Practical Implementation of Queueing Model in Human Resource Development

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Abstract

The multi-server queueing model has been used to calculate approximately the average waiting time, queue lengths, number of servers and service rates. These queueing models estimate the performance of queueing systems with multiple queues. In this paper, we use the queueing theory to identify the most favorable number of required human resource development in a professional institution running in India. The queue analysis is executed for different numbers of employees. The final outcome of this study shows that the employees in the institution must be retrenched.

Key Words: Retrenched, Queueing system, Human resource development, waiting time, Poisson arrival, Exponential service

Introduction:

A long waiting time in any organization and unemployment personnel is considered as an indicator of poor quality and needs for any improvement. Managing waiting lines create a great dilemma for managers seeking to improve the return on investment of their operations. Customers also dislike waiting for long time. If the waiting time and service time are high, customers may leave the queue prematurely and this event may result in customer dissatisfaction. This will reduce customer demands and eventually revenue or profit. On the other hand, unemployed personnel incur the related costs for any organization.

Nowadays, human resource is one of the most valuable factors in production and important asset of any organization and the main source of competitive advantage for generating of fundamental capabilities in any organization. Human resource planning (HRP) is one of the main organizational planning. The important factor for requiring HRP is for achieving the skill that needs training and eventually making human resource optimal [1]. The correct HRP can eliminate lots of problems that are hampering the developing ways. The precise HRP can compensate both reducing costs due to a low productivity rate and job absence. Nowadays, resorting the ideal methods to reach the best results has substantial importance after a period we observe the various transformations, which are taking place in many scientific fields. Negligence to these developments and new tools leads to mistrusting the results and solutions.

A queueing theory called as a random service theory is one of the issues in mathematics so that the existing techniques in the queueing theory have substantial importance in solving mathematical problems and analyzing different systems [3]. One of the important features of this method is to consider a mathematical logic and analysis of the possibility of the precise and logic deduction without noticing to the environmental factors. The successful implementation of this technique in solving the economic and management problems can make a desirable vision towards implementing of quantity techniques in various services and production fields. Among implementations queueing theory, we can indicate assessment parameters, such as operational efficiency, response time and utilizing rate.

Regarding to the role, economic and social importance of the queueing theory in engineering fields, telecommunications, transportation system and service sectors, lots of studies are carried out using a queueing theory. In this paper along with extending the analytical methods, the presentation of new implementations is emphasized. Sample studies use the queueing theory models are the performance of tools selection [4], determination of a optimal number of returned goods sensitive to time, determination of a optimal number of machines analysis of a service system [5], transportation and the analysis of radio networks according to and the evaluating of employee's performance.

This study helps the managers of the Research Centre and Financial Department in DAV Branch at Kanpur University in India and the purpose of this study is to measure the institutional progress in operations with a focus on the organization administration issues. This educational institute is a university promoted by a group of eminent professors and over than 20000 students. The university is located in Kanpur in Uttar Pradesh.

Research Problem

The staff strength plays a very important role in human resource management. Staff inadequacy leads to decreased service rates and makes the queue sizes larger. Here, after an initial checking of Financial Department of DAV Branch, we feel that the personnel unemployment is too long. The study is required to understand the expected waiting time of customers, unemployment time of personnel and the actual waiting time in the studied department of the university, in which the gap between the actual and expected time can be analyzed to know any improvement is required. Therefore, it is necessary to know whether more staff is required to serve the students in an efficient manner.

Providing the human resources is a strategic goal in each organization to achieve their objectives. In this study, according to the information (i.e. the number of patients and physicians), the evidence and documents in 10 years ago, the required number of physicians for the five-year plan is estimated. Li and Liling [6] presented a multi-objective model that includes the flexibility of jobs. They determined the number of required employees for planning period in budget, in which the objectives are in a priority level from each other. Although some attempts are carried out for using quantitative methods in HRP; however, there was an empty room for a research that uses the queueing theory in analyzing the issues.

Research Methodology

Queueing Theory: Erlang's attempt in 1909 for analyzing the density of telephone lines with a uncertain demand in a telephone system of Kopenhak under the name of a new queueing theory was successful. Nowadays, this theory is one of the most important tools in different professions because of its ability to solve and stimulate any problems by the queueing theory. This theory is based on a

Probability theory and random processes. It uses a mathematical logic and the related analysis that provides the analytical logic and precision analysis. In addition, it can provide the suitable solution for them [7].

For this study, two kinds of models are presented. The first model, which is called descriptive model, concentrates on situation explaining and the real problem. The second model, which is called prescriptive model, declines what should be the situation of real world, namely it introduces the optimal behavior for introducing the goal [7]. In other words, they describe the queueing theory in the way that consumers visit to achieve services to a system. Then after waiting in the system for the desired service, consumers go to the related department and after passing the required time for ending the service and finally they depart from it. By the way, the consumer or server is not only a human agent and thoroughly we consider all the parts including applicants, people, human or a server [8]. The queueing theory is extended for describing the phenomenon of waiting in a line in applying units sections to achieve services. Depending on the way of entrance and servicing in the system is probability or certainty function according to the model of queue is [9]. In the considered unit, the arrival and service variables usually are random, so we use the model to describe them.

From the fundamental features of the queueing theory, we can name the consumer's arrival pattern, the service pattern of servers, the discipline of queue, the system capacity and the number of parallel servicing channel which awareness of these features in generating a model is inevitable matter [10]. The service time in a queueing procedure may be one of the probability distributions. A distribution, which often is considered for service time, is a negative exponential distribution. If the distribution of the time service is negative, the consumers' arrival rate is based on Poisson distribution and the consumer's arrival is in terms of the Poisson arrival rate. The service time is negative exponential distribution, which is explained in the queueing theory literature. The service rate is composed of the average number of consumers, in which a time unit gets services from one server. According to the Poisson model the arrival distribution is not without experimental basic.

Statistical studies show the majority of queue procedures. The costumer's arrival rate is the Poisson distribution. It assumes that the costumer's arrival rate is completely random, because the arrival rate of every costumer is completely independent of other system states. So, for real experimental implementations, the queueing analysis is required that the hypothesis is controlled before its use in the system.

Multi-Server Queueing Systems - *M/M/C*: The multi-server queueing mathematical model is known in the Kendall's notation as the *M/M/C* model, where

- *M signifies* a poisson distribution
- C= number of parallel service channels in the system

The *M/M/C* model is one of the most commonly used to analyze queueing problem.
This model computes the average wait times and queue lengths, given arrival rates, number of servers and service rates. For *C*>1 (where *C* is a number of servers), the mathematical model is complex. When *C* = 1, more readily calculable set of equations applies. This particular model applies, in which there are multiple channels served by a single queue, as at a bank teller or many airline tickets counters. The outputs of the model are as follows:

- Expected waiting time per customer in the system
- Expected waiting time of customer in the queue
- Expected number of customers in the system
- Expected number of customers in the queue

The exact calculation of these measures requires knowledge of the probability distribution of the arrival rate and service time. Furthermore, even with that knowledge, the resulting formulae are exceedingly complex. . Thus, some simplifying assumptions are required.

- The most basic of these assumptions is that the arrival rate obeys Poisson distribution, which is equivalent to saying that the inter-arrival times are exponential.

- The second assumption is regarding the nature of the probability distribution of the service times. With a Poisson distribution, the service times are assumed to be exponentially distributed.

Moreover, successive inter-arrival times and service times are assumed to be statistically independent of each other. Collectively, the Poisson assumptions of the *M*/*M*/*C* model make for a reasonably tractable solution. Figure 1 shows a generalization of the simple model of a multi-server queueing system. In this case, there are multiple servers with all sharing a common waiting line. If a user arrives and at least one server is available, then the user is immediately dispatched to that server. A waiting line is being creating when all the servers are busy. As soon as one server becomes free, a customer is dispatched from the waiting line using the dispatching discipline in force.

Research Model: The observation of the visitor's behavior (i.e. arrivals) is very important in determining the queue model. In this paper, the system behavior is considered in a period of time. The study of consumer's behaviors shows that in more than 90% of cases, they select one of the servers from eight options. The acceptable amount is observed as C=8 and the suitable model for the mentioned hypothesis is as *M*/*M*/*C*, this model has C servers with no limitation queue capacity for the consumer's population. The rate of the consumer's arrival is stable and is equal to μ for every amounts of n. In addition, the service rate of each server is supposed as

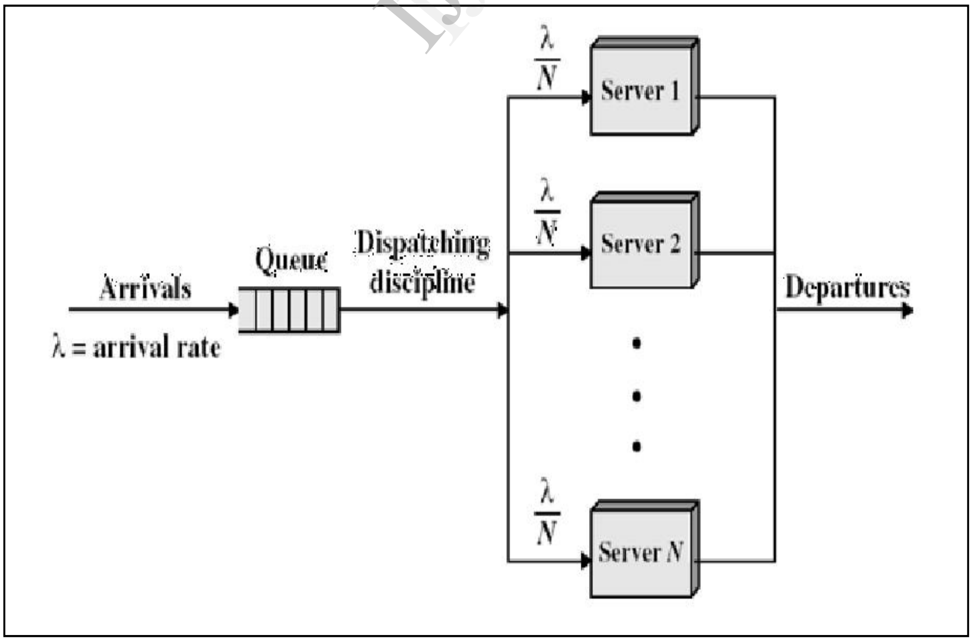

Fig. 1: Multi-servers model

In this system, each time that the number of consumers in the system is less than C (where n=C), the consumers' departure rate is equal to $n\times\mu$. If the number of consumers is more than C (where n>C), the departure rate is equal to $C\times\mu$.

Model Assumptions: The main assumptions are as follows:

- Based on the experiment it can be concluded that the obtained findings from a unit can be valid in other units in the organization.
- Customers are aware of the organizational structure. The arrival rate of customers to queue and the service rate are compatible to Poisson distribution or in another word, the time interval between two consecutive arrivals and time service, both follow from exponential distribution.
- In the case of customer in the queue, none of the servers must be unemployed.
- The queue discipline is in the way that the first customer goes to the first server which is prepared for service.

Model Parameters

Utilization Coefficient: It is obtained from dividing the average arrival rate λ (in time) to the average service rate, μ. in fact for showing the percentage of time which the system is working we used from factor under the name of utilization coefficient of the system or ρ which can be defined by:

$$\rho = \frac{\lambda}{\mu} \qquad (1)$$

Whatever ρ is larger, the demand will be more and the system should work harder and the queue will be longer. On the contrary whatever the $\rho$is smaller, the queue will be shorter; but in this case, the use of system will be low too. If the arrival time of costumer to system were more than service rate, namely λ >μC, then ρ> 1, which means the system capacity, is less than the arriving demand, therefore, the queue length is increased. Eventually, it reaches its highest point namely infinity. So, for this reason, the conditional μ> 1 situation is a stable condition for most systems.

Average Waiting Time in Queue: The average waiting time in queue is equal to the average time which a costumer waits in the queue for getting service, its formula is:

$$Wq=\frac{\lambda}{\mu(\mu-\lambda)} \qquad (2)$$

Average of Time Spent in System: The average spent time in a system is equal to the total time that a costumer spends in a system, which includes the waiting time and service time, namely

$$Ws=\frac{1}{\mu-\lambda} \qquad (3)$$

Average Queue Length: The average queue length is composed of the average number of people who are waiting in the queue:

$$Lq= \frac{\lambda^2}{\mu(\mu-\lambda)} \quad (4)$$

Average Number of Individual in the System: The average number of individual in the system is equal to the average number of individuals who are in the line or server:

$$Ls= \frac{\lambda}{(\mu-\lambda)} \quad (5)$$

The above factors is determined if and only if $\rho < 1$. In the case that $\rho > 1$, the above factors reach infinity. In the case of using multi-servers, the above factors are more complicated. If we suppose that *C* numbers of servers are in the system, the above functions are defined, respectively, as follows:

$$\rho = \frac{\lambda}{c\mu} \quad (6)$$

$$Wq= \frac{Lq}{\lambda} \quad (7)$$

$$Lq= \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^c}{(c-1)!(c\mu-\lambda)^2} P_o \quad (8)$$

$$Ls= \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^c}{(c-1)!(c\mu-\lambda)^2} P_o + \frac{\lambda}{\mu} \quad (9)$$

$$Ws= Wq + \frac{1}{\mu} \quad (10)$$

The possibility of no costumers in the system is computed by:

$$P_0 = \frac{1}{\sum_{n=0}^{c-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^c}{c!} \times \frac{c\mu}{c\mu-\lambda}} \quad (11)$$

RESULTS AND DISCUSSION

In the preliminary observations, the educational and financial units were considered due to the amount of visitors but during further studies the financial unit was selected for minute investigation as a case study (various reasons for this selection shows we can indicate to some conspicuous reasons such as the large amount of visitors in this unit and the close inter- relationship of this unit with other units and so on). For data collecting of mentioned unit the following ways are done:

Some of information (such as the number of financial experts and activities) are observed from the investigation of the mentioned unit.

And other data are collected from interviewing with personnel and staff.

For determining some parameters and digital amounts due to shortage of resources and time constraints, the researcher used of presupposition information. All in all the required information for queue analyzing and for consecutive arrival time of customers for determining the arrival distribution function, the service time for determining the service distribution function and also the queue length and waiting time were collected in 30 working days by close observation. In the current system one person can serve all the activities such as work flowing pay rolling, accounting, property and so on (i.e. the customers of the studied unit are composed of professors, employees and the students of university and even the outside clients from other organization can play as the costumer role in the system.

For collecting the system information in the period of time all days of the week are covered and the time between arriving and departing to the system are determind as $\mu \times \lambda$, the easiest way for the distribution of the data collection is the usage of histogram. The arrival time of costumers and service rate in one store is carried out by the SPSS software, finally related data are illustrated. Kolmogorov-Smirnov test is the precise non-parametric method for determining the assimilation between data and the experimental data, which are collected by a statistical distribution. Here by using the SPSS software, of histogram. The arrival time of costumers and service rate in one store is carried out by the SPSS software, finally related data are illustrated. Kolmogorov-Smirnov test is the precise non-parametric method for determining the assimilation between data and the experimental data, which are collected by a statistical distribution. Here by using the SPSS software, the data are collected, In the rest of the paper, the procedure of parameters determination are presented in Tables 1 and 2.

Table 1: Data related to the consumers' arrival time

Description	Sixth group of collected information	Fifth group of collected information	Fourth group of collected information	Third group of collected information	Second group of collected information	First group of collected information
N	198	174	188	164	100	152
S(sec.)	19610	17604	17156	18498	13536	20680
Mean (sec.)	98	101	91	113	135	136
λ (sec.)	0.010	0.010	0.011	0.009	0.007	0.007
λ (Hrs.)	36	36	40	32	25	25

Table 2: Data of the service time to consumers in the system						
Description	Sixth group of collected information	Fifth group of collected information	Fourth group of collected information	Third group of collected information	Second group of collected information	First group of collected information
N	200	176	190	166	102	154
S (sec.)	35274	36030	38154	44820	25500	30432
Mean(sec.)	176	205	201	270	250	198
μ (sec.)	0.006	0.005	0.005	0.004	0.004	0.005
μ (hrs.)	22	18	18	14	14	18

Table 3: Calculation of the queueing theory criteria according to various amounts of servers						
Number of servers	*I*	ρ	*W* (hour)	*Ls* (person)	*Ws* (hour)	*Lq* (person)
2	0.0588	0.94	0.515152	16.48482	0.456328	14.6025
3	0.372549	0.62	0.079385	2.540311	0.020561	0.657958
4	0.529412	0.47	0.062888	2.012402	0.004064	0.130049
5	0.623529	0.37	0.059726	1.911226	0.000902	0.028873
6	0.686275	0.31	0.059019	1.888612	0.000196	0.006259
7	0.731092	0.26	0.058863	1.883625	3.98	0.001272
8	0.764706	0.23	0.058831	1.882592	7.48	0.000239

$$\lambda = \frac{\sum_{i=1}^{6} \lambda_i}{n} \cong 32$$

Customers with the average number of 32 in an hour are visiting the related unit.

$$\mu = \frac{\sum_{i=1}^{6} \mu_i}{n} \cong 17$$

Customers with the average number of 17 per hour receive service in the related unit.

For data analyzing, we used from QSB and Excel software and the required parameters are determined as the service time, waiting time and the service time. The results are in the table 3. Before explaining of the table 3 some points must be clarified: if the amount average waiting time of costumers is more (it means the system is populated), the average number of consumers will be more. The number of customers in the system is straightly related to the both arrival rate and relationship with *w* which is an average spent time when a costumer spends in the system that is equal to the average spend time in the line plus the time that he/she has been receiving the service.

In Table 3, when the number of server is 2, the number of client in queue is equal to 14, the average waiting time is 0.45, the people in the system are 16 persons, the average time in system is 0.51, the system utilization is 0.197 and the probably unemployment of system is 0.05. With changes in number of servers, the changes of these parameters will be also tangible. Also, regarding to the results of table 3, by increasing the utilization, the average queue length and the waiting time of customers in the line are increased. This matter is completely valid with the issue of utilization factor. In the *M*/*M*/*C* model, because λ and μ are stable, so, the authors just consider those expenses, which are related to the number of servers, namely *C*. So, the cost function is calculated by:

$$T(c) = (C_1 + C_2)c + C_3 L_q \quad (12)$$

Where C_1, C_2 , C_3 are the maintenance cost of unemployed server, investing cost on one server and the wasting time cost of costumer in the line, respectably. In this paper, the cost of server per hour on base of paying and the cost of customers' wasting time per hour are considered approximately. In this paper, C_2 and C_5 are gotten from document and interviewing with relevant employees, who on the base of employees' pay and the wasted time cost of customers are respectively 2000, 3000 Rs. per hour. C_1 is equal to zero. The optimal number of servers can decrease the waiting time for customers in the system and line (*Ws*, *Wq*) and moreover it can decrease the queue langue in service processing. The minor amount of *m* must be in the way that ρ<1. The optimal number of servers are obtained from the less cost spent. If the supposition of identifiable parameters and the optimal capacity of system are determined, the assessment criteria such as queue length, waiting time, utilization factor are identified. According to these findings, two types of costs (i.e. server cost and wasting time cost in a queue) are calculated.

The cost of people and operators in the line is related to their waiting time in the queue. So, the most favorable number of services is the place of curves confluence point as shown in Figure 2.One of these curves is the waiting cost in a queue that is dependent to the number of servers, namely the employees that has descending state. It means that with increasing of staff numbers or servers, the number of queues is increased so the waiting cost in a queue decreases. Another ascending curve dictates the cost of generating a server or hiring a new employee. The waiting cost in a queue has the inversely relationship with the number of employees. In addition, increasing in the number of employees has the straight relationship with increasing in employment costs as well as the cost of generating the servers and employing the new personnel. This point is called *P*, which shows the optimal number of employees. In this point, the waiting cost in a queue for

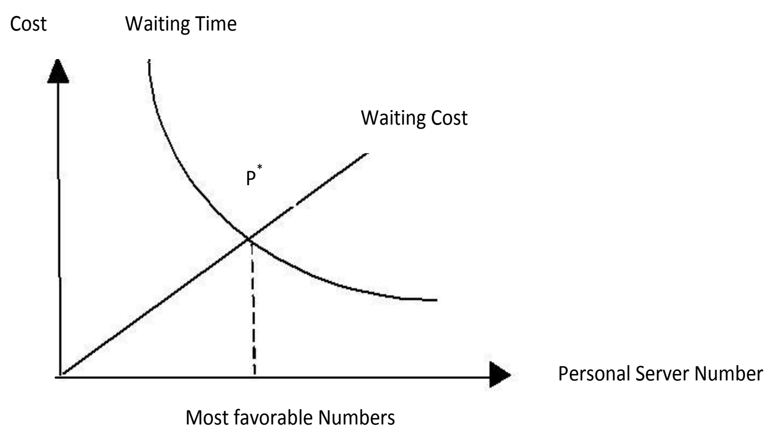

Fig: 2

Most favorable solution for Queueing Model

Table: 4 Calculation of the optimal number of servers with the use of the Queueing Theory

Number of servers	**Lq**	**T(c) Rs.**	ρ
2	14.6025	47807.5	0.94
3	0.6579	7980	0.62
4	0.1300	8390	0.47
5	0.0289	10090	0.37
6	0.0063	12018	0.31
7	0.0013	14003.9	0.26
8	0.0002	14000.6	0.23

Applicants are equal to the cost of generating the servers and employment of personal for the organization. Regarding to the mentioned requirements plan, policy and organizations strategy in this study and with the usage of multiserver queueing model, we answer to the question what the optimal number of employees is in the considered unit.

In this study the influence of changing parameters on assessments criteria and finally on total system cost or reexamined.

Due to these changes the best response of cost reduction is achieved. For every various amount of c regarding to mentioned formula in this case assessment criteria the most optimal amount is determined. According to cost function and result in table 3 the optimal numbers of servers is determined as table.4.

Because ρ must be smaller than 1, the minor amount of servers is c=2. According to result in table 4 cost of 3 servers is 7980 Rs. Lower than the other amounts, the number of servers for considered unit is equal to 3.

Conclusion:

This paper has found the most favorable number of the human resources with data processing, which are attained from the system coefficient and parameters of the considered unit in the organization by the use of the queueing theory. Some of the services demanded by customers in comparison with other services allocated more time to themselves. In addition, based on the calculated criteria for the system and obtained results, the optimizing processes and procedures should be followed. It means the transparency and certainty of the workflow and improvement of work processes and simplification of issues or eliminating of the redundant procedures through engineering process should be done a serious work. Finally, the suitable number of servicing channels (i.e. employee) can improve the calculated system criterion. Thus, the obtained results in this research show in retrenchment of employees, creating the developing opportunities for the employment skills in the organization, training and mastering in technical information competently and proficiently in related opportunities and positions. This research is also applicable for other studies in the field of human resources development and planning.

References:

1. Dr. Sanjay Kumar: New Queueing Theory For Customer-Based Problem, (IJRST) 2011, Vol. No. 1, Issue No. II, July-Sept, International Journal of Research in Science And Technology, ISSN: 2249-0604
2. Thakur Vats Singh Somvanshi, Quazzafi Rabbani, Sandeep Dixit,'Application of queing models in real life', IOSRJM vol 2, issue 1(July August 2012),pp 41-45.
3. Dieter, F., S. Bart and B. Herwig, 2001. Performance evaluation of CAI andRAI transmission modes in a GI-G-1 queue. Computers & Operations Research, 28: 1299-1313.
4. Azmat Nafees Queueing Theory And Its Application: Analysis Of The Sales Checkout Operation In Ica Supermarket: A 'D level' essay in Statistics
5. Ajay kumar Sharma, Dr. Rajiv Kumar, Dr. girish Sharma: Queueing theory approach with queueing model: International journal of engineering science invention(ISSN (online: 2319-6734, volume 2/ issue 2, Februry 2013.
6. Li, L. and X. Liling, 2000. Modeling staffing flexibility: a case of china. European Journal of Operational Research, 124: 255-266,
7. Ji-Hong, L., T. Nai-Shuo and M. Zhan-You, 2008. Performance analysis of GI/M/1 queue with working vacations and vacation interruption. Applied Mathematical Modelling, 32: 1551-1560.
8. Irfan, A., A. Bashir and A. Shakeel, 2007. Performance analysis of networks of queues under active queue management scheme. Simulation Modelling Practice and Theory, 15: 416-425.
9. Wen-Hui, Z., Performance analysis of discrete-time queue GI/G/1 with negative arrivals, Applied Mathematics and Computation, 170: 1349-1355.
10. Joris, W., F. Dieter and B. Herwig, 2008. Time-dependent performance analysis of a discrete-time priority queue. Performance Evaluation, 65: 641-652.
11. János Sztrik :Queueing Theory and its Applications, A Personal View , 8th International Conference on Applied Informatics Eger, Hungary, January 27–30, 2010. Vol. 1. pp. 9–30.
12. Operation Research by Prem Kumar Gupta and Dr. D.S. Hira, S. Chand & company private Ltd. ISBN Number: 81-219-0281-9