H. Amin[1] , A. M. Khourshid[3]
[1, 3] Production Engineering and Mechanical Design department, Faculty of Engineering, Tanta University, Tanta, Egypt

E. Saber[2]
[2] Mechanical Engineering department, Faculty of Engineering and Technology, Arab Academy for Science and Technology, Alexandria, Egypt

***Abstract*— In this study thermo-mechanical analysis of function graded rotating disks of variable thickness and with temperature dependent material properties is presented. Four types of variable thickness disk-profiles are considered: namely convex, linear, concave, and constant thickness disk profile. The temperature field considered is assumed to be uniformly distributed over the disk surface and varied in the radial direction. Numerical solutions for the displacement field are obtained by using finite difference method for solid disk and annular disk with free-free and fixed-free boundary conditions. The effects of the material grading index and the parameters related to the geometry of the disk are investigated.**

Keywords—function graded material; Rotating disk; Finite difference method.

I. INTRODUCTION

Analysis of a rotating disc is an important subject due to its wide range of applications in mechanical engineering such as in steam and gas turbine rotors, turbo generators, turbojet engines, centrifugal compressors and components like gears, flywheels, etc. and in many such industrial applications**.**

In many applications, the disc is working at high temperature which presenting thermal loading. The temperature gradient presents throughout the disc resulted in changes in material properties throughout the disc. The reinforcement in composites used as structural materials in many aerospace and automobile applications is generally distributed uniformly. Functionally graded materials (FGMs) are being used as interfacial zone to improve material performance are usually made by continuous gradation of different material phases where one or two of them are ceramics and the others are metal alloy phases. Functionally graded composite materials are characterized by a spatially variable microstructure. For the purpose of analysis, functionally graded materials are modeled as inhomogeneous materials with continuously varying properties.

Generally, there are two approaches for the solution of rotating discs, namely, theoretical and numerical methods. For homogeneous rotating disc with constant thickness, closed-form analytical solution is available. However, for nonhomogeneous rotating disc with variable thickness, analytical solution is not possible to obtain. Hence, many numerical attempts have been presented to solve such a problem.

An analytical model for the thermo-mechanical behavior of FG hollow circular cylinders that are subjected to the action of an arbitrary steady state or transient temperature field has been developed in [1]. The solutions of temperature, displacements, and thermal/ mechanical stresses in a functionally graded circular hollow cylinder by using a multi-layered approach based on the theory of laminated composites were presented in [2]. Thermo-mechanical analysis of functionally graded hollow circular cylinders subjected to mechanical loads and linearly increasing boundary temperature was studied. Thermo-mechanical properties of functionally graded material (FGM) are assumed to be temperature independent and vary continuously in the radial direction of cylinder and the solution for the time-dependent temperature and thermo-mechanical stresses is obtained by employing Laplace transform techniques and series solving method for ordinary differential equation [3]. The elastic stress analysis of annular discs made of functionally graded materials subjected to both uniform pressures on the inner surface and a linearly decreasing temperature distribution was studied [4].

The stress analysis of functionally graded rotating annular disks subjected to internal pressure and various temperature distributions in radial direction was studied. The model is obtained by using infinitesimal deformation theory of elasticity and power law functions for graded parameters [5]. An analytical thermo elasticity solution for a disc made of functionally graded materials was presented. Infinitesimal deformation theory of elasticity and power law distribution for functional gradation are used in the solution procedure. It is found that the grading indexes play an important role in determining the thermo mechanical responses of FG disc and in optimal design of these structures [6]. Two solutions to design a thin annular disc of variable thickness subject to thermo-mechanical loading were proposed. The initiation of plastic yielding is controlled by Mises yield criterion. The design criterion for one of the solutions proposed requires that the distribution of stresses is uniform over the entire disc. The other solution is obtained under the additional requirement that the distribution of strains is uniform. This solution exists for the disc of constant thickness at specific values of the loading parameters [7]. The static analysis of rotating isotropic and functionally graded solid and annular disks with and without thermal load was investigated in [8]. The stresses on rotating rectilinearly or polar orthotropic discs subjected to various temperature distributions were investigated in [9]. Stresses and strains in variable-thickness annular and solid rotating elastic disks subjected to thermal loads and having a variable density along the radius were studied in [10]. Thermo-elastic solutions for rotating functionally graded disk with variable thickness under a steady temperature field and related material grading index and the geometry of the disk to thermo-elastic solutions were reported by [11]. The analytical and numerical solutions for rotating variable-thickness solid disk and numerical solution for rotating variable-thickness annular disk was presented in [12].

In this study the problem of nonhomogeneous rotating disk with variable thickness is presented. It is demonstrated that the analytical solution for such a problem is not possible to be obtained. Finite difference method is used to display result.

II. MATHEMATICAL FORMULATION

The problem may be considered as plane stress with variable thickness. Hence $\sigma_z = 0$, as this is a static problem, the solution has to satisfy equilibrium, compatibility and constituve law of the material properties.

A. *Equilibrium:*

Figure (1) shows an infinitesimal element of the disc in the radial direction, where r, h, σ_r, σ_t *and* ω are radius, thickness of the disc, radial stress, tangent stress and angular velocity respectivelv.

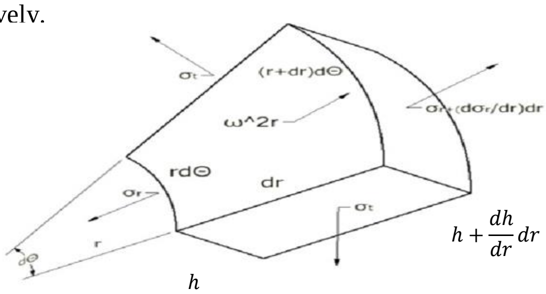

Fig. 1. An infinitesimal element of the disc in radial direction.

Assuming that the weight of the disc is neglected, the equilibrium of the element in radial direction may be obtained as follows:

$$\frac{1}{h}\left(\frac{dh}{dr}\right)\sigma_r + \frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_t}{r} + \rho\omega^2 r = 0 \qquad (1)$$

B. *Compatibility:*

Displacement occurs in the disc is shown in figure (2). For this axisymmetric case, the displacement field is of the form $u = u_r(r)$ and $u_t = 0$ .The strain field is given by:

$$\varepsilon_r = \frac{\left(u + \frac{du}{dr}dr\right) - u}{dr} = \frac{du}{dr} \qquad (2)$$

$$\varepsilon_t = \frac{(r+u)d\theta - rd\theta}{rd\theta} = \frac{u}{r} \qquad (3)$$

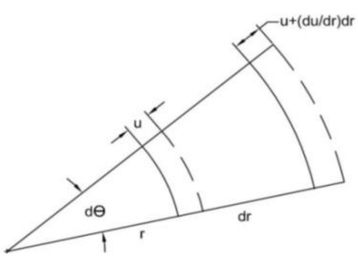

Fig. 2. Displacement in infinitesimal element of the disc

Where $\varepsilon_r, \varepsilon_t$ and u are the strains in radial and tangential directions and displacement component in the radial direction. Eliminating u from equations (2) and (3), the simple compatibility condition may be obtained as follows,

$$\varepsilon_r = \frac{d}{dr}(r\varepsilon_t) = r\frac{d\varepsilon_t}{dr} + \varepsilon_t$$

Thus,

$$\varepsilon_r = \varepsilon_t + r\frac{d\varepsilon_t}{dr}$$

Hooke's law is used to get the stress-strain relation. For plane stress case and due to only mechanical loading with $\sigma_z = o$, then

$$\varepsilon_r = \frac{1}{E(r)}(\sigma_r - \nu\sigma_t) \qquad (5)$$

$$\varepsilon_t = \frac{1}{E(r)}(\sigma_t - \nu\sigma_r) \qquad (6)$$

Because these is also thermal loading, then

$$\varepsilon_{total} = \varepsilon_{elastic} + \varepsilon_{thermal} \qquad (7)$$

and,

$$\varepsilon_{thermal} = \alpha(r)T(r) \qquad (8)$$

Hence, the total strains become,

$$\varepsilon_r = \frac{1}{E(r)}(\sigma_r - \nu\sigma_t) + \alpha(r)T(r) \qquad (9)$$

$$\varepsilon_t = \frac{1}{E(r)}(\sigma_t - \nu\sigma_r) + \alpha(r)T(r) \qquad (10)$$

Where E, α and T are elasticity modulus, thermal expansion coefficient and temperature change respectively. It is assumed that the material properties (E, ρ and α) and temperature change T are varying through the radial direction as shown in figure (3). The temperature distribution assumed to be:

$$T(r) = fn\big(r, h(r), k(r)\big) \qquad (11)$$

The boundary conditions at inner and outer surfaces of the disc are T_a and T_b illustrated in figure (3). Poisson's ratio υ is assumed as a constant because its variation has much less practical significance than variation in other material properties.

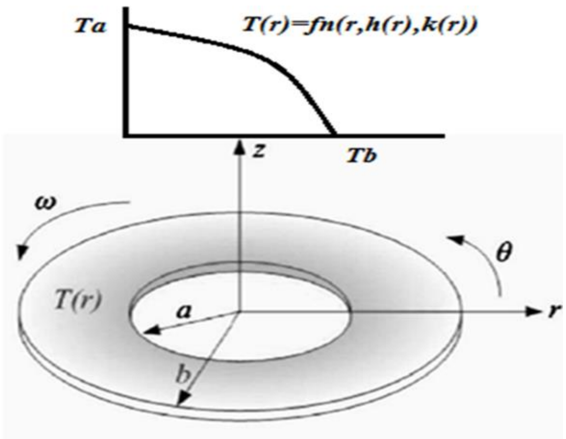

Fig. 3. Functionally graded mechanical and thermal loads.

Re-avenging equations (9) and (10), we get:

$$\sigma_r = E\varepsilon_r + v\varepsilon_t - E\alpha T \tag{12}$$

and,

$$\sigma_t = E\varepsilon_t + v\varepsilon_r - E\alpha T \tag{13}$$

Substituting equation (13) into (12) to get σ_r *as*:

$$\sigma_r = \frac{E}{1-v^2}(\varepsilon_r + v\varepsilon_t) - \frac{E\alpha T}{1-v} \tag{14}$$

Now substituting equation (12) into (13) to get σ_t *as*,

$$\sigma_t = \frac{E}{1-v^2}(\varepsilon_t + v\varepsilon_r) - \frac{E\alpha T}{1-v} \tag{15}$$

Substituting (2) and (3) into equation (14), we get:

$$\sigma_r = \frac{E}{1-v^2}\left(\frac{du}{dr} + v\frac{u}{r} - (1+v)\alpha T\right) \tag{16}$$

Similarly, substituting (2) and (3) into equation (15) we get σ_t *as*,

$$\sigma_t = \frac{E}{1-v^2}\left(\frac{u}{r} + v\frac{du}{dr} - (1+v)\alpha T\right) \tag{17}$$

Now substituting (16) and (17) into equation (1) yields:

$$\frac{d^2u}{dr^2} + \left(\frac{1}{h}\frac{dh}{dr} + \frac{1}{E}\frac{dE}{dr} + \frac{1}{r}\right)\frac{du}{dr} + \left(\frac{v}{rh}\frac{dh}{dr} - \frac{1}{r^2} + \frac{v}{rE}\frac{dE}{dr}\right)u = \alpha(1+v)\left[\frac{1}{h}\frac{dh}{dr}T + \frac{dT}{dr} + \frac{T}{\alpha}\frac{d\alpha}{dr} + \frac{T}{E}\frac{dE}{dr}\right] - \frac{(1-v^2)}{E}\rho\omega^2 r \tag{18}$$

The coefficients of Eq.(18) contain variable parameters r, h=h(r), E=E(r), α=α(r), ρ=ρ(r) and T=T(r). Because the coefficients contain many variable parameters, there is no exact solution for Eqn. (18). However, there are many numerical approaches to solve such a problem. . In this study we used finite difference method.

C. *Boundary conditions:*

The following traction conditions on the inner and outer surfaces of rotating disc must be satisfied.

- Hollow disc Free-Free:

$$\sigma_r = 0 \text{ at } r = a\,;\ \sigma_r = 0 \text{ at } r = b \tag{19}$$

- Hollow disc Fixed-Free:

$$u = 0 \text{ at } r = a;\ \sigma_r = 0 \text{ at } r = b \tag{20}$$

- Solid disc:

$$u = 0 \text{ at } r = 0;\ \sigma_r = 0 \text{ at } r = b \tag{21}$$

D. *Material gradation relation:*

Generally the variation of the property ϕ of the material in the FGM circular disc along the radial direction is assumed to be represented by the simple power-law the same as considered in [11, 13, 14].

$$\phi(r) = (\phi_o - \phi_i)\left(\frac{r-r_i}{r_o-r_i}\right)^n + \phi_i \tag{22}$$

Where ϕ_o *and* ϕ_i are the corresponding material properties of the outer and inner faces of the disc, r_o *and* r_i are the outer and inner radius of the disc respectively; $n \geq 0$ is the volume fraction exponent (also called grading index). Since FGMs are generally used in high temperature environment, the material properties may be written as

$$\phi(r,T) = (\phi_o(T) - \phi_i(T))\left(\frac{r-r_i}{r_o-r_i}\right)^n + \phi_i(T) \tag{23}$$

Where $\phi_o(T)$ and $\phi_i(T)$ are taken of the form [17].

$$\phi_j(T) = \beta o_j\left(\frac{\beta_{-1_j}}{T} + 1 + \beta_{1_j}T + \beta_{2_j}T^2 + \beta_{3_j}T^3\right) \tag{24}$$

and $j = i$ *or* o

T indicates the absolute temperature, $\beta_{o_j}, \beta_{-1_j}, \beta_{1_j}, \beta_{2_j}$ and β_{3_j} are constants in the cubic fit of material property. The modulus of elasticity E and the coefficient of thermal expansion α may be considered as,

$$E(r,T) = (E_o(T) - E_i(T))\left(\frac{r-r_i}{r_o-r_i}\right)^n + E_i(T) \tag{25}$$

and

$$\alpha(r,T) = (\alpha_o(T) - \alpha_i(T))\left(\frac{r-r_i}{r_o-r_i}\right)^n + \alpha_i(T) \tag{26}$$

Whereas the density ρ and the thermal conductivity K are assumed to be temperature independent and are taken as,

$$\rho(r) = (\rho_o - \rho_i)\left(\frac{r-r_i}{r_o-r_i}\right)^n + \rho_i \tag{27}$$

$$K(r) = (K_o - K_i)\left(\frac{r-r_i}{r_o-r_i}\right)^n + K_i \tag{28}$$

The Poisson's ratio v will be assumed to be constant throughout [15]. In the present study, Copper alloy as inner-surface metal and tungsten as outer-surface ceramic, the same as considered by [16].

The materials properties of Copper alloy are:

$$E_i(T) = 129.2 - 0.03452\,T - 4.125 * 10^{-5}\,T^2 \ (GPa) \tag{29}$$

$$\alpha i(T) = (15.52 + 8.241 * 10^{-3}T - 4.125 * 10^{-6}T^2) * 10^{-6} \ (°C^{-1}) \tag{30}$$

$\rho_i = 8960\ (kg/m^3)$, $K_i = 383.3\ (W/m°C)$, $v = 0.3$

and the material properties of Tungsten are:

$$E_o(T) = 397.9 - 2.307 * 10^{-3}\,T - 2.716 * 10^{-5}\,T^2\,(GPa) \tag{31}$$

$$\alpha_o(T) = (3.923 + 5.835 * 10^{-4}T + 5.705 * 10^{-11}T^2 - 2.046 * 10^{-14}T^3) * 10^{-6} \ (°C^{-1}) \tag{32}$$

$\rho_o = 19300\ (kg/m^3)$, $K_o = 174.9\ (W/m°C)$, $v = 0.3$

Then the distribution of these material properties are shown in figures (4), (5), (6), (7).

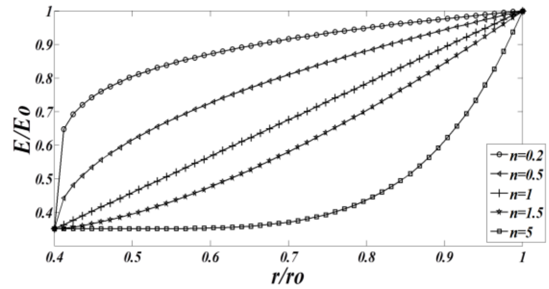

Fig .4.The modulus of Elasticity distribution for different value of grading Index (n).

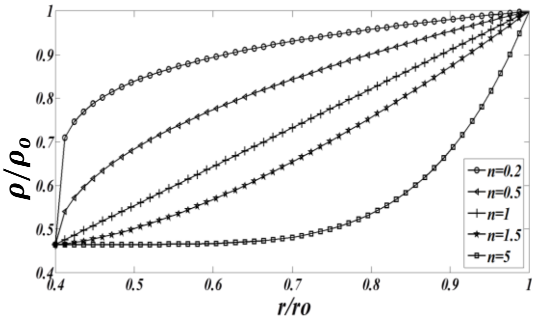

Fig. 5. Thermal expansion coefficient distribution for different value of grading index (n).

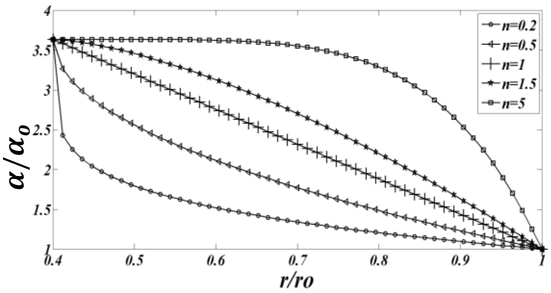

Fig. 6. Thermal conductivity distribution for different value of grading index.

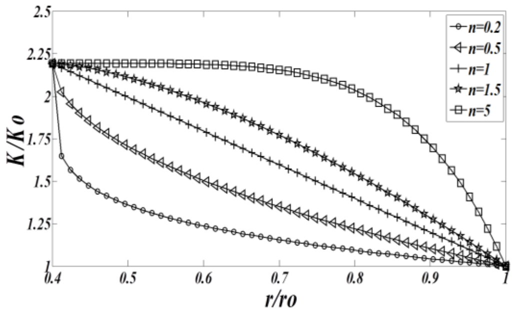

Fig. 7. Density distribution for different values of grading index.

E. Thickness profile of the disc:

The thickness profile of the disc h(r) is assumed to be vary radially according to

$$h(r) = h_o\left(1 - q\left(\frac{r}{r_o}\right)^{m1}\right) \tag{33}$$

Where q and m1 are geometric parameters such that ($0 \le q < 1$, $m_1 > 0$); h_o is the thickness at the center of the disk ($r = 0$). A uniform thickness disk can be obtained by setting q=0. A linearly decreasing thickness can be obtained for $q \neq 0$ and $m_1 = 1$. The profile is concave if $m_1 < 1$ And it is convex if $m_1 > 1$. Different forms of the thickness profile are shown in Figure (8).

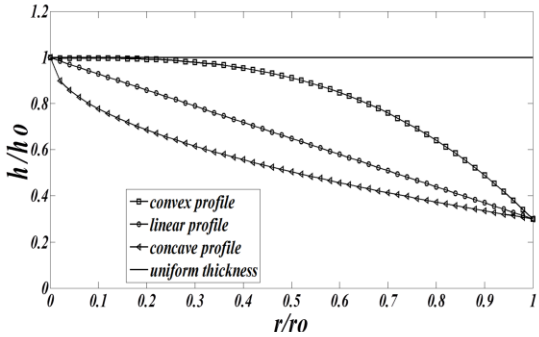

Fig. 8. Thickness profile of rotating FG disc

The chosen values of the geometric parameters q and m_1 are illustrated in table (1).

TABLE 1. Different cases of thickness profiles [11]

$q = 0$ Constant thickness	$q \neq 0$ $m_1 > 1$ convex	$q \neq 0$ $m_1 = 1$ linear	$q \neq 0$ $m_1 < 1$ Concave
$q = 0$	$q = 0.7$ $m_1 = 2$	$q = 0.7$ $m_1 = 1$	$q = 0.7$ $m_1 = .5$

F. Non-dimensionality:

The following dimensionless parameters may be introduced,

$$\bar{r} = \frac{r}{r_o}, \quad \bar{h} = \frac{h(r)}{h_o}, \quad \bar{u} = \frac{u(r)}{u_o}, \quad \bar{E} = \frac{E(r,T)}{E_o},$$

$$\bar{\rho} = \frac{\rho(r)}{\rho_o}, \quad \bar{\alpha} = \frac{\alpha(r)}{\alpha_o}, \quad \bar{T} = \frac{T(r)}{T_o},$$

$$where\ u_o = \frac{\rho_o {r_o}^3 \omega^2}{E_o} + r_o\alpha_o T_o$$

E_o *and* α_o are modulus of elasticity and coefficient of thermal expansion at $r = r_o$ while $T = T_o$ is the temperature at outer surface. The non-dimensional form of equation (18) may be obtained as follows,

$$\frac{d^2\bar{u}}{d\bar{r}^2} + \left(\frac{1}{\bar{h}}\frac{d\bar{h}}{d\bar{r}} + \frac{1}{\bar{E}}\frac{d\bar{E}}{d\bar{r}} + \frac{1}{\bar{r}}\right)\frac{d\bar{u}}{d\bar{r}} + \left(\frac{v}{\bar{r}}\frac{1}{\bar{h}}\frac{d\bar{h}}{d\bar{r}} - \frac{1}{\bar{r}^2} + \frac{v}{\bar{r}\bar{E}}\frac{d\bar{E}}{d\bar{r}}\right)\bar{u} = \frac{\alpha_o T_o r_o}{u_o}(1+v)\left[\frac{\bar{\alpha}\bar{T}}{\bar{h}}\left(\frac{d\bar{h}}{d\bar{r}}\right)\bar{T} + \frac{\bar{\alpha}(d\bar{T})}{d\bar{r}} + \frac{d\bar{\alpha}}{\partial\bar{r}}\bar{T} + \frac{\bar{\alpha}\bar{T}}{\bar{E}}\frac{d\bar{E}}{d\bar{r}}\right] - \frac{\rho_o {r_o}^3(1-v^2)}{u_o E_o \bar{E}}\bar{\rho}\omega^2\bar{r} \tag{34}$$

- Non-dimensional stresses $\overline{\sigma_r}$ and $\overline{\sigma_t}$:

$$\overline{\sigma_r} = \frac{\sigma_r}{\left(\frac{E_o u_o}{r_o}\right)} = \frac{\bar{E}}{(1-v^2)}\left(\frac{d\bar{u}}{d\bar{r}} + v\frac{\bar{u}}{\bar{r}} - \left(\frac{\alpha_o T_o r_o}{u_o}\right)(1+v)\bar{\alpha}\bar{T}\right) \quad (35)$$

$$\overline{\sigma_t} = \frac{\sigma_t}{\left(\frac{E_o u_o}{r_o}\right)} = \frac{\bar{E}}{(1-v^2)}\left(v\frac{d\bar{u}}{d\bar{r}} + \frac{\bar{u}}{\bar{r}} - \left(\frac{\alpha_o T_o r_o}{u_o}\right)(1+v)\bar{\alpha}\bar{T}\right) \quad (36)$$

- Non-dimensional Boundary Conditions:

Hollow disc free-free:

At $r = r_i \;\; or \;\; r = r_o \; : \; \sigma_r = 0$

Then,

$$\frac{\partial\bar{u}}{\partial\bar{r}} + \frac{v}{\bar{r}}\bar{u} - (1+v)\frac{T_o r_o \alpha_o}{u_o}\bar{\alpha}\bar{T} = 0 \quad (37)$$

Hollow disc fixed-free:

At $r = r_i : \; u = 0$

Then, $\bar{U} = 0$

At $r = r_o : \sigma_r = 0, Eqn. (37)$

Solid disc:

$At \; r = 0 : \; u = 0$

Then, $\bar{U} = 0$

$At \; r = r_o : \sigma_r = 0, Eqn. (37)$

G. Thermal analysis:

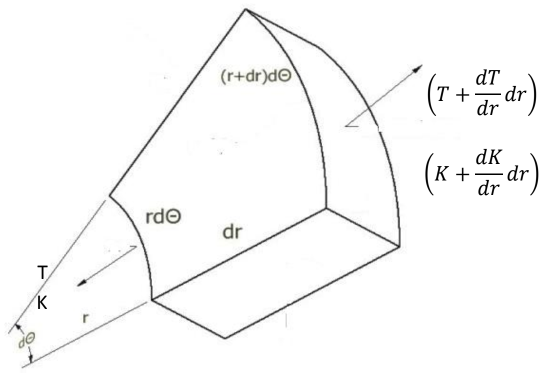

Fig. 9. Infinitesimal element of the disc

The steadt steady state heat transfer equation may be written followes;

$$krh\frac{d^2T}{dr^2} + \left(kh + kr\frac{dh}{dr} + rh\frac{dk}{dr}\right)\frac{dT}{dr} = 0 \quad (38)$$

dimenssionless form

$Eq\underline{n}.$ (40)becomes,

$$\bar{K}\bar{h}\bar{r}\frac{d^2\bar{T}}{d\bar{r}^2} + \left(\bar{K}\bar{h} + \bar{K}\bar{r}\frac{d\bar{h}}{d\bar{r}} + \bar{r}\bar{h}\frac{d\bar{K}}{d\bar{r}}\right)\frac{d\bar{T}}{d\bar{r}} = 0 \quad (39)$$

Boundary conditions of $eq\underline{n}$ (39):

$$\left(\bar{T_i} = \frac{T_a}{T_o} \;\; at \; \bar{r} = \frac{r_i}{r_o}\right), \; \left(\bar{T_o} = \frac{T_b}{T_o} \;\; at \; \bar{r} = 1\right) \quad (40)$$

By solving eqn (39) with boundary conditions (40), temperature distribution for the case of increasing the temperature change from 0 at inner surface to T_b at outer surfacebe can be obtained as shown in figure (10).

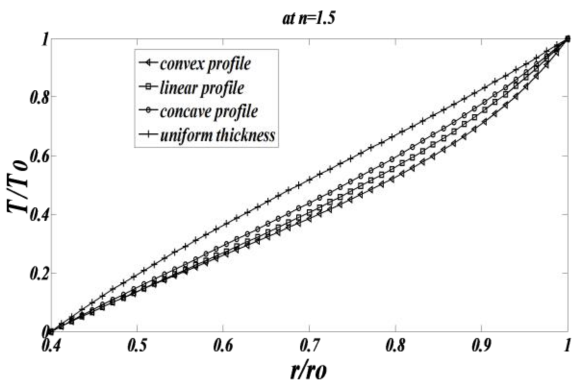

Fig. 10. Non-dimensional temperature distribution for different disk thickness profiles at n=1.5.

III. RESULTS AND DISCUSSION

A. The effect of different thickness:

Hollow and solid FG disks with variable thickness has smaller radial stress, tangent stress and radial displacement compared to those with uniform thickness at the same material index as shown in figures (11), (12), and (13). According to the geometric parameters q and m_1, it is observed that FG disk with convex profile has smaller radial stress, tangent stress and radial displacement.

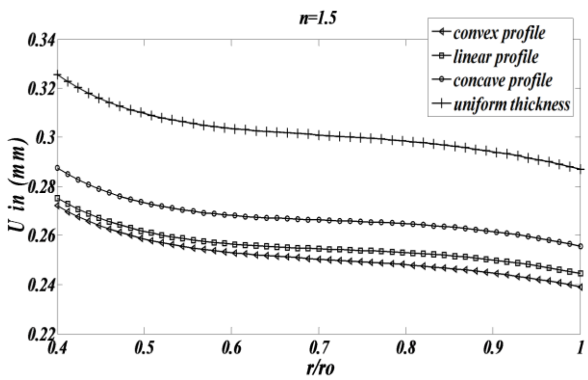

Fig.11.a. Radial displacement distribution under mechanical load in the hollow disk (free-free) with variable disk thickness and grading index n =1.5.

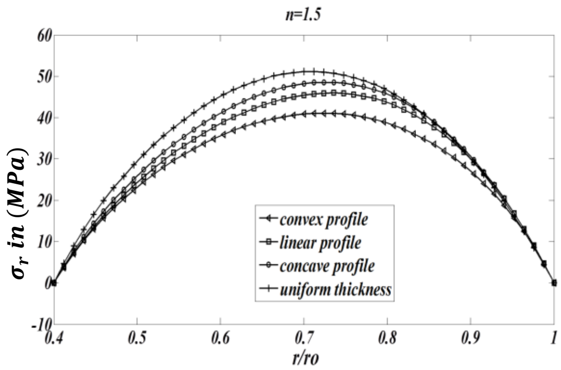

Fig.11.b. Radial stress under mechanical load in the hollow disk (free-free) with variable disk thickness and grading index n =1.5.

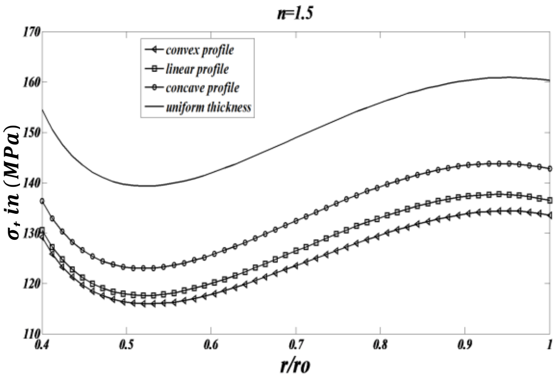

Fig. 11.c Tangent stress under mechanical load in the hollow disk (free-free) with variable thickness and grading index n=1.5

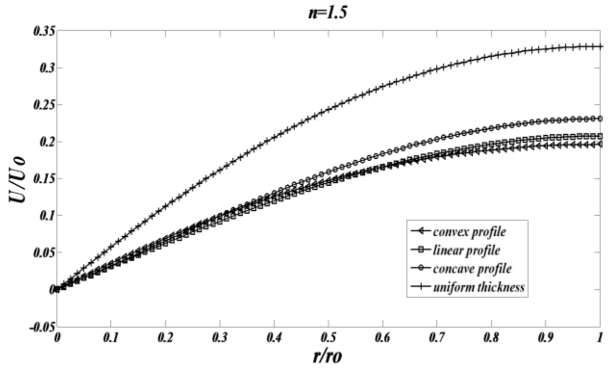

Fig. 12.a Non-dimensional radial displacement under mechanical load in the solid disc with variable thickness and grading index n=1.5

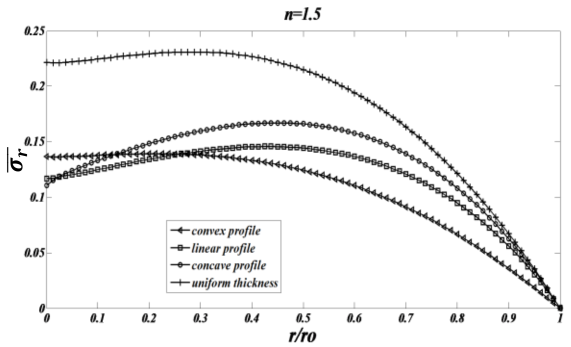

Fig.12.b Non-dimensional radial stress under mechanical load in the solid disc with variable thickness and grading index n=1.5

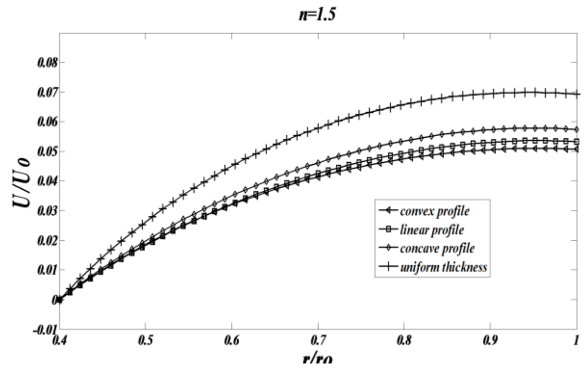

Fig. 13.a Non-dimensional radial displacement under mechanical load in the hollow disc (fixed-free) with variable thickness and grading index n=1.5

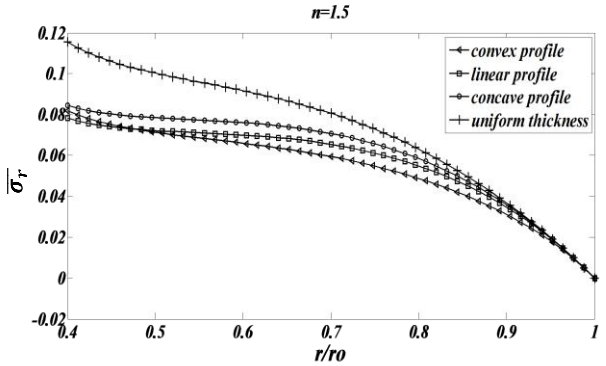

Fig. 13.b Non-dimensional Radial stress under mechanical load in the hollow disc (fixed-free) and grading index n=1.5.

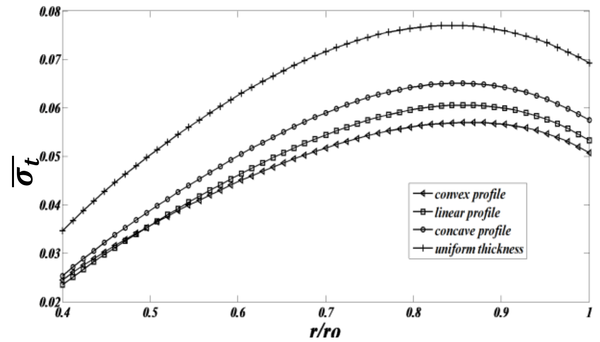

Fig. 13.c Non-dimensional tangent stress under mechanical load in the hollow disc (fixed-free) and grading index n =1.5

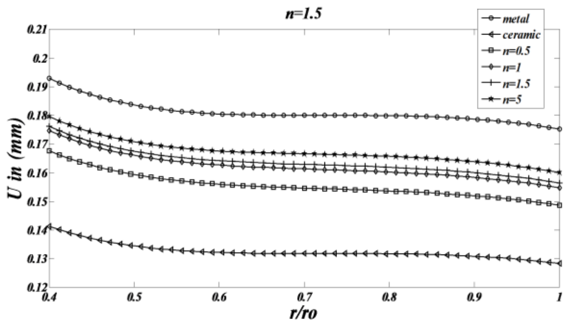

Fig. 14.a. Radial displacement distribution under mechanical load in the hollow disc (free-free) with linear profile for different grading indexes.

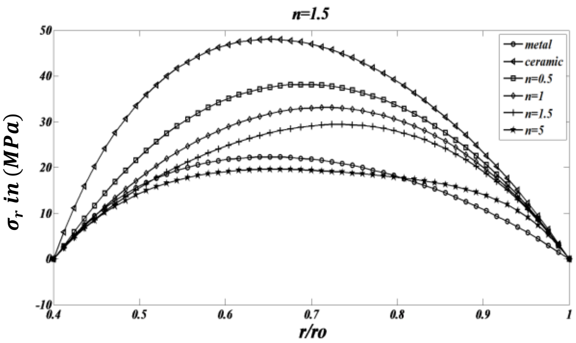

Fig. 14.b Radial stress distribution under mechanical load in the hollow disk (free-free) with linear profile and different grading indexes.

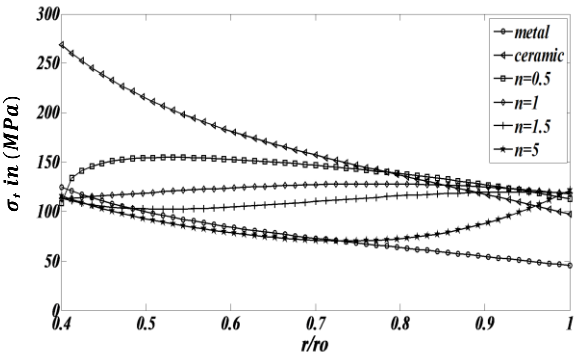

Fig. 14.c Tangent stress distribution under mechanical load in the hollow disc (free-free) with linear profile and different grading indexes.

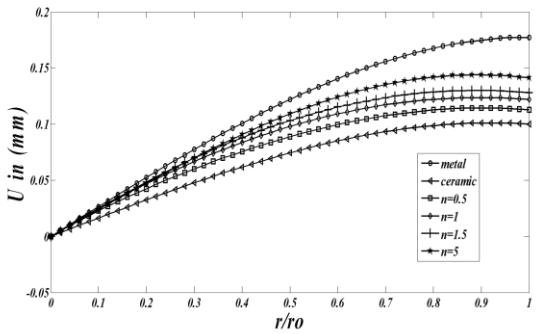

Fig. 15.a Radial displacement under mechanical load in solid disc with uniform thickness and different values of grading index.

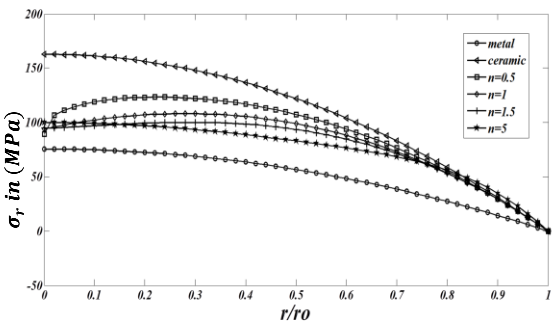

Fig. 15.b Radial stresses under mechanical load in solid disc with uniform thickness and different values of grading index.

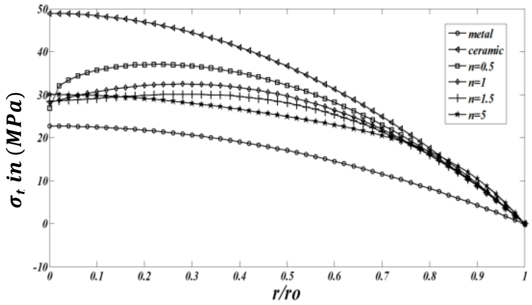

Fig. 15.c Tangent stresses under mechanical load in solid disc with uniform thickness and different values of grading index.

B. The effect of material index:

The radial displacement increase with increasing material index in both solid and hollow disc as shown in figures (14.a, 15.a). The highest value of displacement at metal disk and the lowest value at ceramic disk at the same profile shape. The radial and tangent stresses decrease with increasing material index as shown in figures (14.b), (14.c), (15.b), (15.c). The highest value of stresses at ceramic disk.

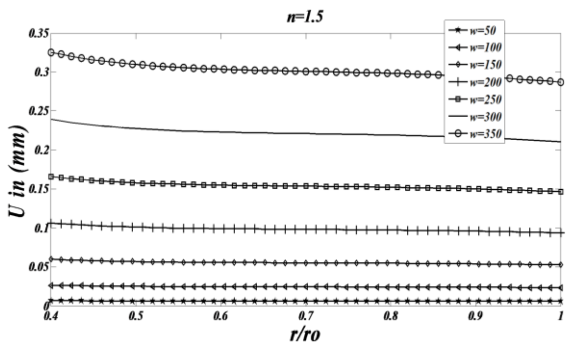

Fig. 16.a Radial displacement under mechanical load in hollow disk (free-free) for uniform thickness and n=1.5 for different angular velocities

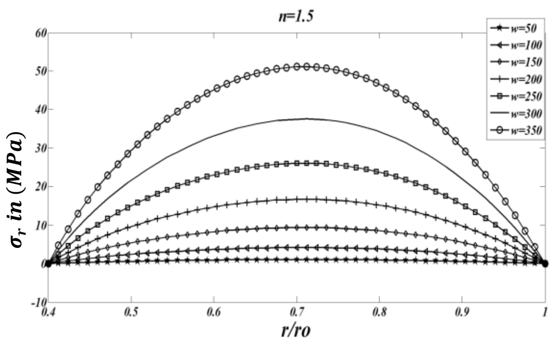

Fig. 16.b. Radial stress under mechanical load in hollow disk (free-free) with uniform thickness and n=1.5 for different values of angular velocity

C. The effect of angular velocity:

As a result of increasing angular velocity, the radial displacement, radial and tangent stresses are increasing as shown in figures (16.a), (16.b), and (16.c). The position of the maximum radial stress is unaffected by increasing the value of the angular velocity. see figure (16.b). The same results are obtained in other cases: hollow disk (fixed-free) and solid disk.

D. The effect of temperature change:

steady state temperature distributions in the radial direction creat a thermal loading in addition to the mechanichal load, it is assumed that we have temperature change from inner to outer surfaces.

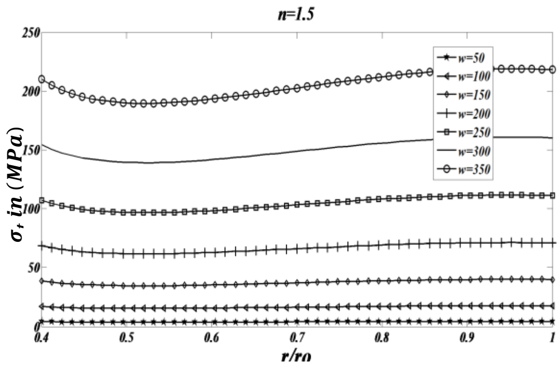

Fig. 16.c. tangent stress under mechanical load in hollow disk (free-free) with uniform thickness and n=1.5 for different values of angular velocity.

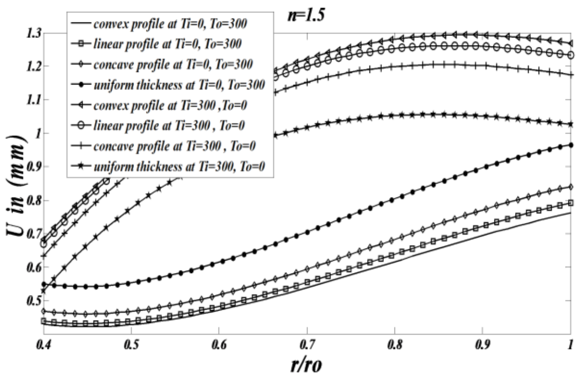

Fig. 17.a. Radial displacement under thermo-mechanical loading at rotating speed of 300 1/s.

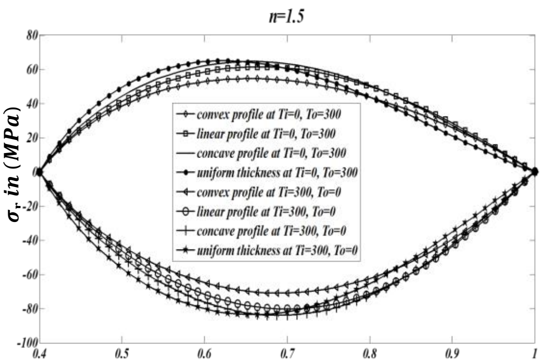

Fig. 17.b. Radial stress under thermo-mechanical loading at rotating speed of 300 1/s.

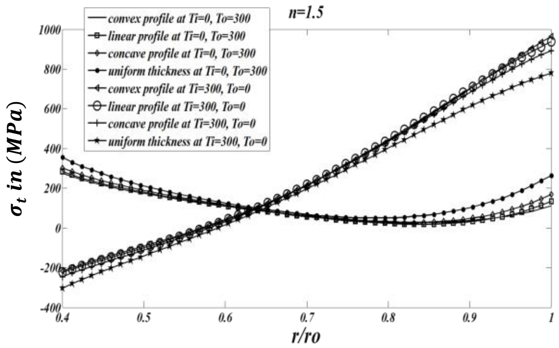

Fig. 17.c. Tangent stress under thermo-mechanical loaing t rotating speed of 300 1/s.

To study the effect of thermal loading on the displacement, radial and tangential stresses, various temperature distributions were introduced such as increasing the temperature in the radial direction from 0 at the inner surface to maximum temperature value of 300 ^{o}C at outer surface. The second temperature distribution in the radial direction was represented by decreasing the temperature from maximum value of 300 ^{o}C at inner surface to 0 at outer surface. Figure (17.a) shows the variations of radial displacements in both cases, the radial displacement increases in the radial direction. For the case of decreasing temperature from maximum value at inner surface to 0 at outer surface, the radial displacement values are greater than those for the case of increasing temperature. It is noted that, the convex disk profile has the smaller value of radial displacement in the case of temperature increasing, while the uniform disk thickness has the smaller radial displacement in the case of decreasing temperature. Figure (17.b) illustrates the variations of radial stresses at cases of increasing and decreasing temperatures. The convex disk profile has the smaller radial stress in both cases. Figure (17.c) shows the tangential stresses of rotating disc subjected to increasing and decreasing temperatures. The results show that the in the case of increasing temperature, the tangential stresses at the inner surface decrease gradually and they increase again at the outer surface. In the case of decreasing temperature, the tangential stresses have a smaller value at the inner surface and increase to larger values at the outer surface.

CONCLUSION

Thermo-analysis of function graded rotating disks of variable thickness and with temperature dependent material properties is presented by using numerical method (FDM). Four types of variable thickness disk-profiles are considered: namely convex , linear, concave, and constant thickness disk profile. Thermo elastic stresses for solid disk and the hollow disks with free-free, fixed-free boundary conditions are obtained. The effects of the material grading index (n) and the geometric parameters q and m_1 are investigated. Some general observations of this study can be summarized as follows:

- Maximum values of radial stresses due to mechanical loading in FG hollow disk with variable thickness under different boundary conditions remain between the maximum values for homogenized disks. Both the location and the value of maximum stress depends on the disk profile.
- For the conidered values of the geometric parameters q and m_1, FG disk with convex profile has smaller stresses than those with linear or concave profiles.
- For each of disk-thickness profile, the radial stress in FG disks decreases with increase in grading index n. Furthermore, radial displacements in pure metal disks are larger than in FG disks, while it is smaller values at ceramic disks.
- Under thermo-mechanical loading, FG disks with variable thickness has smaller stresses and radial displacements than those in the case of mechanichal loading .
- For the case of decreasing temperature from maximum value at inner surface to $T_b = 0$ at outer surface, the radial displacement values are greater than those for the case of increasing temperature. It is noted that, the convex disk profile has the smaller value of radial displacement in the case of temperature increasing, while the uniform disk thickness has the smaller radial displacement in the case of decreasing temperature.
- The convex disk profile has the smaller radial stress in both cases; this is true in increasing or decreasing temperatures.
- In the case of increasing temperature, the tangential stresses at the inner surface decrease gradually and they increase again at the outer surface. In the case of decreasing temperature, the tangential stresses have smaller values at the inner surface and increase to larger values at the outer surface.

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