Performance of Multi-server Infinite Source Retrial Queueing System in Operational Research for the application of GSM Networks

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***Abstract*:A standard queueing models gives a survey of main results for both single server M/G/1 type and multiserver M/M/C type retrial queues and discuss similarities and differences between the retrial queues and their standard counterparts.Queueing theory is usually assumed that a customer who can't get service immediately, after arrival either joins the waiting line or leaves the system forever, Retrial Queues, that is, queues with returning customers, repeated orders, etc. have been introduced to solve this deficiency.Most queueing systems with retrials are motivated by computer and telecommunication applications where a repeated attempt appears due to blocking in a system with limited service capacity.**

Keywords: Queueing Theory Models, Retrial Queues,

1.INTRODUCTION :

Operations Research is a scientific approach to problems solving for executive management. In 1951, the first book on the subject methods of operation research by Morse and Kimball was published. Operation Research uses the method of science to understand and explain the phenomena of operating systems. In Operation Research, we are going to discuss about the Queueing systems.Queue is a common word that means a waiting line or the act of joining a line. It is formed when the number of customers arriving is greater than the number of customers being served during a period of time. Queueing theory is usually assumed that a customer who can't get service immediately after arrival either joins the waiting line or leaves the system forever. Retrial queues that is, queues with returning customers, repeated orders, etc. have been introduced to solve this deficiency.Retrial queueing systems are characterized by the feature that arriving calls who find the server busy join the retrial group for their requests in random order and at random intervals. Retrial queues have been widely used to model many problems in telephone switching systems, computer and communication systems. I consider a trial queueing systems with batch arrivals in which the server is subject to controllable interruptions and random interruptions. The main characteristic of retrial queues is that if an arriving customer finds all servers busy, he leaves the service area, but after some random time repeats his demand.

2. <u>STANDARD AND RETRIAL QUEUEING SYSTEMS</u>

A standard queueing models gives a survey of main results for both single server M/G/1 type and multiserver M/M/C type retrial queues and discuss similarities and differences between the retrial queues and their standard counterparts.

Queueing theory is usually assumed that a customer who can't get service immediately, after arrival either joins the waiting line or leaves the system forever, Retrial Queues, that is, queues with returning customers, repeated orders, etc. have been introduced to solve this deficiency.

In the most general form these networks contain two nodes :

- The main node where blocking is possible and a delay node for repeated trials.
- To describe specific retrial queues with a certain structure and queueing discipline more nodes have to be introduced.

The single server has intrinsic interest for the stochastic modeling of communication protocols arising from local area networks.The classical retrial policy assumes that the probability of a repeated attempt during the interval (t, t+dt), given that j calls are in orbit at timetis j μ dt + o(dt).Most queueing systems with retrials are motivated by computer and telecommunication applications where a repeated attempt appears due to blocking in a system with limited service capacity.It is clear that there exists a rich variety of different single server and multiserver queueing systems with retrials. In many other cases an extended investigation based on the methods developed for the M/M/C and M/G/1 retrial queue may be carried out for structural complex retrial models.

2.1The M_1, M_2/G/1/K Retrial Queueing Systems with priority :

Consider an M_1, $M_2/G/1/K$ retrial queueing system with a finite priority queue for type I calls and infinite retrial group for type II calls where blocked type I calls may join the retrial group.

Retrial queueing systems are characterized by the feature that arriving calls who find the server busy join the retrial group for their requests in random order and at random intervals.

Retrial queues have been widely used to model many problems in telephone switching systems, computer and communication systems.

Retrial queues with two types of calls are the typical model of telephone exchange with subscriber line modules and base station in a mobile cellular radio communication system.

Consider M_1, $M_2/G/1/K$ retrial queue with two type calls where blocked type I calls may allow to join the retrial group. Type I calls and type II calls arrive independently of each other according to poisson processes with rate λ_1 and λ_2 respectively.

An arriving type I call joins the priority queue if there is a waiting position, but if there are no waiting positions in the priority queue, he enters the retrial group with probability α or leaves the system with probability 1 - α . If an arriving type II call finds the server busy, then he joins the retrial group in order to seek service again after random random amount of time. A call in the retrial group always returns to the retrial group when he find the server busy on his retrial attempt to the server.

The retrial time (the time interval between two consecutive attempts made by a call in the retrial group) is exponentially distributed with mean 1/ν and is independent of all previous retrial times and all the other stochastic process in the system.

The service times of calls are independent and identically distributed with distribution function B(x) and mean 1/μ.

$$b^*(\theta)=\int_0^\infty e^{-\theta x}\, dB(x)$$

And

$$b^{*(i)}(\theta)=\frac{d(i)\,(b^*(\theta))}{d\theta^i}$$

2.2Applications of Ergodicity

- For an irreducible and aperiodic Markov chain $\{Z_n\}$ with state space S, a sufficient condition for ergodicity is the existence of a non-negative function f(s), s ∈S and ε > 0 such that the mean

$$x_s = E\left[f(Z_{n+1}) - f(Z_n) \mid Zn = s\right] < \infty$$

for all s ∈S and $x_s < -\in$ for all s ∈S except perhaps a finite number.

- Let $\{Z_n\}$ be a irreducible Markov chain with countable state space S. If there exists a non-constant function $f : S \to [0,\infty)$ such that

a) $E\ \{f(Z_{n+1}) - f(Z_n) \mid Zn = i\} \geq 0$ for all i∈ S

b) there is an M > 0 such that $E\ \{| f(Z_{n+1}) - f(Z_n) \| Z_n = i\} \leq M$ for all i∈ S

then $\{Z_n\}$ is not ergodic.

- The imbedded Markov chain $\{Z_n = (X_n, Y_n) \mid n = 1,2,.....\}$ is ergodic and $h(1) = a_0^{-K} \det(\hat{A}) < 1$.

2.3. Queue size distribution in steady state

If the distribution of service time is not exponential, then the stochastic process $\{(N_q(t), N_r(t)\ ; t > 0\}$ is not Markov process. Let X(t) and I(t) be a random variables. Where X(t) is the elapsed service time of the call in service at time t and I(t) is the server state, I(t) = 0 if the server is idle at time t and I(t) = 1 otherwise then

$$\{(N_q(t), N_r(t), X(t), I(t)\ ; t > 0\}$$

is Markov process with state space

$$\{(i, j, x, \ell)\ ; i = 0, 1, ... K,\ j = 0, 1, ...,0 \leq x < \infty, \ell = 0,1\}$$

define the probabilities,

$$q_j(t) = P\ \{N_q(t) = 0, N_r(t) = j, I(t) = 0\}$$

$$P_{i,j}(t, x)\, dx = P\ \{N_q(t) = i, N_r(t) = j, x < X(t) \leq x + dx, I(t) = 1\}$$

$$i = 0, 1, ... K$$

2.4. NUMERICAL EXAMPLES:

Assume that the mean service time is 1 and the retrial rate ν=0.3. the service time distribution was taken as hyper-expomential with parameter(1/3,2/3).The loss probability of type I calls for two cases (α=0 and α=0.3) verses the capacity K and arrive rate of type I calls under a fixed λ_2=0.1. the loss probability decreases as the capacity K increases and the arrival rate of typeI calls decreases.The loss probability of type I calls as functions of the arrival rate λ_2 under the parameters: K=8 and $\lambda_1=0.4\lambda_2$. the loss probability increases as the arrival rate of type II calls increases, and decreases as the probability α of entering group increases.

The mean waiting time of type I calls in priority queue as functions of the arrival rate λ_1 under the parameters: K=8 and $\lambda_2 = 0.2\lambda_1$. the mean waiting time of type I calls increases as the arrival rate of type I calls increases,but has no a great difference according to the probability α.

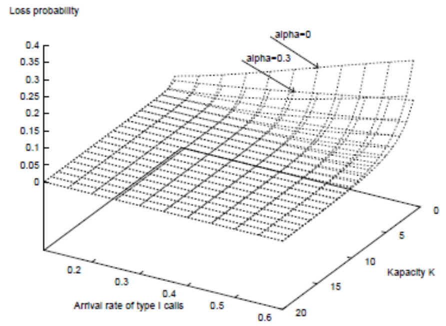


Figure Loss probability of type I calls H exp (1/3, 2/3) service time, v = 0.3 , λ_2=0.1

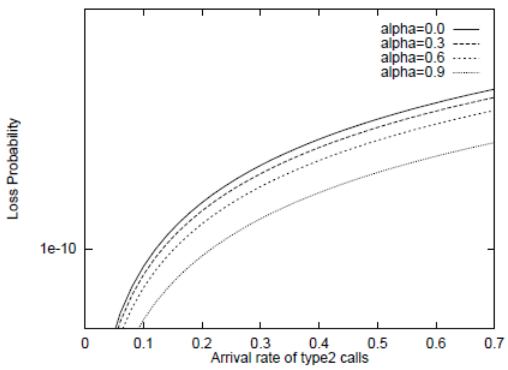


Figure Loss probability of type I calls H exp (1) service time, K = 8 , λ_1=0.4λ_2

3. CELLULAR MOBILE NETWORKS USING MOSEL

The retrial queues investigates a multiserver infinite - source retrial queueing system for the performance modeling of cellular mobile communication networks.

The objective is to demonstrate how performance tool MOSEL (Modeling specification and Evaluation Language) can be efficiently used in the modeling of cell based networks. In our analysis the blocked and dropped users are treated separately, i.e. redial with different probabilities and different rates, with reducing the state space by maximizing the number of redialing customers with appropriately large values.

Queueing network models are widely used in the traffic modeling of cellular mobile systems, such as GSM (Global System for Mobile communications). GPRS (General Packet Radio Service) and UMTS (Universal Mobile Tele Communication System).

Tran-Gia and Mandjes described a model which demonstrated in the context of cellular systems that the retrial phenomenon is not neglectable because of the significant negative influence on the system performance measures into consideration in their cellular mobile network model.

The main characteristic of retrial queues is that if an arriving customer finds all servers busy, he leaves the service area, but after some random time repeats his demand.

Cellular systems with customer redials are treated in [MARSAN ET AL,2001] , where an approximate technique is proposed for finite and infinite Markovian models. The authors reduce the state space of the continuous time Markov chain model by registering only that if there are retrying blocked and dropped customers in the system or not .

In the works [ONUR ET AL,2002; ALFA AND LI ,2002], various infinite –source queueing models are studied.In [ONUR ET AL,2002], not only customer redials,but also automatic retrials by the cellular system are taken into consideration , but the dropped customers redials handled as generating new fresh call attempts in the new cell and incase of blocking the cell is treated as a blocked fresh call. It is probably less realistic , because an interrupted customer may try to restablish the call with higer probability in shorter time intervals. In [ALFA AND LI ,2002], the blocked new and dropped handoff calls are not distinguished, but the involved random variables have general phase type distribution.

The blocked and dropped users are treated separately , that is they redial with different probabilities and different rates , like in [MARSON ET AL,2001], but reduce the state space by maximizing the number of redialing customers with appropriately large values (i.e. when the ignored probability mass can be neglected).

In [TRAN-GIA AND MANDJES ,1997;ONUR ET AL,2002; ALFA AND LI,2002], these two types of redialing customers were not distinguished. Furthermore , in our model we allow not only the active but also both types of redialing cutomers to depart to other cells , the current stydy can be considered as an initial step towards the analysis of more complex third generation systems focusing on the quality of service issues.

In cellular networks, the most important quality of service measures are the following :

- The fresh call blocking probability (P_f), i.e. the fraction of new all requests in the cell that can't be served due to the lack of free channels.
- The handoff call dropping probability (P_h), that is the average fraction of incoming handoff calls that are terminated because of the lack of free channels.

The grade of service (GOS) is generally defined as the combination of these two probabilities, for example as

$$GOS = \frac{p_f + 10p_h}{11}$$

Because of the fact , that handoff call dropping probability has more significant impact on the grade of service, It is important to reduce it even at the expense of prioritize handoff calls , several channel allocation schemes are utilized. One of the most popular policies is the guard channel scheme]

[DHARMARAJA ET AL ,2003;TRAN-GIA AND MANDJES,1997; MARSAN ET AL ,2002;ALFA AND LI,2002], where some channels are reserved for the calls that move across the cell boundary , that is if there are g reserved channels in the cell, a new fresh call is only accepted if there are at least g+1 available channels . A handoff call is rejected only if all the channels in the cell are occupied.

3.1. Model Description :

The model description is translated step by step into the description language of MOSEL, and it is automatically converted into the other tool - specific system descriptions and analyzed by the appropriate tools.

In cellular network model treat only one cell.the cells are considered identical and to have the same traffic parameters , so it is enough to investigate one cell, and the handoff effect from the adjacent cells to this cell and from this cell to adjacent cells is described by handoff processes. Instead of the frequently used single arrival stream model distinquish the fresh call and handoff call arrivals . if investigate complex call handelling policies.

The number of channels in the cell is C, and the number of guard channels is g, where g<C.

The arrival process of the fresh calls is a poisson process with rate λ_f. If the number of the active users is smaller than c - g, the incoming call starts to be served. It is blocked and it starts generation of a poisson flow of repeated calls (redialing) with probability θ_1 or leaves the system with probability $1 - \theta_1$.

A blocked customer repeats his call after a random time which is exponentially distributed with mean $1/\nu_{bl}$, and it can be served or blocked again like the fresh calls. The call duration time is exponentially distributed with mean $1/\mu$.

The arrival process of the handoff calls is a poisson process with rate λ_h. If the number of the active users is smaller than C, the incoming call starts to be served. It is dropped and it starts generation of a poisson flow of repeated calls with probability θ_2 or leaves the system with probability $1 - \theta_2$.

A dropped customer tries to repeat his call after a random time which is exponentially distributed with mean $1/\nu_{dr.}$ if it is blocked it continues redialing with probability $\theta_{2.}$ the call duration time for handoff calls is also exponentially distributed with mean $1/\mu$.

The active, redialing blocked and dropped customers leave the cell after an exponentially distributed time with mean $1/\mu_a$, $1/\mu_b$, $1/\mu_d$ respectively.

The number of redialing users because of blocking and dropping is limited to an appropriately large values of N_{bl} and N_{dr} to make the state space finite in orders to the tools in the steady state.

3.2.The underlying Markov Chain

The stochastic process X(t) = (C(t); N(t); M(t) where
C(t) is the number of active customers,
N(t) is the number of blocked new customers
M(t) is the number of dropped customers.

the exponentiality of the involved random variables the describing process is a Markov chain with a finite state space S = {0, ...C} x {0,..............N_{bl}} x {0,..............N_{dr}}
Since its state space is finite, the process is ergodic for all values of the rate arrival of handoff calls.

Define the stationary probabilities :

$P(i ; j ; k) = \text{Lim } P(C(t) = i, N(t) = j, M(t) = k)$
$i = 0, \ldots C,\ j = 0, \ldots N_{bl},\quad k = 0, .N_{dr.}$

Because of the fact the state space of (x (t), t $\geq$ 0) with sufficiently large N_{bl} and N_{dr} is very large and the functioning of the system is complex . it is very difficult to calculate the steady state probabilities. To simply these calculations and to make our study more usable in practice, we use the software tool MOSEL to formulate the model and to calculate these probabilities and the system measures. MOSEL has already been used ,and it has proved its applicability for the modeling of serval computer and communication system. The MOSEL description can be translated automatically into the language of various performance tools and then analyzed by the appropriate tools (at present SPNP-stochastic perti net package and time NET are supported and suitable for this model) to get these measures.

4. MODEL CONVERSION TO MOSEL:

We discuss the translation of the model into the language of the MOSEL tool. The full MOSEL program can be assembled from the following program parts among the model description in the order of the part numbers.

The number of channels in the cell is C, which is denoted as N_CHS in the program, and the number of guard channels is g , which is denoted as N_G_CHS.

In the first part of the MOSEL description , we have to define some other system parameters too, these will be introduced at the appropriate program parts.

```
CONST  N_CHS := 15;
CONST  N_ G_ CHS := 1;
CONST  MAX_BL_ USERS :=25;
CONST  MAX_DR_USERS :=25;
CONST call_arrive :=1.5;
CONST call_retry_bl :=5;
CONST call_retry_dr :=6;
CONST call_duration :=0.05;
CONST handoff_ arrive :=0.4;
CONST handoff_dep_ac :=1/3;
CONST handoff_dep_bl :=1/3;
CONST handoff_dep_dr :=1/3;
CONST p_retry_ bl :=0.7;
CONST p_retry_dr :=0.9;
```

The state of the system is described by the number of active users , the number of blocked users who redial after some random time, and the number of users whose calls are dropped at handoff and who are redialing.

It can be wrote down in MOSEL as defining the nides of the system . the number of active users is denoted by active _users. Its maximum value is the number of channels , and it is 0 at the starting time.the number of redialing users because of blocking and dropping is limited to MAX_BL_

USERS and MAX_DR_USERS, which are defined in (1).

```
NODE active_users [N_CHS] : =0;
NODE redialing _users_br
[MAX_BL_USERS]: =0;
NODE redialing _users_dr
[MAX_DR_USERS]: =0;
```

The arrival process of the fresh calls is a poisson process with rate λ_f , that is denoted in the program as call_arrive , that is defined in (1) like the other parameters. If the number of active users is smaller than c-g, the incoming call starts to be served . otherswise it is blocked and it starts generation of a poisson flow of repeated calls (redialing) with probability θ_1 (denoted by p_retry_ bl) or leaves the system with probability 1-θ1.

```
IF active_users_N_CHS_N_G_CHS
FROM EXTERN TO active_users
RATE call_arrive ;
IF active_users> - N _CHS_ N_G_CHS
FROM EXTERN RATE  call_arrive THEN  {
TO  redialing _ users_ bl
WEIGHT  p_retry_bl ;
TO EXTERN WEIGHT 1- P_retry_bl ;
}
```

The blocked user redials can be handled similar to the fresh call arrivals. If a user is blocked , he repeats his call after a random time which is exponentially distributed with mean $1/\nu_{br}$.$\nu_{br}$is denoted as call_retry_bl.

It can be served or blocked as the fresh calls in the previous part .

```
IF active_users< N_CHS_N_G_CHS
FROM  redialing_users_bl  TO active_users
RATE   call_retry_bl*  redialing _users_bl ;
If active_users>= N_CHS_N_G_CHS
FROM  redialing_users_bl
RATE call_retry_bl*  redialing _users_bl
THEN  {
TO  redialing _users_bl
                  WEIGHT P_retry_bl ;
TO EXTERN WEIGHT 1- P_retry_bl ;
}
```

The call duration time is exponentially distributed with mean $1/\mu$. μ is denoted as call_duration .

```
FROM active_users TO EXTERN
RATE call_duration * active_users ;
```

The arrival process of the handoff calls is a poisson with rate $\lambda_{h.}$ λ_h is denoted in the program as handoff_arrive. If the number of active users is smaller than C, the incoming call starts to be served . otherwise it is dropped and it starts generation of a poisson flow of repeated calls with probability θ_2 (denoted by p_retry_dr) or leaves the system with probability $1-\theta_2$.

IF active_users< N_CHS
FROM EXTERN TO active_users
RATE handoff_arrive ;
IF active_users = N_CHS
FROM EXTERN RATE handoff_arrive
THEN {
TO redialing _ users_dr
WEIGHT P_retry_dr ;
TO EXTERN WEIGHT 1-P_retry_dr ;
}

The dropped user redials can be handled like the blocked fresh call redials. The customer repeats his call after a random time which is exponentially distributed with $1/\nu_{dr}$. ν_{dr} is denoted as call_retry_ dr. If it is blocked it continues retrying with probability θ_2 (p_retry_dr).

IF active_users< N_CHS-N_G_CHS
FROM redialing_users_dr TO active_users
RATE call_retry_dr* redialing_users_dr ;
IF active_users>= N_CHS-N_G_CHS
FROM redialing_users_dr
RATE call_retry_dr* redialing_users_dr
THEN {
TO redialing_users_dr
WEIGHT p_retry_dr ;
TO EXTERN WEIGHT 1-p_retry_dr ;
}

The active and redialing customers leave the cell after an exponentially distributed time with parameters μ_a, μ_b ,and μ_d . denoted as handoff_dep_ac ,handoff_dep_bl and handoff_dep_dr , respectively.

FROM active_users TO EXTERN
RATE handoff-dep-ac* active_users ;
FROM redialing_users_bl TO EXTERN
FROM handoff_dep_bl* redialing_users_bl;
FROM redialing_users_ dr TO EXTERN
RATE handoff_dep-dr* redialing_ users_dr ;

After describing the system functioning , we can define the system measures we would like to calculate, such as the mean number of active and redialing customers because of blocking and handoff failure, the fresh call blocking and handoff call dropping probabilities .

PRINT mean_active_users_bl=
MEAN (active-users) ;
PRINT mn_redialing_users_bl=
MEAN (redialind_users_bl) ;
PRINT mn_redialing_users-dr=
MEAN (redialing_users_dr) ;
PRINT call_blocking_prob=
PROB (active_users>= N_CHS-N_G_CHS) ;
PRINT handoff_call_dropping_prob=
PROB (active_users>= N_CHS)

Finally, We define two pictures that show the changing of the blocking and dropping probabilities depending on the number of channels. If we use N_CHS as parameter, we have to define it in (1) as follows:

PARAMETER N_CHS:= 6,7,8,9,10 ;
PICTURE “Blocking probability vs N_CHS “
PARAMETER N_CHS
CURVE call_blocking_prob;
PICTURE “Dropping probability vs N_CHS “
PARAMETER N_CHS
CURVE handoff_call_dropping_prob ;

2.5 Numerical Examples

Consider a sample numerical results to illustrate graphically the sysem measures depend on variable system parameters.

The fresh call blocking and hand-off call dropping probabilities are displayed versus the number of channels with and without user redials. The system parameters belonging to the curves without redials are the same as in [Dharmaraja et al,2003]. Where a similar model is studied without customer redials

($g = 3$, $\lambda_f = 05$, $\mu = 0.05$, $\mu_a = \mu_b = \mu_d = 1/3$
$\lambda_h = 0.4$, $\nu_{bl}=\nu_{dr}=10^6$, $\theta_1,= \theta_2=10^{-6}$
and for other curve

$\nu_{bl}=\nu_{dr}=6$, $\theta_1=\theta_2=0.9$

Further more the maximum number of redialing customers is 25 respectively). These results are in agreement with theirs in the exponential case.

The fresh call blocking and handoff call dropping probabilities are displayed versus the mean handoff call arrival rate. The system parameters are the same as in figure,except of that C=8,and $\lambda_h$is on the x axis .like in [Dharmaraja et al,2003].

The negative influence of the retrial phenomenon is shown in each figures, and we can see that it increases as the handoff call arrival rate increases.

The fresh call blocking probability, the handoff call dropping probability and the grade of service as the mean fresh call arrival rate increases. The following system parameters were used:

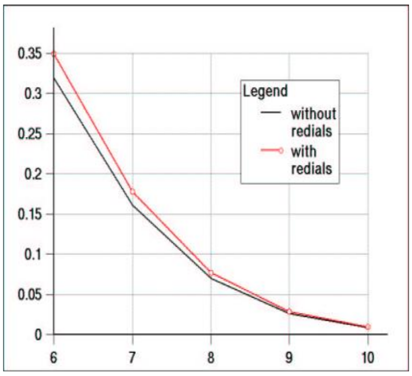
$C=7$, $g=1$, $\mu=0.05$, $\mu_a=\mu_b=\mu_d=1/3$,

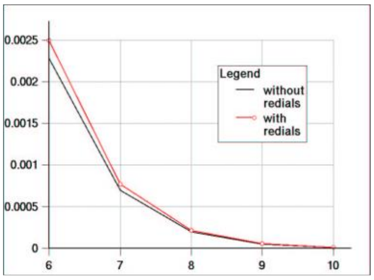
$\lambda_h=0.4$, $\nu_{bl}=6$, $\nu_{dr}=7$, $\theta_1=0.8$, $\theta_2=0.9$

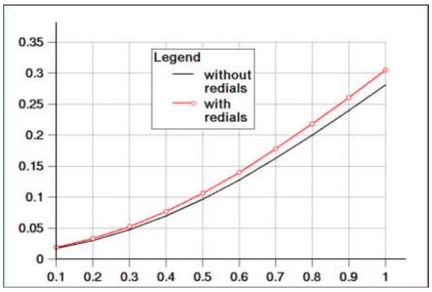
The fresh call blocking and handoff dropping probabilities and the GoS are displayed versus the number of guard channels. We can see that a very few number of guard channels can improve the grade of service significantly , but then only very small handoff dropping advance can be achieved on the great expense of fresh call blocking probability, and the GoS declines. The system parameters are the following:

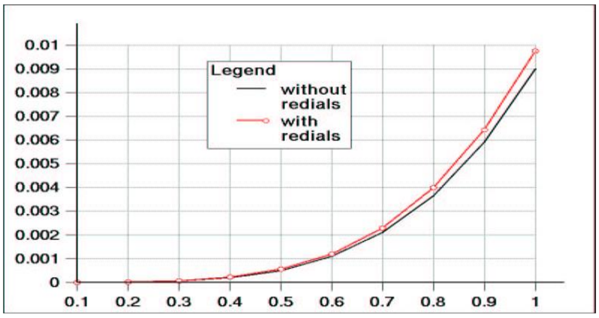
$C=15$, $\lambda_f=3$, $\mu=0.05$, $\mu_a=\mu_b=\mu_d=1/3$,

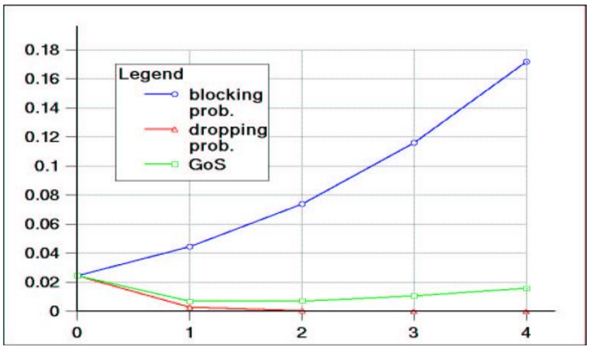
$\lambda_h=0.4$, $v_{bl}=6$, $v_{dr}=7$, $\theta_1=0.8$ and $\theta_2= 0.9$











5. NUMERIAL ILLUSTRATIONS :

The effect of parameters (retrial, vacation, and breakdowns) on system performances. In the remainder of the basic data of [ARTALEJO , 1997] :

$\lambda = 1$, $g_1= 1$, $g_1 = 0$, $h_1= 0.25$, $h_2 = 1$ maintenance parameters $W_1 = 0.1$, $W_2 = 1$.

The effect of failure rate on the retrial parameter δ . the function $\delta(\theta)$ for different retrial PDF with mean $r_1 =1$.

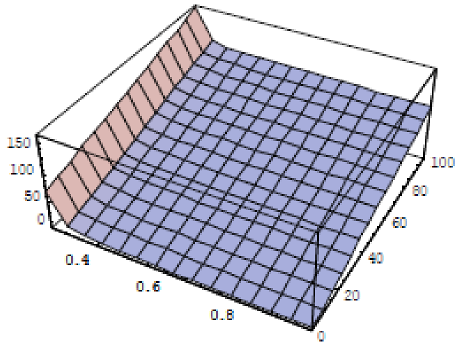
(i) Hyperexponential (H_2).

(ii) Exponential (EXP) :

(iii) Determinist (D) :

We observe that parameter δ increases in the case (i) and decreases in the case (iii) as the failure rate increases. (ii) the parameter δ is independent of the failure rate. This can be easily understood from exponential nature of retrial time.

The expontation E(M) versus failure rate θ and ratio ν_2 /ν_1. E(M) decreases when θ and $\nu_2/\nu 1$ increases and increases otherwise.

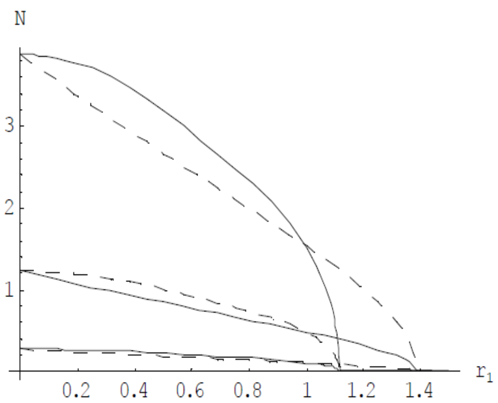


Effect of breakdowns and vacations on Mean system size

The effect of failure rate on the optimal threshold for different values of C_S/C_h=10,50 and 100. we have considered a 2-Erlangian retrial distribution (E_2) with r_1=0.5; the optimal threshold increases with the ratio C_S/C_h .

Lower and upper bounds on the optimal value N* for different parametric (Exp,D,H_2) and non parametric (NBCE) retrial PDF which typify some PDF observed in Practice. For each of these choices we varied the ratio C_S/C_h from 0.5 to 10^5 .

Behaviour of the bounds as a function of the mean retrial time for different values of C_S/C_h =10,1,0.1. For a given value of this ratio , the dot-dashed curve corresponds to a lower bound and the continous curve to an upper bound. The lowest pair of curve corresponds to the case C_S/C_h=0.1. We see that lower bound tends to be more closed to the upper bound curve for small values of r_1 and C_S/C_h.



Finally, the joint effect of retails and breakdowns upon the optimal value N* and its corresponding minimum expected cost. The optimal value N* increases and the cost decreases when both δ and θ increases.

6. QUEUEING SYSTEM

ON optimal and equilibrium retrial rates in a single-server queueing model. Calls arrive according to a poisson process with average rate per unit time. Compare the two rates and suggest ways in order to equate the equilibrium rate with the socially optimal one. The rate minimizes the total expected cost by a customer.

A retrial rate defines a Nash equilibrium it is used by all customers then an individual minimizes its own expected cost by using the rate itself. The rest of the costs as structural costs that can't be changed by the decision maker, that excluding the part of the costs, and waiting costs, retrial costs are coincide. The Nash equilibrium rate coincides with the social optimal rate. This resembles the economic order quantity inventory control model, where holding costs and the setup costs coincide under the optimal ordering policy.

4.2 THE EQUILIBRIUM RETRIAL RATE :

The social optimal and the equilibrium rates depend on the ratio w/c and not an the individual cost parameters.

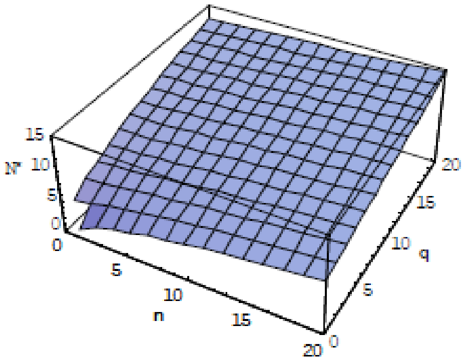
Let $\rho = \lambda\tau$ be the system's utilization factor and denote $\sigma^2 + \tau^2$ by S^2. We denote τ by $^1/_\mu$, where μ is the service rate, λ be the poisson process with average rate per unit time.

The server is busy, the call is repeated later, between retrials, the call is said to be orbit. The times between retrials are independent and exponentially distributed with an expected value of $^1/_\theta$ (θ is the retrial rate). Each retrial costs C and the cost of waiting is W per unit of time.

CONCLUSION

A Multiserver infinite – source retrial queueing system is studied for the performance modeling of GSM networks. It is easily efficiently the tool MOSEL can be used, and some numerical examples are presented to the impact of the retrial phenomenon and some system parameters on the quality of service measures and on the grade of service.The current study is an initial step towards the analysis of more complex third generation cellular systems. These hierachical systems may consist two or more layers, and varius dynamic channel allocation schemes can be utilized and analyzed.Furthermore, other than exponential distributions can be treated that are supported by both MOSEL and the applied tools .

I have studied the effect of retrials, vacations and breakdowns on the performance metrices of queueing service systems. I have showed how to control the vacation and retrial mechanisms. A similar study can be provided to control the maintenance actions.



Effect of retrial rate δ and failure rate of the optimal threshold N*

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