

# Predicting Reliability of Lithium Ion Batteries

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**Abstract**— Over the last decade, electric vehicles (EVs) have evolved rapidly due to their energy saving and environmentally friendly components. This has led to the improvement of EV batteries. One of the most useful batteries that has been significantly developed for the vehicle applicant is the lithium-ion battery. Due to its high energy and power densities, long life cycle, and environmentally friendly components, many kinds of lithium-ion batteries have been developed. In this paper, two types of commercial lithium ion batteries are studied: LiFePO<sub>4</sub> (LFP), and LiMn<sub>2</sub>O<sub>4</sub> (LMO). The reliability of these two batteries was presented by using Weibull analysis. With Weibull distribution, Beta or shape parameter, and Eta or scale parameter were calculated and used to compare their characteristic lives.

**Keywords**—lithium-ion;batteries; Weibull; reliability

## I. INTRODUCTION

No one denies that reliability is the most important need for engineered products. In every organization, there is a keen awareness of the cost of unreliability due to the high costs of failure under warranty, and customers endure the inconvenience. Furthermore, the manufacturer will also probably incur a loss of reputation, possibly affecting future business. Reliability engineering is engineering that emphasizes the system's availability, and reliability in the life cycle management of a product. Reliability is theoretically defined as the probability that a part will last at least a specified time under a specified experimental condition. Reliability can also be expressed as the number of failures over a period of time.

## II. RELIABILITY THEORY

As stated above, reliability  $R(t)$  is theoretically defined as the probability that a product or service will operate its intended function during a specified period of time  $t$  (define life) under stated conditions without failure. Statistically, this may be expressed as:

$$R(t) = Pr\{T > t\} = \int_t^{\infty} f(x) dx$$

Where:

$R(t)$  is the probability of no failures before time  $t$   
 $f(x)$  is the failure probability density function and  
 $T$  is the lifetime of the device

$t$  is the length of the period of time starting from time zero  
However, even if no individual part of the system fails, but the system as a whole does not work as intended, then it is still considered against the system reliability. From the formula above, a system has a specified chance that it will

operate without failure before time  $t$ . Reliability engineering ensures that a product will meet the requirements during the specified time and be restricted to operation under stated (or explicitly defined) conditions. This is also necessary because it is impossible to design a system for unlimited conditions.

Reliability can be defined in another way by using parameters. The most common reliability parameter is the mean time to failure (MTTF) for items that cannot be repaired. It is the measurement of an average time to failure with the modeling assumption that the failed system is not repaired (infinite repair time). It can also be specified as the failure rate or the number of failures during a given period. These parameters are very useful for systems that are operated frequently, such as most vehicles, machinery, and electronic equipment. Reliability increases as the MTTF increases. The MTTF is usually specified in hours, but can also be used with other units of measurement, such as miles or cycles. For repairable systems, it is specified either with failure rate, mean time between failures (MTBF), and mean time to repair (MTTR).

## III. RELATED WORK

The founder of Weibull Distribution was the Swedish physicist W. Weibull during 1950s. This distribution has become very popular in reliability studies of the electronic devices [1] due to its well-known versatile distribution fitting characteristics based on its shape and scale parameters with different values. De [2] and Folk [3] have demonstrated the application of failure probabilities using Weibull distribution in ceramics domain under certain loading.

Kishimoto discusses the advantage of Weibull distribution due to its small larger sample size requirement [4], which makes failure the analysis of the electronics devices practical with limited sample sizes.

In the area of semiconductor fabric reliability, Oliver has investigated the application of Weibull, Gamma, and Bimodal distributions in the area of machine reliability [5]. However, the developments of the coefficients of Weibull and Gamma distributions need further work.

Weaver analyzes reliability of Sodium/Sulfur batteries by using Monte Carlo methodology with Weibull statistical distributions [6]. The accuracy of the algorithm used to generate the Weibull distribution is discussed in this paper. Power supply system reliability taking battery failure into account is studied by Koizumi by Weibull plotting method [7].

The focus of this paper is to develop a framework for estimating the lives of two types of batteries using Weibull parameter estimation methods via simulation of the data presented in proceeding sections.

Battery lifetime estimation of the Galileo spacecraft is demonstrated in the research of Michael [8]. He presents Bayesian Weibull analysis using a Monte Carlo solution technique to estimate the spacecraft lifetime. Leung's research presents the application of the Weibull analysis of switching contact resistance in laboratory and commercial circuit breakers [9]. The proximity analysis of life time distribution in high speed machines and their comparative studies using Weibull, Lognorm and Exponential distribution is presented by Zhou [10]. The search summary presented above is used to build the necessary foundation for research methodology developed in this paper.

#### IV. BATTERY MODELING

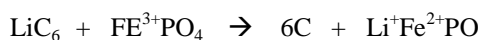
A battery is a device that converts chemical energy into electrical energy through a spontaneous chemical reaction. There are three components in batteries, which are cathode, electrolyte, and anode. Cathode is characterized as the positive electrode, while anode as the negative. The electrolyte serves as a medium to transport ions from one electrode to the other. There are two types of batteries, which are primary and secondary. Primary batteries are not rechargeable since the electrode reactions are not reversible. After one discharge, they are discarded. On the other hand, the electrode reactions of secondary batteries are reversible, therefore; the cells are rechargeable.

In a lithium battery, the anode and cathode are separated by lithium-ion electrolyte. The most common commercial anode material is carbon (C), which can alloy with lithium to compound  $\text{LiC}_6$ . In this paper, we will focus on  $\text{LiFePO}_4$  (LFP), and  $\text{LiMn}_2\text{O}_4$  (LMO). Both of these lithium batteries use carbon(C) as their anodes, but different cathodes. Since these two lithium-ion batteries have different cathode, they have different performances, especially aging mechanism and cycle life. Furthermore, battery life can be changed from many factors such as: loss of lithium inventory (LLI), loss of active material (LAM), and an increase in resistance.

##### A. LITHIUM IRON PHOSPHATE BATTERY

Lithium iron phosphate battery ( $\text{LiFePO}_4$ , LFP) is a lithium ion rechargeable battery for high power applicants. LEP cell has 3.2V nominal working voltage and its energy density is 90-120 Wh/kg, which is lower than normal Li-ion cells. LEP has an average cycle life approximately 1,000 – 2,000 cycles and it is the safest lithium-based battery.

The overall reaction occurring in  $\text{LiFePO}_4$  (LFP):

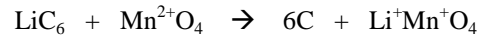


##### B. LITHIUM MANGANESE OXIDE

Lithium manganese oxide ( $\text{LiMn}_2\text{O}_4$ , LMO) is also useful for high power applicants such as power tools, medical instruments and electric vehicles. It has a 3.7V nominal working voltage and its energy density is 100-135 Wh/kg.

LMO has an average cycle life approximately 500–1,000 cycles. Its advantage is low internal cell resistance, which is the key to fast charging and high-current discharging.

The overall reaction occurring in  $\text{LiMn}_2\text{O}_4$  (LMO):



First, the batteries were charged at 1/3 °C, and discharged at 1.5 °C for 90 cycles at 45 °C, and 90 cycles at 5 °C. A reference performance test (RPT) was conducted after every 30 cycles [11]. From the result, the data were simulated at resistant between 9 to 11 mΩ.

#### V. METHODOLOGY

Among those reliability distributions, Weibull distribution is one of the most widely used probability distributions in the reliability engineering study. Since it does not need a large amount of data, Weibull distribution becomes a standard in reliability for modeling time-dependent failure. Weibull distribution is a very useful tool to check the failure components and the life cycle of systems. It is also a very flexible reliability model that approaches different distributions. It is the generalization of the exponential distribution when  $\beta = 1$ . When  $\beta = 2$ , it represents Rayleigh distribution. When  $\beta = 2.5$ , then the shape of the density function is similar to the Lognormal shape of function. When  $\beta = 3.4$  then the shape of the density function is similar to the normal shape of function.

##### A. Weibull Models Analysis

###### 1) Weibull Probability Distribution

There are three parameters in Weibull distribution, which are shape parameter  $\beta$ , scale parameter  $\eta$ , and location parameter  $\gamma$ . In this paper, we let  $\gamma = 0$ , then the Weibull distribution is a two-parameter distribution. Weibull can be used to represent the failure probability density function (PDF) with time by the following formula

$$f(t) = \left(\frac{\beta}{\eta}\right) \left(\frac{t}{\eta}\right)^{\beta-1} e^{-\left(\frac{t}{\eta}\right)^\beta}$$

where  $\beta > 0$ ,  $\eta > 0$ , and  $t \geq 0$ . Here  $\beta$  is the shape parameter representing the different pattern of the Weibull PDF and  $\eta$  is a scale parameter representing the characteristic life at which 63.2% of the population can be expected to have failed.

###### 2) Cumulative Distribution Function

Cumulative Distribution Function (CDF) of Weibull distribution is defined as:

$$F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^\beta}$$

where  $\beta > 0$ ,  $\eta > 0$ , and  $t \geq 0$ . CDF is the plot between cumulative probability and time (t).

###### 3) Reliability Function

Reliability Function (RF) also know as the survivor function, is defined as

$$R(t) = e^{-\left(\frac{t}{\eta}\right)^\beta}$$

where  $R(t) = 1 - F(t)$ ,  $\beta > 0$ ,  $\eta > 0$ , and  $t \geq 0$ . If we know  $\beta$  and  $\eta$ , we can find the reliability of systems at time  $t$ .

4) Hazard Function

Hazard Function (HF) of the Weibull distribution, also known as instantaneous failure rate, is defined as:

$$h(t) = \left(\frac{\beta}{\eta}\right) \left(\frac{t}{\eta}\right)^{\beta-1}$$

where  $h(t) = f(t)/R(t)$ ,  $\beta > 0$ ,  $\eta > 0$ , and  $t \geq 0$ . When  $\beta < 1$ , the hazard function is continually decreasing which represents early failures. When  $\beta > 1$ , the hazard function is continually increasing which represents wear-out failures. In particular, when  $\beta = 2$ , it is known as the Rayleigh distribution. When  $\beta = 3.4$ , the shape of the PDF is similar to the normal PDF.

VI. SELECTED METHOD

There are many ways to compute (estimate?) the parameters of the Weibull distribution. In this paper, Weibull parameters were calculated by using the Weibull formula in an Excel program. From the data that we simulated from RPT as shown in Error! Reference source not found. we put the failure cycles from lowest to highest in the Excel sheet by using Sort Ascending button. Secondly, we calculated the median rank by using formula:

$$\text{Median Rank} = \frac{i - 0.3}{N + 0.4}$$

where  $i$  = rank number,  $N$  = sample size. LFP and LMO data were prepared. To estimate Weibull parameters, we derived the Cumulative Distribution Function (CDF) as shown below.

$$F(x) = 1 - e^{-\left(\frac{x}{\alpha}\right)^\beta}$$

$$1 - F(x) = e^{-\left(\frac{x}{\alpha}\right)^\beta}$$

$$\ln(1 - F(x)) = -\left(\frac{x}{\alpha}\right)^\beta$$

$$\ln\left(\frac{1}{1 - F(x)}\right) = \left(\frac{x}{\alpha}\right)^\beta$$

$$\ln\left[\ln\left(\frac{1}{1 - F(x)}\right)\right] = \beta \ln\left(\frac{x}{\alpha}\right)$$

$$\ln\left[\ln\left(\frac{1}{1 - F(x)}\right)\right] = \beta \ln x - \beta \ln \alpha$$

Comparing this equation with the simple linear equation:  $Y = mX + b$ , we see that the left side of the equation corresponds to  $Y$ ,  $\ln X$  corresponds to  $X$ ,  $\beta$  corresponds to  $m$ , and  $-\beta \ln \alpha$  corresponds to  $b$ . Now we can plot the graphs of these two lithium-ion batteries by using  $\ln(\text{LFP, LMO Cycles})$  as X-axis, and  $\ln(\ln(1/(1-\text{Median Rank})))$  as Y-axis. Therefore, when we perform the linear regression, the estimate for the Weibull  $\beta$  parameter comes directly from the slope of the

line. The estimate for the  $\alpha$  parameter, which is Eta  $\eta$  in this paper, must be calculated as follows:

$$\alpha = e^{-\left(\frac{b}{\beta}\right)}$$

The results of the graphs we plotted between  $\ln(\text{LFP, LMO Cycles})$  as X-axis, and  $\ln(\ln(1/(1-\text{Median Rank})))$  as Y-axis are shown in Fig. 1 and Fig. 2.

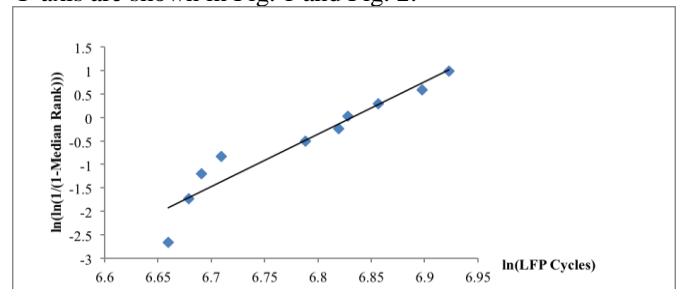


Fig. 1. Linear Regression of ln(LFP Cycles)

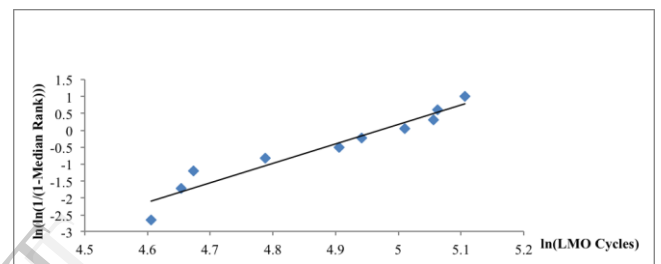


Fig. 2. Linear Regression of ln(LMO Cycles)

VII. ANALYSIS

From Weibull calculations of LFP and LMO life cycles, we can compare  $\beta$  and  $\eta$  of those batteries as shown in the table below.

TABLE I. WEIBULL PARAMETERS OF LFP AND LMO.

Battery	Beta	Eta
LFP	11.17	926.78
LMO	5.75	144.20

The value of  $\beta$  has an effect on the failure rate of the Weibull distribution, and also on the shape of the distribution. From the result shown in Table, both LFP and LMO have  $\beta > 1$ . This indicates an increasing failure rate. As stated above in the Hazard function, these two types of batteries are continually increasing in failure rate, which represents wear-out failures. Since LFP has  $\beta$  greater than LMO, LFP is increasing in failure rate more than LMO.

Eta ( $\eta$ ) represents the characteristic life of an item, defined as the time at which 63.2 percent of the population has failed. That means about 36.8 percent of LFP should run at least 926.78 cycles. In comparison, about 36.8 percent of LMO should run at least 144.20 cycles.

From Table, we can create Weibull reliability formulas for LFP as shown below:

$$\text{PDF: } f(t) = 8.07 \times 10^{-33} \times t^{10.17} e^{-(t/926.78)^{11.17}}$$

$$\text{CDF: } F(t) = 1 - e^{-(t/926.78)^{11.17}}$$

$$\text{RF: } R(t) = e^{-(t/926.78)^{11.17}}$$

$$\text{HF: } h(t) = 8.07 \times 10^{-33} \times t^{10.17}$$

For LMO, Weibull formulas are shown below:

$$\text{PDF: } f(t) = 2.22 \times 10^{-12} \times t^{4.75} e^{-(t/144.20)^{5.75}}$$

$$\text{CDF: } F(t) = 1 - e^{-(t/144.20)^{5.75}}$$

$$\text{RF: } R(t) = e^{-(t/144.20)^{5.75}}$$

$$\text{HF: } h(t) = 2.22 \times 10^{-12} \times t^{4.75}$$

### VIII. CONCLUSION

Weibull analysis can be particularly helpful in analyzing the root cause of specific design failures, such as unanticipated or premature failures. Furthermore, we can then look for unusual circumstances that will help uncover the cause of these failures, which could include a bad production run, poor maintenance practices, or unique operating conditions, even when the design is good. By the formulas derived above, engineers can now predict reliability parameters in the selected domains.

### IX. FUTURE WORK

In this paper, the reliability formulas of lithium-ion batteries have been developed for help in battery designs. The study was based on the characteristic life of LFP and LMO by using Weibull analysis. Even though battery data is available, the limitations of the studies suggests that the characteristic life of batteries is difficult to interpret via Weibull analysis. In this report, we have developed a methodology to simulate data in order to calculate Weibull parameters in an efficient manner.

While this paper has demonstrated Weibull calculations, there are many more opportunities for extending the scope of this work. We can create reliability formulas from other

reliability theories. Furthermore, we can also investigate the application of the other reliability formulas of other batteries, such as lead-acid, nickel-metal hydride, and zebra batteries.

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