Performances of Kalman filter and LMS Algorithm in Channel Estimation

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Abstract

In this paper, we consider the estimation of communication channel using Kalman filter. The lower order kalman filter estimates the radio channel with Gaussian distribution. The same channel is used to estimate by using LMS algorithm. The performance of the both adaptive filter is compared in this paper.

1. Introduction

The approach of the problem of predicting and analyzing the observable properties of transmission, it is must first define what we mean by a channel. In its most general sense, a channel can describe everything from the source to the sink of a radio signal. This includes the physical medium (free space, fiber, waveguides etc.) between the transmitter and the receiver through which the signal propagates. The word channel refers to this physical medium throughout this work. An essential feature of any physical medium is, that the transmitted signal is received at the receiver, corrupted in a variety of ways by frequency and phase-distortion, inter symbol interference and thermal noise.
A channel model on the other hand can be thought of as a mathematical representation of the transfer characteristics of this physical medium. This model could be based on some known underlying physical phenomenon or it could be formed by fitting the best mathematical / statistical model on the observed channel behavior. Most channel models are formulated by observing the characteristics of the received signals for each specific environment. Different mathematical models that explain the received signal are then fit over the accumulated data. Usually the one that best explains the behavior of the received signal is used to model the given physical channel. Channel estimation is simply defined as the process of characterizing the effect of the physical channel on the input sequence. If the channel is assumed to be linear, the channel estimate is simply the estimate of the impulse response of the system. It must be stressed once more that channel estimation is only a mathematical representation of what is truly happening. A "good" channel estimate is one where some sort of error minimization criteria is satisfied.

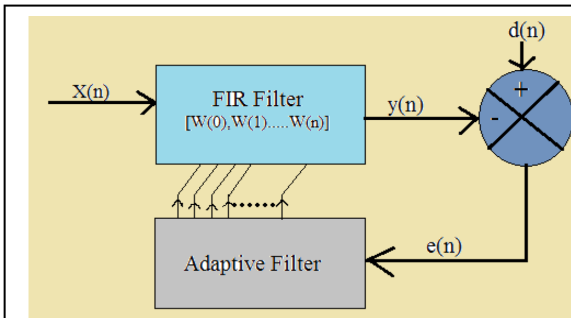

Figure: 1: The general system for Adaptive Algorithm

In the Figure:1 above e*(n)* is the estimation error. The aim of most channel estimation algorithms is to minimize the mean squared error, $E[e^2(n)]$ while utilizing as little computational resources as possible in the estimation process. Channel estimation algorithms allow the receiver to approximate the impulse response of the channel and explain the behaviour of the channel. This knowledge of the channel's behaviour is well-utilized in modern radio communications. Adaptive channel equalizers utilize channel estimates to overcome the effects of inter symbol interference.

2. Literature Survey

Jones et al. [1] have proposed that this module introduces adaptive filters through the example of system identification using the LMS algorithm. The adaptive filter adjusts its coefficients to minimize the mean square error between its output and that of an unknown system.
Haykin [2] discussed the concept of the adaptive filter algorithms that are implemented with FIR filter structures and their variety of applications in those systems where minimal information is available about

the incoming signal. He discussed several applications of adaptive filters based on FIR filter structures such as system identification, adaptive equalization for data transmission, echo cancellation for speech-band data of transmission, linear predictive coding of speech signals and array processing. The concept to design an adaptive equalizer using the mean square error(MSE) criterion and also explained the concept to minimize the cost function adaptively by applying the stochastic gradient(SG) or the least mean square(LMS) algorithm are discussed in Werner et al.[3]. Hayes et al.[4] discussed the RLS algorithm. Shen et al.[5] described the basics of channel estimation in ofdm system. Balakrishnans et al. [6] proposed an iterative Kalman filtering algorithm for estimation of the time-variant Rayleigh fast fading channel. Feldbauer et al. [7] discussed the LMS (least mean squares) and the RLS (recursive least-squares) algorithm for the design of adaptive transversal filters. These algorithms are applied for identification of an unknown system. For inverse modeling or equalization the adaptive filter is used in series with the unknown system. Rojas et al [8] discussed about the application of the Kalman filter and its ability to guess the best possible value for the next measurement by taking into account the present and the previous values, and further discussed a numerical method that can be used for sensor fusion or for calculation of trajectories. Welch et al. [9] discussed that the Kalman filter is a mathematical power tool that is playing an increasingly important role in computer graphics as we include sensing of the real world in our systems. Simon et al [10] proposed the concepts that are needed to know to design and implement a Kalman filter. He introduced the Kalman filter algorithm and observed the use of this filter to solve a vehicle navigation problem. Safaya et al [11] proposed that the data based channel estimation methods offer low complexity and good performance and are thus quite widely used in communications systems today. McGuire et al. [12] This paper presents a method for creating low-order Kalman filters to accurately track the Rayleigh fading radio channel. Olama et al. [13] have proposed an algorithm which consists of filtering based on the Kalman filter to remove noise from data.

3. Channel estimation

Mainly two types of adaptive algorithms are used in channel estimation purpose. The algorithms are Least-Mean Square (LMS) & Kalman filter.

3.1. LMS algorithm

LMS algorithm uses the estimates of the gradient vector from the available data. LMS incorporates an iterative procedure that makes successive corrections to the weight vector in the direction of the negative of the gradient vector which eventually leads to the minimum mean square error. Compared to other algorithms LMS algorithm is relatively simple [17].
Input: A random process x(n);
FIR filter of weight: (w0, w1…wN-1);
Filter output: $Y(n) = w^T x(n)$;
Error signal: $e(n) = d(n) - y(n)$
Where d(n) is the desired output.
From the method of steepest descent, the weight vector equation is given by:

$$W(n) = W(n) + 1/2\mu[-\nabla(E\{e^2(n)\}] \qquad (1)$$

Where μ is the step-size parameter and controls the convergence characteristics of the LMS algorithm.
In the method of steepest descent the biggest problem is the computation involved in finding the values r and R matrices in real time. The LMS algorithm on the other hand simplifies this by using the instantaneous values of covariance matrices r and R instead of their actual values i.e.

$$R(n) = x(n)x^T(n) \qquad (2)$$

$$r(n) = d^*(n)x(n) \qquad (3)$$

Therefore the weight update can be given by the following equation:

$$w(n+1) = w(n) + \mu x(n)[d^*(n) - x^T(n)w(n)]$$
$$= w(n) + \mu x(n)e^*(n) \qquad (4)$$

$$e(n) = d(n) - y(n) \qquad [n = 0\, to \quad final] \quad (5)$$

$$Y(n) = w^T(n)x(n) \qquad (6)$$

Equation number (4) & (6) are respectively known as weight update & filtering operation equation.

3.2. Kalman filter

The Kalman filter not only works well in practice, but is theoretically attractive because it can be shown that of all possible filters, it is the one that minimizes the variance of the estimation error. In order to use a Kalman filter to remove noise from a signal, the process that we are measuring must be able to be described by a linear system. A linear system is simply a process that can be described by the following two equations:

State equation: $x_{k+1} = Ax_k + Bu_k + w_k \qquad (1)$

Output equation: $y_k = Cx_k + z_k$ (2)

In the above equations *A*, *B*, and *C* are matrices;
A is transition matrix, B is the input matrix and C is the measurement matrix.

K is the time index; *x* is called the state of the system; *u* is a known input to the system; *y* is the measured output; and *w* and *z* are the noise. The variable *w* is called the process noise (PN), and *z* is called the measurement noise. Each of these quantities are (in general) vectors and therefore contain more than one element. The vector *x* contains all of the information about the present state of the system, but we cannot measure *x* directly. Instead we measure *y*, which is a function of *x* that is corrupted by the noise *z*. We can use *y* to help us obtain an estimate of *x*, but we cannot necessarily take the information from *y* at face value because it is corrupted by noise. We can use the information that it presents to a certain extent, but we cannot afford to grant it our total trust.

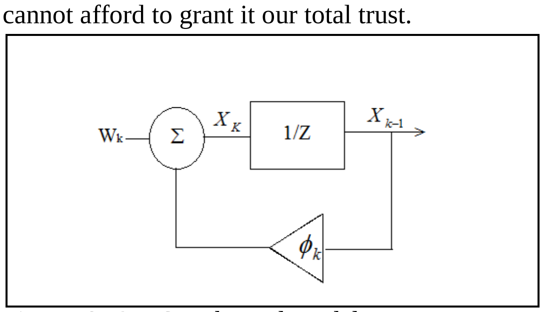

Figure: 2: AWGN channel model

For example, suppose we want to model a signal power is increasing with time. We can say that the state consists of the signal power *p*. The input *u* is the zero and the output *y* is the measured power. Let's say that the signal power is changes in every *T* seconds.
Say the AWGN channel is introduced one delay for the signal. The channel model is shown in Figure:2.

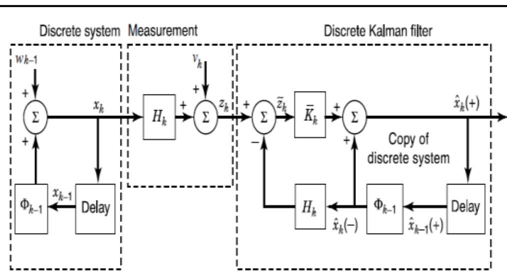

Figure:3: Block diagram of system, measurement model and discrete time Kalman filter.

4. Performance of adaptive filter over AWGN channel:

For the Additional White Gaussian Noise (AWGN) channel the received signal is equal to the transmitted signal with some portion of white Gaussian white noise added. This channel is particularly important for discrete models operating on a restricted number space, because this allows one to optimize the circuits in terms of their noise performance. The AWGN channel is represented by a series of outputs Y_i at discrete time event index i. Y_i is the sum of the input X_i and noise Z_i. Z_i is independent and identically-distributed and drawn from a zero-mean normal distribution with variance n. The Z_i is further assumed to not be correlated with the X_i.

$$Z_i \approx N(0,n)$$
$$Y_i = X_i + Z_i$$

A Gaussian random variable X is one having the PDF

$$f(x) = \frac{1}{\sqrt{2\Pi}\sigma} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}} \qquad -\alpha < x < \alpha \qquad (3)$$

Where $\bar{x}$ and σ are mean and variance respectively.

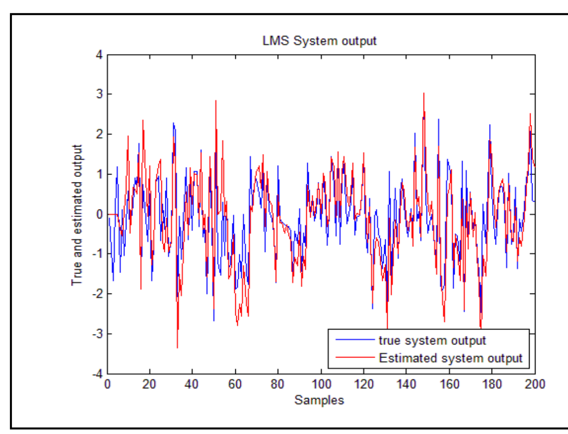

Figure: 4 True and Estimated output of LMS filter.

The Figure: 4 show that the true and estimated value of LMS filter. The red line indicates the estimated value and the blue line indicates the true value. In lower samples there are some differences between these two values but in higher samples the differences gradually decreases.

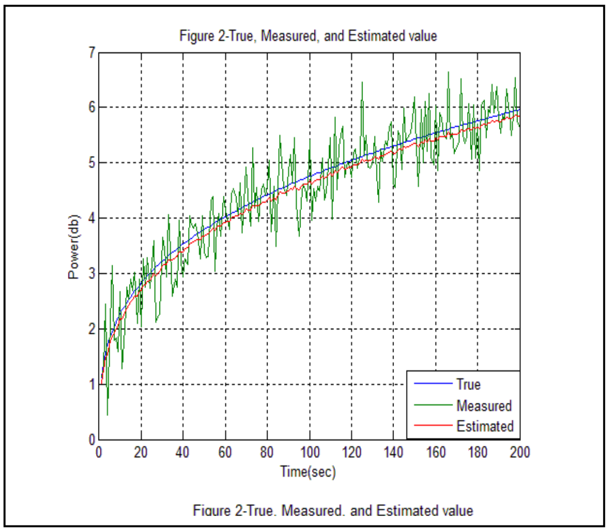

Figure: 5 True, Measured and Estimated output of Kalman filter

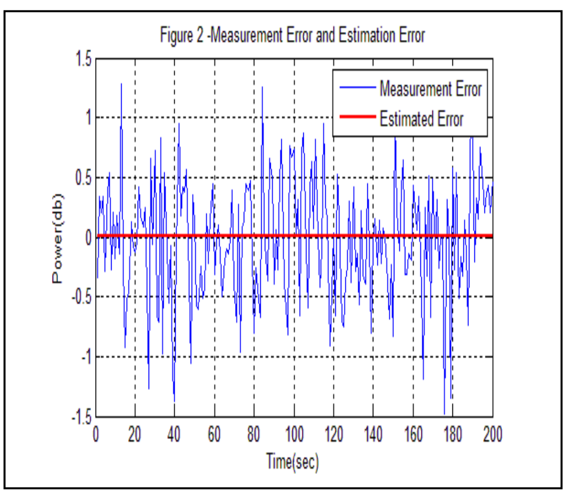

Figure: 6 Measurement error and estimated error of kalman filter.

In the Figure: 5 the true value of signal and estimated value by the kalman filter are similar. It proves that kalman filter estimates very close to the true value. The green line indicates the measurement value of kalman filter. The blue line indicates the true value and the red line denotes estimated value. The measurement error and estimated error is shown in the Figure: 6.

The bit error rate of kalman filter is compared to LMS algorithm in Figure: 7. The performance of kalman filter is better than LMS algorithm.

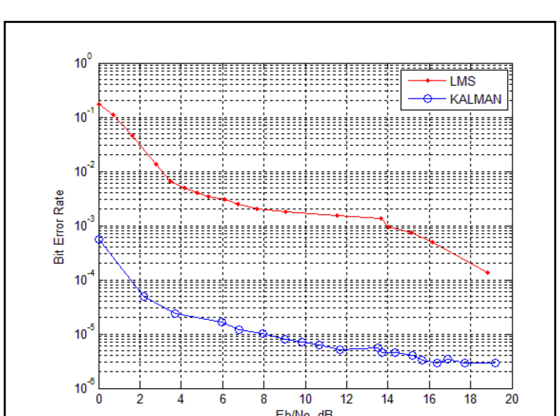

Figure: 7 BER Vs Eb/No plot of Kalman filter

Kalman filter generates very low bit error rate. As SNR is increased bit error rate is decreased. For high SNR the error rate become stable for kalman filter.

The next Figure: is throughput vs time. In communication networks, such as Ethernet or packet radio, throughput or network throughput is the average rate of successful message delivery over a communication channel.

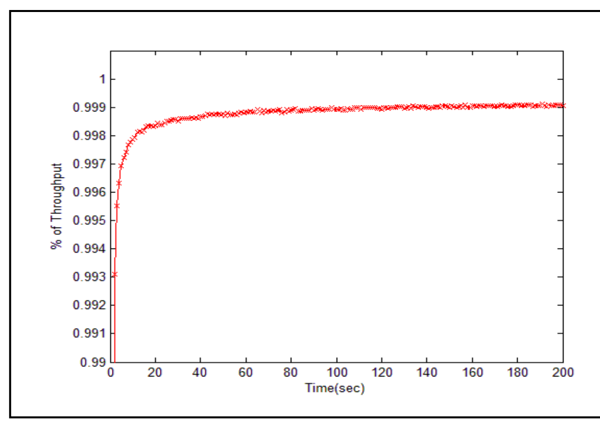

Figure: 8: Throughput plot of kalman filter.

The throughput is usually measured in bits per second (bit/s or bps), and sometimes in data packets per second or data packets per time slot. The system throughput or aggregate throughput is the sum of the data rates that are delivered to all terminals in a network.

5. Conclusion

The channel estimation process is done by using LMS and Kalman filter. Figure: 7 BER Vs Eb/No plot proves that kalman filter produce better result compare with LMS in high SNR. Bit error rate reaches multiple of 10^{-6}. For the purpose of LMS in the same SNR the bit error rate is multiple of 10^{-4}. In the Figure: 8, it is shows that the successful data received by the receiver by using kalman filter. So Kalman filter is the better estimator compare to the LMS algorithm.

6. References

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