

# Potential of Periodic Foundation as an Efficient Multi-Dimensional Seismic Isolator: Review

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**Abstract** - This paper provides an overview of a novel concept in the field of earthquake engineering. Recent investigations in the field of solid state physics revealed the possibility of manipulating the unique dynamic property of 'frequency band gaps' possessed by phononic crystals or 'periodic structure/materials'. When the frequency of a wave falls within the range of the frequency band gap of a periodic material, the wave, and hence its energy, cannot propagate through the periodic material. So, incorporating periodic structures in foundation and hence termed as, 'periodic foundation' can remarkably reduce the seismic response of superstructure over it. There are a number of variables that can affect the frequency band gaps and their widths of the periodic foundation viz. the density and Young's modulus of materials, filling fraction. Parametric investigations enable the design of efficient and economical periodic foundations. Since the concept of periodic materials dates back to even centuries, there are numerous applications of periodic material in different domains of practical life, which are also reviewed.

**Keywords**- Frequency band gap; Phononic Crystals; Periodic Structure / Materials; Periodic Foundation

## I. INTRODUCTION

### A. General

Nature's one of the most inevitably destructive phenomenon is earthquake. An earthquake is a sudden movement of the Earth, caused by the abrupt release of strain that has accumulated over a long time. If the earthquake occurs in a populated area, it may cause many deaths and injuries and extensive property damage. So, man-made structures are prone to heavy destruction during an earthquake event.

In this background, researchers, through several decades were keen on design of buildings and other structures that are seismic resistant. As a result, many methods have evolved that could appreciably reduce the earthquake impact on structures. Methods ranging from those based on the static structural strength with impact factors accounting for dynamic loads to concepts of structural element ductility and those understanding importance of shear resistance were developed. But, owing to several parameters viz. geological characteristics of the land, structural characteristics, intensity and pattern of earthquake events etc., a commonly accepted method is still behind the light.

### B. Background

As an advancement, installation of active and passive systems to structures were proposed to enhance the structural resistance against an earthquake event. Seismic isolation technologies have received great attention, among which base isolation technique was noteworthy and implemented in many number of structures around the world. The underlying principle in this system is that it modifies the fundamental frequency of the structure and in turn reduces its acceleration response. One significant drawback of a traditional seismic isolation system, however, is that the isolation system will usually have residual (permanent) horizontal displacements even after earthquake events, demanding for better re-centering capability or supplementary dampers that may increase both internal deformations and absolute accelerations of the superstructure. Another concern is that vertical seismic forces are important in high-rise buildings and large-span structures, such as bridges. The resistance of rubber isolation bearings to vertical deformation is usually low and can result in the collapse of the structure, especially when the ratio of structure height to width is relatively large [1].

Many researchers conducted works to extend base isolation technology to high-rise, even ultrahigh-rise buildings, and for structures subjected to near-field ground motions. Especially, with the development of structure vibration control, some researchers proposed methods to improve the performance and safety of base isolation structures by using semi-active base isolation systems. Attempts were made to improve the properties of isolators with fiber reinforced. Seismic responses of base-isolated structures including the dynamic interaction between soil and structure were also investigated. However, owing to the tremendous cost of implementing base isolation technique especially for developing countries, applications can only be seen in structures with critical or expensive contents. So it is very necessary and meaningful to find new more technically efficient and cost-efficient isolation systems.

## II. PERIODIC MATERIAL

Researches in the field of solid-state physics revealed the potential of certain crystal arrangements to manipulate the patterns of acoustic (mechanical) wave. Those materials are termed as phononic crystals/periodic material, which can be designed to produce specific gaps in the frequency

response of the material. A unique dynamic property, to be specifically mentioned, these gaps are termed as “frequency band gaps” (FBG). Exclusive attention has been given for FBG through decades. When the frequency of a wave falls within the range of the frequency band gap of a periodic material, the wave, and hence its energy, cannot propagate through the periodic material as shown in Fig. 1. Hence, periodic material incorporated periodic structures have numerous potential engineering applications, ranging from noise suppression to the control or isolation of vibrations [2].

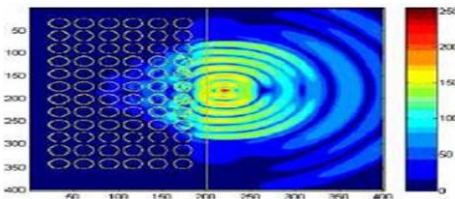


Fig. 1 Wave Propagation in Periodic and Non-periodic materials (Torres and Montero de Espinosa (2004))

#### A. Periodic Material based Seismic Isolation System

Enlightened by the inherent properties of attenuation zones, a periodic foundation was proposed to block seismic waves by Jia and Shi (2010) [4]. If the foundation has periodicity in three dimensions, theoretically speaking, it can prevent seismic waves that arrive from all directions. Hence, this innovative material can be employed in structures to obstruct or change the earthquake energy. This will result in the total isolation of the structure from the earthquake wave energy since no energy will be passing through it.

The total isolation potential of periodic foundations will be of special significance to structures that house equipment that are highly sensitive to vibration such as research laboratories, medical facilities with sensitive imaging equipment, or manufacturing facilities specializing in the fabrication of electronic components. Further, the full isolation of emergency-critical structures such as bridges, facilities housing emergency response units or equipment, and power generation or distribution structures will result in better earthquake emergency response.

It is well known that both the fundamental frequency of civil engineering structures and the frequencies of the main components of seismic waves are very low. Therefore, the periodic foundation needs to be designed with a lower-bound frequency (LBF) and a wider band of frequency gap.

### III. TYPES OF PERIODIC FOUNDATION

Initially, researchers considered three configurations of periodic isolation systems. The first type was experimented by Xiang and Shi (2009) on a periodic beam made of two alternating materials of different properties [1]. The second type is the layered periodic foundation, with concrete and rubber. The third type is the two-dimensional periodic foundation, where steel cylinders were embedded periodically into concrete matrix.

[5] experimented on the feasibility of concrete and rubber layered periodic foundation. A frame on a periodic foundation was fabricated and shake table tests were

performed, in which great attenuations were found when the exciting frequencies fell into the frequency band gaps. The peak horizontal acceleration was reduced by as much as 50% compared to that of the frame without the periodic. The same model was studied for the influences of the incident angle on the band gaps and the effectiveness of the layered periodic foundations to isolate the seismic load in the horizontal and vertical direction through numerical simulations by Bao et al.(2011) [6].

[4] studied the feasibility of a two-dimensional three-component cell composite of concrete, rubber and Pb and obtained the frequency band gaps in the range of 2.49 and 3.72Hz, which can be used in foundation to effectively isolate seismic waves with frequencies in this range. Studies were also done on the frequency band gaps for two-component and three-component two dimensional period materials using concrete, rubber and steel.

Jing Bao et al. (2012), based on a parametric study and optimization of the locations and widths of the frequency band gaps of the periodic structure, designed a one dimensional periodic isolated foundation whose validity was tested by the dynamic responses of a seven-story frame structure with three different foundation configurations. The results of these analyses demonstrate that when the frequencies of a seismic wave fall into the band of frequency gaps of the periodic foundation, the dynamic responses of the upper structure can be significantly reduced. The seismic responses of the structure to the Big Bear earthquake are then simulated. As expected, the results showed that the periodic foundation can attenuate the seismic input in both vertical and horizontal directions indicating the strong potential for the successful application of periodic foundations in the area of seismic design [1].

H.J. Xiang et al. (2012) studied a one dimensional layered periodic foundation. Firstly, based on the theory of elasto dynamics, the band gaps in the periodic foundations were analyzed by employing the Bloch-Floquet theorem. Subsequently, a parametric study was conducted to achieve a frequency band gap below 20Hz. A steel frame on the periodic foundation was then employed subjected to seismic loading on a shaking table to simulate earthquake motion in three axes. The dynamic response of the frame showed that vibration can be attenuated significantly [2].

Jia and Shi (2010) studied the band of frequency gap of a 2D three-component periodic structure with infinite length in the z direction. Effect of the physical and geometrical parameters such as density and elastic modulus as well as filling fraction of the components were also studied. Numerical simulations and comparisons of a six-story frame structure with different foundations subjected to seismic waves were also conducted [4].

Z.B. Cheng et al. (2012) investigated the feasibility of a two dimensional periodic foundation. Numerical analyses was done on reinforced concrete unlike in other studies. Typical dispersion relations of periodic foundation are obtained by the improved plane wave method and the influence of steel bars on the dispersion curves were investigated. It was found that the effect of reinforcement on the attenuation zones can be ignored, which is very convenient for engineering applications [7].

Y.Q. Yan et al. (2013) conducted experimental program on 2D periodic foundations with an upper structure. Steel frames were set on two types of foundation: namely, concrete foundation and periodic foundation. Vibrations and seismic wave inputs generated by the truck shake (T-Rex) were induced for on specimens. Under the 1984 Bishop Earthquake record, whose main frequency was modified into the frequency band gap of 50Hz, acceleration response at the top of the steel frame with periodic foundation is much lower than that of the frame with concrete foundation. It was validated by scanning frequency analysis in ABAQUS. Also, parametric study was conducted. There are a number of variables that can affect the frequency band gaps and their widths of the 2D periodic foundation. In order to obtain the lower frequency band gap and wider band width, the finite element model of one unit cell, was set up to conduct the parametric analysis. The results are summarized below. 1) Higher core cylinder density yields lower frequency in the first band gap.

2) Lower Young's modulus of the rubber yields lower frequency in the first band gap.

3) Higher filling ratio yields lower frequency in the first band gap.

4) When the ratio of the radius of the core cylinder to the thickness of rubber layer equals 1.5, the starting frequency of the first band gap reaches its minimum value [8].

The periodic foundations are able to block the input waves transmitting in the directions of periodicity. Therefore, the 3D periodic foundations are superior to 1D and 2D periodic foundations, and will change the pattern of the waves traveling along three directions.

With references from above works, Z.B. Cheng et al. (2013) made an attempt to study the feasibility of 3D periodic foundations based on the theory of elastodynamics. Two types of 3D three-component periodic structure, cubic lattice with cube or sphere, were investigated for which frequency band gaps were found in a low frequency region  $< 20\text{Hz}$ . The influences of the material parameters and the geometrical parameters on the frequency band gaps were studied. The complex 3D periodic foundation system was simulated by using a simple model, in which three unit cells were set in z direction. The upper structure is simplified as a lumped mass with a beam. Comparison was made with a concrete supporting the lumped mass structure. Conclusions were highly favourable for the establishment of 3D periodic foundation as seismic isolator. Comparing the case with coated cube core, the frequency band gap was found to be wider and lower than for the case with coated sphere core, with same values for side length of the core and the diameter of the sphere core [9].

Zhifei Shi and Jiankun Huang (2013) proposed a new three-dimensional periodic foundation, which can be described as a three-dimensional typical cell consisting of a high density core, a soft coating and a concrete matrix. The seismic waves with the frequencies in the attenuation zones (AZs) cannot propagate across the foundation. Thus, the seismic responses on the top surface of the periodic foundation can be reduced significantly. In this paper, the dispersion curves of the three-dimensional three-

component (3D-3C) periodic foundations were analyzed by the finite element method (FEM). The influencing factors such as physical and geometrical parameters of the typical cell were investigated. The following conclusions can be drawn from the present investigations: 1. High density of the core, soft rubber and increasing filling fraction can result in a rapid decrease in the Lower-Bound Frequency (LBF). 2. High density of the core, increasing filling fraction and size of the core can increase the width of AZ. 3. The seismic responses of periodic foundations can be greatly reduced in both the vertical and horizontal directions, if the incident wave's frequencies fall into the AZ [3].

Y Yan et al. (2015) conducted experimental program on 3D periodic foundations. The theoretical frequency band gaps of 3D periodic foundations were calculated first using the finite element method using the commercial software COMSOL MULTIPHYSICS. Then the dynamic responses of the 3D periodic foundations under both transverse wave (S-wave) and longitudinal wave (P-wave) excitations from 5 Hz to 100 Hz were studied, and test specimens were designed. The field tests were conducted to verify the frequency band gap effect. The field tests showed that the acceleration of a superstructure with a periodic foundation was reduced by more than 90% when compared to one with a traditional reinforced concrete foundation. Finally, the test results were compared with the finite element simulation results in both the frequency and time domains. The results from the field tests and analysis are consistent, which shows that 3D periodic foundations are an effective way to mitigate seismic waves during earthquakes [10]. Fig.3 shows the different types of periodic foundations.

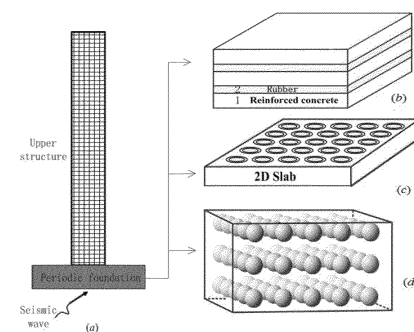


Fig. 3 Different types of periodic foundations

#### IV. OTHER APPLICATIONS OF PERIODIC MATERIALS

Various structures and compositions of composite materials have been investigated using different approaches. The special isolation property of periodic structures has many potential interesting applications, such as reducing engine noise in car cabins, controlling the noise in industrial halls, protecting electronic equipment on which sound waves can have a damaging effect and reducing traffic noise along roads.

Researchers are exploring possible application of this new type of structure in engineering. Redondo et al. (2011) explored the potential of sonic crystals as efficient sound diffusers for applications in room acoustics [11]. Based on the idea of locally resonant sonic materials, Yang et al. (2010) developed a light-weight, relatively thin acoustic attenuation panel, which can barrier a broad frequency range of 50~1000Hz effectively [12].

Xinnan Liu et al. (2016) proposed periodic material-based vibration isolators or periodic isolators for satellites. The vibration environment of a satellite is very severe during launch. The exhaust steams of the engines and solid rocket boosters will produce vibrations transmitted to the satellite through the launch vehicle, which may cause severe consequences, such as fatigue and failure of satellites. Various methods have been proposed to suppress or isolate the vibration transmitted to the satellites, including passive vibration isolation and active vibration isolation. A parametric study was conducted to provide guidelines for the design of periodic isolators. With proper design, frequency band gaps of the periodic isolator will cover main frequency range of vibrations that the satellite is subjected to. The feasibility of vibration isolation by the designed periodic isolators was validated by the dynamic response analysis of a micro-satellite with a set of periodic isolators [13].

Saeed A. Asiri et al. (2009) presented the concept of periodic legs in offshore platforms. There are many sources of vibration that affect the offshore platform structures, including machinery, water waves, wind and impact boats. The dynamic response of periodic legs incorporated platforms to wave excitation was determined theoretically and experimentally. The behavior of legs with geometric and material periodicity was studied. It was found that the periodic legs are capable of attenuating considerably the vibration transmitted from the water to the platform both in the axial and lateral directions over a broad frequency band [14].

A periodic structure consists of an assembly of identical elements connected in a repeating array which together form a completed structure, which are prevalent in many engineering applications. These include bulkheads, airplane fuselages, and apartment buildings with identical stories. Each such structure has a repeating set of stiffeners which are placed at regular intervals. The study of periodic structures has a long history. Wave propagation in periodic systems has been investigated for approximately 300 years. In the words of Mead, a periodic structure is a structure that consists fundamentally of a number of identical structural components that are joined together to form a continuous structure [15].

Gupta (1970) presented an analysis of periodically-supported beams that paved the way for the concepts of the cell and the associated transfer matrix. He presented the propagation and attenuation parameters' plots which was the stepping stone for further studies of one-dimensional periodic structures [16].

Faulkner and Hong (1985) presented a study of mono-coupled periodic systems. Their study analyzed two types of mono-coupled systems that are considered as numerical

examples: a spring-mass oscillating system and a continuous Timoshenko beam resting on regularly spaced knife-edge supports. Their study analyzed the free vibration of the spring-mass systems as well as point-supported beams using analytical and finite element methods [17].

Mead and Yaman (1991) presented a study for the response of one dimensional periodic structures subjected to periodic loading. Their study involved the generalization of the support condition to involve rotational and displacement springs as well as impedances [18].

Gry and Gontier (1997) concluded the insufficiency of both Euler-Bernoulli and Timoshenko beam theories for the analysis of railway tracks. Thus, they developed a generalized cross-section displacement theory for periodic beams with general support conditions for the study of the dynamic characteristics and vibration attenuation of railways [19].

Xu and Huang (2002) showed that the introduction of a finite number of nearly-periodic supports into an infinite beam introduced a large band of localized vibration and reduced the amount of energy transferring through the nearly-periodic segment [20]. Using the same concept, Ruzzene and Baz (2002) used shape memory inserts into a one dimensional rod, and by activating or deactivating the inserts they introduced a periodicity which in turn localized the vibration near to the disturbance source. Later, they used a similar concept to actively localize the disturbance waves traveling in a fluid-loaded shell [21].

Asiri, Baz (2005) and Pines developed a new class of these periodic structures called passive periodic struts, which can be used to support gearbox systems on the airframes of helicopters. When designed properly, the passive periodic strut can stop the propagation of vibration from the gearbox to the airframe within critical frequency bands, consequently minimizing the effects of transmission of undesirable vibration and sound radiation to the helicopter cabin [22].

Asiri and Aljawi (2006) presented a new class of periodic mounts for isolating the vibration transmission from vehicle engine to the car body and seats [23]. Advanced fast nuclear power plants (AFR) and small modular fast reactors (SMFR) are composed of thin-walled structures such as pipes. For this type of structures, seismic isolation is an effective solution for mitigating earthquake hazards.

Zheng-Hua Qian et al. (2008) revealed the existence of several very large complete band gaps in a material of practical interest such as PZT rods reinforced polythene composite. Knowledge of the band structures of piezoelectric phononic crystals can facilitate the design of acoustic transducers [24].

S.I. Fomenko et al. (2014) studied the in-plane wave propagation in layered phononic crystals composed of functionally graded interlayers obtained from the solid diffusion of homogeneous isotropic materials of the crystal. Wave transmission and band-gaps due to the material gradation and incident wave-field were also investigated [25].

Methods based on elastic waves were proven to have high potential for damage detection. Ultrasonic non-destructive methods require precise and accurate wave excitation and

signal reception techniques, which can be realized in particular by the introduction of special elements like phononic crystals with good filtering properties into the actuators and sensors. An example is the acoustic sensor system using resonances of two-dimensional (2D) phononic crystals made up of a steel plate having two regular arrays of holes and a cavity in-between (Zubtsov et al. (2012) [26].

An experimental investigation of phononic band-gaps for a normal wave incidence in a 1D periodic SiO<sub>2</sub>/poly multilayered film at gigahertz frequencies using Brillouin spectroscopy was performed by Gomopoulos et al. (2010) [27].

## V. SUMMARY OF LITERATURE REVIEW

From the literature survey, following conclusions can be made:

- Researches on periodic materials dates back to centuries, and hence wide range of applications are evident ranging from noise reduction to seismic isolation.
- Diligent study of material and geometric parameters of periodic inclusions can result in efficient and economically feasible seismic isolators that can resist the seismic waves generated and hence protect the superstructure.
- Among the three types of periodic foundations, three dimensional periodic foundation is the most effective since it can resist seismic waves from all directions.
- About 90% response reduction was obtained in the literature, assuring the potential of periodic foundations in multi-dimensional structural vibration isolation.

## VI. CRITICAL ISSUES FOR FUTURE RESEARCH

- Implementation of periodic foundation in any real structures has not taken place yet. Studies have to be carried out in this aspect.
- Experimental study on 3D periodic foundation with real seismic data needs to be conducted.
- Construction practise of a 3D periodic foundation has to be detailed.
- Optimization studies should be done to obtain wider and lower band gaps.
- For actual structural application, the length of the unit cell in the vertical direction have to be reduced and hence directional band gaps need to be analysed.
- Replacement of soft coating with springs and using high-density concrete as the core mass should be considered.
- Focus required on soil- periodic foundation interaction.

## VII. CONCLUSIONS

Apart from the renowned base seismic isolation techniques, an innovative approach taking advantage of the unique attenuating property of periodic materials incorporated periodic structures, is reviewed in this paper. Researches based on periodic materials dates back to even centuries. And hence, many applications are found in various

domains of practical life. It was only recently that solid state physics threw light on the possibility of periodic materials to be used to manipulate acoustic and mechanical waves. From the various literatures, periodic foundations of one dimension, two dimension and three dimension have proven high potential in reducing the response of the superstructure under a seismic event. Three dimensional periodic foundation, due to its periodicity in all direction showed maximum response reduction. Design of periodic materials to suit for specific purposes require great knowledge of materials that could be used and their behavior. Physical and geometrical parameters greatly influence the frequency band gap. Efficient parametric studies are highly recommended. The frequency of civil engineering structures and main components of earthquake usually fall within the range of 20 Hz. Efficient configurations satisfying this frequency range have to be the concern.

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