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Performance of Golden Code in MIMO System

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***Abstract*— The optimal design of perfect space time codes constructed from cyclic division algebra (CDA) on the quasi-static uncoded MIMO channel has received lots of attention in industry over the last few years. However, the recent standards that uses multiple antennas terminals such as IEEE 802.11n or IEEE 802.16e, are based on more realistic assumptions involving the use of outer codes, and multi-taps channels. In this paper, we study the performance of these perfect codes in a standard context. We show that although these codes are optimal, the asymptotical gain of these codes is only significant at a very high SNR range and cannot be observed for a moderate Packet Error Rate (PER) probability.**

***Index Terms*— Bit Interleaved Coded Modulation (BICM), Orthogonal Frequency Division Multiplexing (OFDM), Space Time Coding (STC), Pairwise Error Probability (PEP)**

I. Introduction

The objective of IEEE802.11n standardization is to achieve 100Mbps in top of MAC layer while still being backward compatible with IEEE802.11a/g, which results in a maximum PHY rate of 130Mbps. This significant rate increase compared to other IEEE802.11 standards such as IEEE802.11a, whose maximum PHY rate is 54Mbps, is enabled by the introduction of multiple antennas at the Access Point and at the Mobile Terminal. These multiple antennas are used to increase the peak data rate, but also to derive benefit from spatial diversity in order to ensure for instance a larger range of operation for full home coverage, or to better address outdoor hotspot environments.

Another feature of IEEE802.11n consists in addressing handsets specificities, such as a small number of antennas. To accommodate various antenna configurations, the definition of PHY modes is based on the transmission of a number of spatial steams, varying from one to four, that is limited by the minimum number of transmit and receive antennas. For range increase, Space-Time Block Coding (STBC) or/and Cyclic Delay Diversity (CDD) can be applied to map the spatial streams on different transmit chains. Most of pre 802.11n products use two transmit and two or three receive antennas, which limits the number of spatial streams to two. In this configuration, only full diversity or full rate modes have been defined, which motivates the study of full rate full diversity space-time codes.

The Golden code was proposed in [1] for 2 × 2 MIMO configuration, in order to fulfill the design criteria proposed by Tarokh in [3] and in order to achieve the Diversity Multiplexing Tradeoff (DMT) [2]. This code is the optimal 2×2 space time code since it achieves full rate and full diversity, preserves the mutual information and achieves the DMT: by construction, the Golden code has a non vanishing determinant [1].

In this paper, we focus on the 2X2 MIMO-BICM transmission scheme. The main purpose of this paper is to study the performance of the golden codes using such transmission system. For this end, we derive the pairwise error probability for both cases of flat fading and frequency selective channels. From the PEP expression, we derive space time design criterion that minimizes this PEP. We show that in both cases, perfect codes are optimal as they fulfil the space time design criterion in a BICM-MIMO system.

II. PRELIMINARIES ON SYSTEM MODEL

A. *The Golden Code*

The Golden code was proposed for a 2 X 2 MIMO system as an optimum 2 X 2 linear dispersion space-time block codes. The code is constructed using cyclic division algebras which is a particular family of division algebras. It is built over a quadratic extension of the base field Q(i), where i2= -1, thus use of arbitrary Q-QAM constellations is possible. Efficient constellation shaping is provided due to inherent integer lattice structure which is leading to the information lossless property. Q(i) which is the base field provides the non-vanishing determinant (NVD) property for the Golden code, i.e., for any QAM size, the minimum determinant remains constant. It has been also proved that the NVD guarantees to achieve the fundamental performance limit of the multiple-input multiple-output (MIMO) systems, given by the diversity multiplexing tradeoff (DMT).

The Golden code is optimal in several senses. It has:

– Full rate (2 symbols per channel use)
– Full diversity (d = 4)
– Non-vanishing determinant, it is clear that ±min is independent of the constellation size.
– It preserved mutual information (unitarity of the vectorized codeword matrix).

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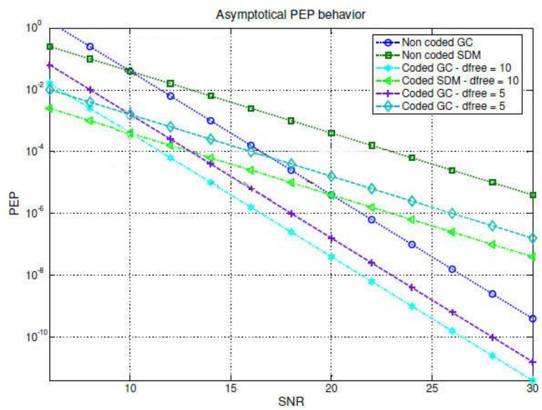

Fig.1. Asymptotical behaviour of the PEP over a flat fading channel

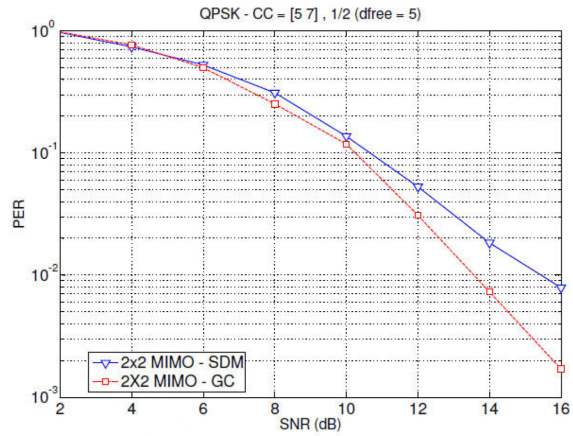

Fig.2. Simulation results for a packet of 1 ko: SDM vs. the Golden code in a 2X2 BICM system for a QPSK modulation with [5 7] encoder ($d_{free} = 5$)

B. Spatial Division Multiplexing

The spatial division multiplexing corresponds to the case when no space time code is used. The spreading factor in this case is s = 1, and the codeword C is a n_t. For SDM schemes, properties used in the following are

- The rank is equal to 1.
- The minimum determinant of the nt x nt matrix is equal to d_{min}^2, where d_{min} is the distance between two constellation points. It is equal to the $\frac{2}{\sqrt{E_s}}$.

C. General pairwise error probability derivation

Expression for pairwise error probability is given for general case using a space-time code with a spreading factor s. It is extension of The pairwise error probability (PEP) of BICM-MIMO-OFDM using an orthogonal space time code such as the Alamouti code[4]. We assume that the energy per coded symbol at each antenna and on each time slot is equal to one and that SNR is given by 1/N0. The PEP over a general channel model is upper bounded by,

$$P(c \to \hat{c}) \leq E_H \exp\left(-\frac{1}{4N_0}\left\{\sum_{k',d_{free}} \|H(k)C(k)\|_F^2\right\}\right)$$

Where, d_{free} is the free distance of the convolutional code, C(k) is a non-zero space time codeword matrix. The notation $\sum k', d_{free}$ means that we only consider the d_{free} bits indexed by k', interleaved such that $\pi: k' \to (k, i)$ Format (PDF).

III. CHANNELS

In this paper, we address the two cases where the channel is either flat fading or selective in frequency. The flat fading channel models the case when the channel remains constant over all realizations during the duration of the transmission, i.e.

$$H(k)=H, \ \forall k$$

We assume in this case that the transmission is done during a period of Ns time slots, and we refer the coded space-time code index by k, with k = 1……N.

The frequency selective fading models the case when the channel. For a frequency selective channel with L taps, a MIMO-OFDM system is considered. The channel is therefore decomposed into N parallel channels,

$$H(k) = H(e^{j2\pi f_k}); \ k = 1\ldots.N$$

that are statistically correlated where f_k is the subcarrier and k refers in this case to the channel frequency index. We assume in this case that the sequence of symbol x is coded into N space-time block codeword, and each space-time codeword is transmitted over one subcarrier k, where k = 1…..N.

A. BICM-MIMO with flat fading channel

For a MIMO-BICM system with a flat fading channel, perfect space time code allows to extract the full diversity order of $n_t n_r$ and the PEP is upper-bounded by,

$$PEP \ \leq \ (d_{free} \ \delta)^{-n_r} \ (E_s)^{n_t n_r} \ SNR^{n_t n_r}$$

It is clear that the convolutional code over a flat fading channel improves the coding gain.

B. Golden codes versus SDM

When no space time code is used, the PEP can be bounded by,

$$PEP \ \leq \ (d_{free} \ \delta)^{-n_r} \ (E_s)^{n_r} \ SNR^{n_r}$$

In Figure 1, the asymptotical behaviour of the coded SDM versus the coded 2X2 golden code scheme is depicted for the 2X2 MIMO configuration using QPSK constellation. As it can be shown, when no convolutional code is used, the coding gain of SDM is dominant at very low SNR. However, the diversity gain of the Golden code became quickly dominant, and can be easily observed at reasonable PER range.

When a MIMO BICM is used, it can be observed that the coding gain of SDM is largely enhanced compared to the Golden code coding gain especially when the free distance of the convolutional code increases. At low SNR, this coding gain dominates the high diversity gain that can be achieved by the GC. However, at high SNR, the diversity gain became dominant.

As shown in Figure 2 for the 2 X 2 MIMO flat fading channel and a convolutional code [5 7], Rc = 1/2 with d_{free} = 5

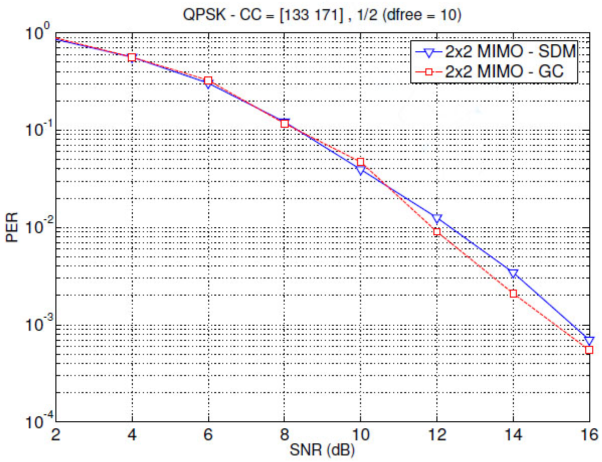

Fig.3. Simulation results for a packet of 1 ko: SDM v/s the Golden code in a 2X2 BICM system for a QPSK modulation with [133 171] encoder (d_{free} = 10)

and a packet of 1000-bits, the gain provided by the full diversity of the GC dominates the coding gain for low SNR as well as for the high SNR order. A gain of 1.9 dB is obtained for a PER of 10^{-2}. However, when using a convolutional code with a large free distance, e.g. [133 171] - Rc = 1/2, with d_{free}=10, as shown in Figure 3 the coding gain dominates the diversity gain. The impact of the additional diversity cannot be observed at a moderate range of PER.

C. BICM-MIMO with frequency selective channels

For a MIMO-BICM system with frequency selective fading channel, golden code used over subcarriers allows to extract the full diversity order of $n_t n_r \min(L, D)$ and the PEP is upper-bounded by,

$$PEP \leq G\ SNR^{n_t n_r \min(L,D)}$$

where, $D \leq d_{free}$ denotes the number of different subcarriers on which erroneous bits are received. The interleaver design allows to maximize the parameter D. The parameter α is a constant that depends on the covariance matrix.

It can be noticed here that the full diversity order for the frequency selective fading channel can be extracted here without requiring to code across all the frequency subcarriers as shown in [8] for the case when no outer code is used. As we show here, by coding the symbols in a MIMO BICM system using a golden code over each subcarrier, the maximal diversity order can be achieved.

D. Golden code versus SDM

Let,

$$\ominus = \left(V_{L\,X\,d_{free}} \otimes I_{nt}\right) CC' \left(V_{L\,X\,d_{free}} \otimes I_{nt}\right)'$$

When no space time coding is used, the minimal rank of the effective codeword $\ominus$ in above equation arises when all the erroneous codewords at the d_{free} subcarriers are received on the same antenna. This implies that, C is proportional to $I_D \otimes C(k)$, where C(k) is a non zero SDM codeword received on an arbitrary subcarrier k, with rank equals to 1 for the SDM case. Therefore, the rank of the effective codeword matrix is equal to,

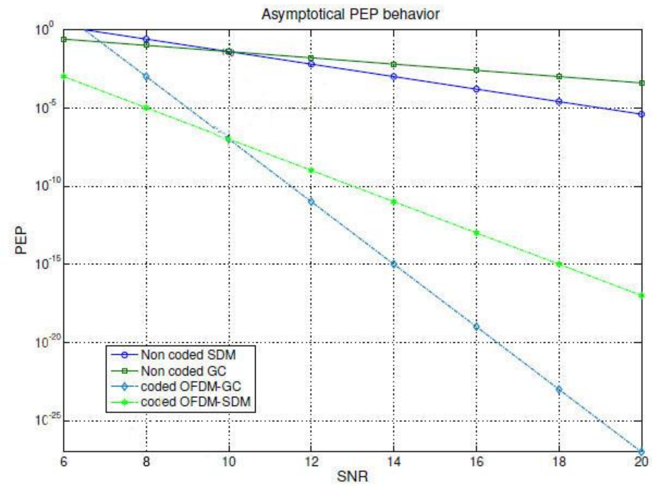

Fig. 4. Asymptotical behavior of the PEP over a frequency selective channel

$$rank\,\{\ominus\} = rank\,\{(V \otimes I_{nt})\,(I_D \otimes c(k))\},$$
$$rank\,\{\ominus\} = \{V \otimes c(k)\} = \min(L, D)$$

In Figure 4, the asymptotical behavior of SDM versus coded GC is depicted for the 2 X 2 MIMO configuration using QPSK constellation, and a convolutional code [5 7] -1/2, with d_{free} = 5, and a over a multi-tap channel with L = 18. It can be observed that at low SNR, the coding gain of SDM is largely enhanced compared to the Golden code coding gain. At low SNR, this coding gain dominates the high diversity gain that can be achieved by the GC. Both schemes gain in diversity compared to the non coded case. At high SNR, the diversity gain became dominant. The diversity gain can be observed at a very low PER rate (range of 10^{-7}).

The performance of the Golden code versus SDM has been evaluated in the IEEE 802.11n context in terms of packet error rate (PER) versus SNR, for a packet length of 1000-bits. In the following, SNR gain will be related to a PER of 10^{-2}. The packet error rates in Figure 5 and Figure 6 are evaluated over channel D using QPSK and 16QAM constellation. The channel D is characterized by a 50ns rms delay spread and 18 taps, and then by significant frequency diversity. In the IEEE 802.11n context, the convolutional code [133 171] with a coding rate of Rc = 1/2 is used with d_{free} = 10. No additional gain is observed at a PER = 10^{-2}. The channel B in the IEEE 802.11n standard can be assimilated to a flat fading channel, for which the additional gain using a convolutional code with high free distance (d_{free} = 10) cannot be observed at reasonable PER.

IV. CONCLUSION

In this paper, we studied the performance of non-vanishing determinant code constructed from cyclic division algebra in a standard context. The PEP is derived for BICM-MIMO for the cases of flat fading and frequency selective channels when respectively golden codes and spatial division multiplexing schemes are used at each subcarrier. When the channel is flat, we show that the diversity order remains the same as the non-coded case. However, the coding gain is improved compared to the non coded case. We noticed that the coding gain of the SDM is largely enhanced compared to the coding gain of perfect code especially for a convolutional with a large free distance. For the frequency selective channel case, we show that using perfect codes at each subcarrier allow to extract the transmit diversity. We show then that the diversity order of BICM-OFDM system can be also extracted when no space time code is used. In a practical context, such as in IEEE 802.11n, the numerical results we provide show that this gain in diversity cannot be observed at a moderate range of PER.

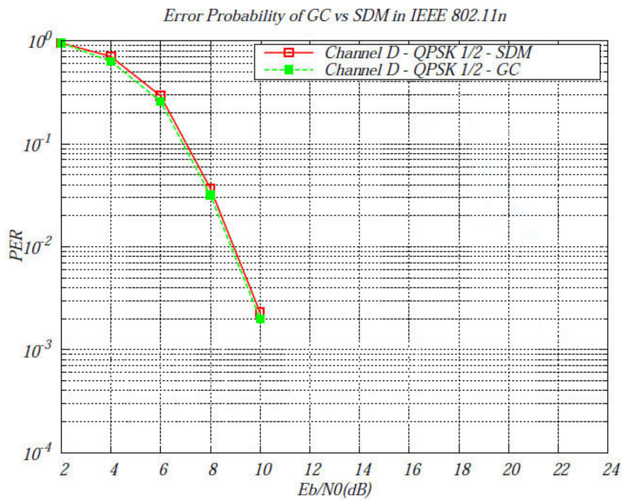

Fig. 5. Golden Code vs SDM in IEEE 802.11n context for a QPSK - 1/2 Modulation with spectral efficiency = 2bpcu

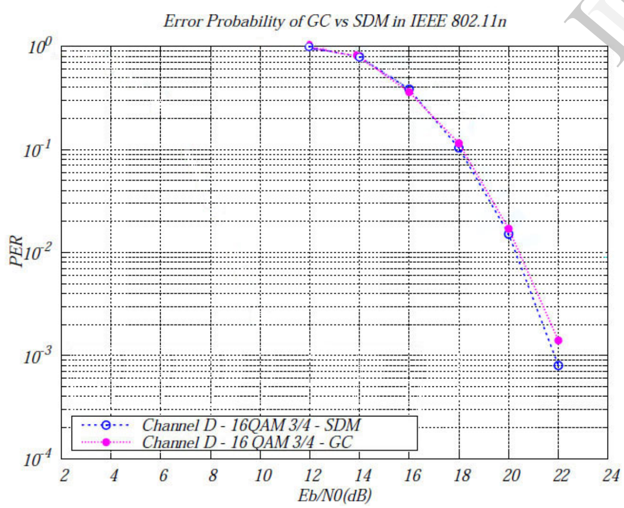

Fig.6. Golden Code vs SDM in IEEE 802.11n context for a 16 QAM Modulation

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