Prime Labeling For Some Fan Related Graphs

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Abstract:

A graph with vertex set V is said to have a prime labeling if its vertices are labeled with distinct integer 1,2,3…$|V|$ such that for edge xy the labels assigned to x and y are relatively prime. A graph which admits prime labeling is called a prime graph. In this paper we investigate prime labeling for some fan related graphs. We also discuss prime labeling in the context of some graph operations namely fusion and duplication, Switching in fan F_n

Keywords: Prime Labeling, Fusion, Duplication and Switching.

1. Introduction:

In this paper, we consider only finite simple undirected graph. The graph G has vertex set $V = V(G)$ and edge set $E = E(G)$. The set of vertices adjacent to a vertex u of G is denoted by $N(u)$. For notations and terminology we refer to Bondy and Murthy [1].

The notion of a prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout. A (1982 P 365-368). [2] Many researches have studied prime graph for example in Fu.H.(1994 P 181-186) [5] have proved that path Pn on n vertices is a Prime graph.

In Deretsky.T(1991 p359 – 369) [4] have proved that the C_n on n vertices is a prime graph. In Lee.S (1998 P.59-67) [2] have proved that wheel W_n is a prime graph iff n is even. Around 1980 Roger Etringer conjectured that all tress have prime labeling which is not settled till today. The prime labeling for planner grid is investigated by Sundaram.M(2006 P205-209) [6]

In (2010) S.K.Vaidhya and K.K.Kanmani have proved the prime labeling for some cycle related graphs [7]

Definition 1.1

Let G = (V(G), E(G)) be a graph with p vertices. A bijection f:V(G)→{1,2,…p} is called a prime labeling if for each edge e=uv, gcd{f(u), f(v)}=1. A graph which admits prime labeling is called a prime graph.

Definition 1.2

Let u and v be two distinct vertices of a graph G. A new graph G1 is constructed by identifying (fusing) two vertices u and v by a single vertex x in such that every edge which was incident with either u or v in G now incident with x in G.

Definition : 1.3

Duplication: Duplication of a vertex v_k of a graph G produces a new graph G_1 by adding a vertex v_{k^I} with N(v_{k^I})=N(v_k)

In other words a vertex v_{k^I} is said to be a duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to v_{k^I}

In this paper we prove that the graphs obtained by identifying any two vertices of degree 2 in the fan graph Fn and two vertices which are adjacent to vertices of degree 2 (u_2 and v_2 or u_{n-1} or v_{n-1}) and the graph obtained by duplication the vertex of degree 2 admit prime labeling.

Definition: 1.4

Switching: A vertex switching G_v of a graph G is obtained by taking a vertex v of G, removing the entire edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition: 1.5 (Fan graph)

A fan graph obtained by joining all vertices of F_n, $n \geq 2$ is a path Pn to a further vertex, called the centre.

Thus F_n contains n+1 vertices say C, $v_1, v_2, v_3 \dots v_n$ and (2n-1) edges, say cv_i , $1 \le i \le n$ and $v_i v_i + 1$, $1 \le i \le n-1$.

Theorem 1:

The graph Obtained by duplicating arbitrary vertex of fan Fn is a Prime graph.

Proof:

Let G = F_n be the graph

Let V(G) be $v_1, v_2, v_3 \dots v_n, C$

and edge of G E(G) = { cv_i , $1 \le i \le n$ } U {$v_i v_i + 1$, $1 \le i \le n-1$ }

Let v_1 be the vertex duplicated to $v_1{}'$. Let the new graph be G_1

$$|V(G_1)| = n + 2$$

Define a labeling f by

f: $V(G_1) \rightarrow \{1,2,3 \dots n+2\}$ as follows

Case (i)

For n≠3k+1, K is an integer.

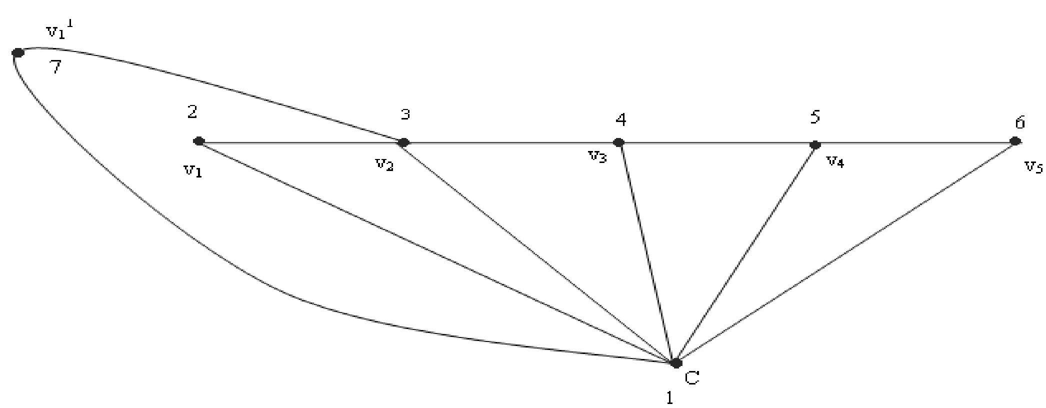
Let f(c) = 1

$f(v_i) = i + 1$ $\qquad 1 \le i \le n$

$f(v_k') = n + 2$

then f admits prime labeling. T herefore G_1 is prime graph.

Example F_5 Vertex duplication of v_1



Case (ii)

For n= 3k+1 k=2,4,6…

Define f as

f(c)=1

f($v_I{}'$)=2

f(v_2)=3

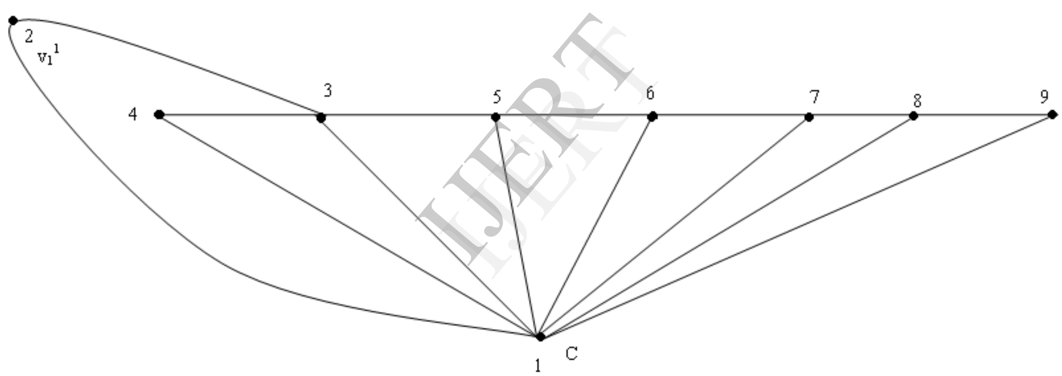
f(v_I)=4

f(v_i) = i+2, 3$\leq i \leq n$

Then of admits prime labeling

G_1 is a prime graph

Example F_7 Vertex duplication of v_1



Theorem 2:

The switching of any vertex v_i in a fan graph F_n produces a Prime graph

for n=k+1 k is a Prime numbers

Proof:

Let G = F_n and $v_1, v_2, v_3 \dots v_n$ be the successive vertices of F_n and let C be the centre vertices of F_n

$$|V(G_1)| = n + 1$$

Define a labeling f: V(G_1) →{1,2….n+1} as follows

f(c)=1

$f(v_i)=i \qquad 2\leq i \leq n$

$f(v_1)=n+1$

then f results a prime labeling

Therefore G_1 is a prime graph

In general for the above value of n, we can generalize the switching of vertex as v_i for i = 1,2…n we get the prime labeling.

Here we consider the new graph as G_i

When the vertex switching is v_i

Define f: V(G_i) →{1,2….n+1}

As followed When i=2 Switching v_2

Let f(c)=1

$f(v_i)=i-1 \qquad$ for $3\leq i \leq n$

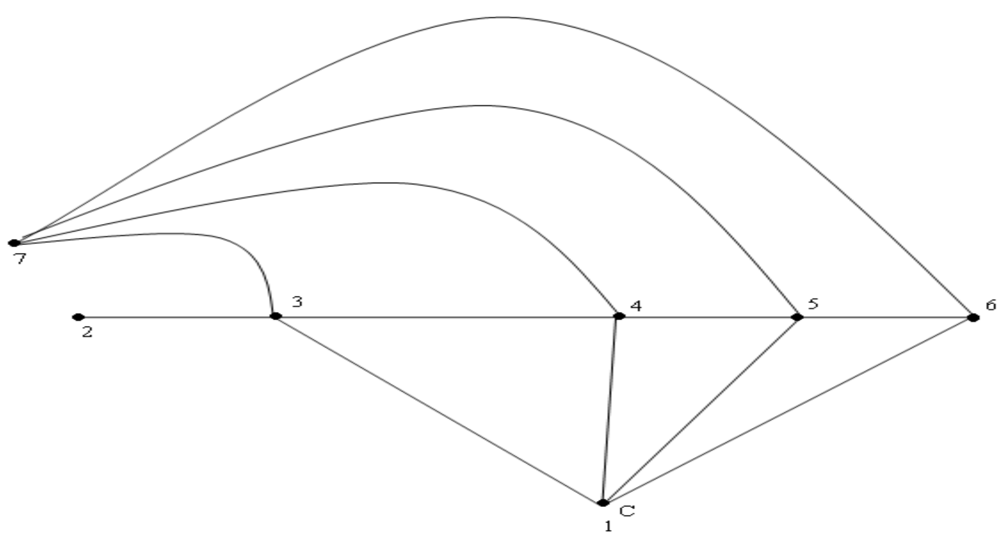
$f(v_1)=n$

$f(v_2)=n+1$

In general fix the number 1 for the centre vertex and assign the remaining number from the next vertex of the switching vertex in clockwise direction, then f permits prime labeling.

Therefore resulting graph G_i is a Prime graph

Example F_5 Vertex Switching v_1



Theorem 3:

The duplication of the vertex v_1 in a fan graph F_n produces a prime graph.

Proof:

Let G=F_n be fan graph.

Let V(G) = $\{v_1, v_2, v_3 \dots v_n, C\}$ and edge of G E(G) = $\{ cv_i , 1 \leq i \leq n \}$

U $\{v_i v_i + 1 , 1 \leq i \leq n - 1 \}$

Let v_1 be the vertex duplicated to v_i. Let the new graph be G_1

$$|V(G_1)| = n + 2$$

Define a labeling f by

f: $V(G_1) \rightarrow \{1,2,3 \dots n + 2\}$ as follows

Let f(c) = 1

$f(v_1) = 2$

$f(v_2) = 3$

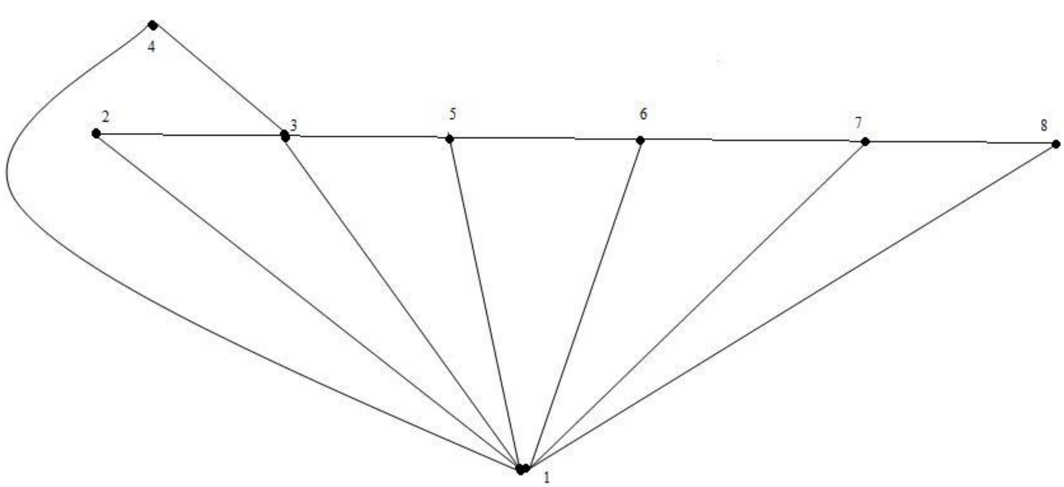
$f(v_1{}') = 4$

$f(v_i) = i + 2$ for $3 \leq i \leq n.$

then f admits prime labeling.

Therefore G_1 is a prime graph

Example: Duplication of v_1 in F_6



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Theorem 4:

The duplication of the vertex v_2 in a fan graph F_n produces a prime graph.

Proof:

Let G=F_n be fan graph.

Let V(G) = $\{v_1, v_2, v_3 \ldots v_n, C\}$ and edge of G E(G) = $\{ cv_i , 1 \leq i \leq n \}$

U $\{v_i v_i + 1 , 1 \leq i \leq n - 1 \}$

Let v_2 be the vertex duplication to v_2'.

Let the new graph be G_1

$|V(G_1)| = n + 2$

Now define a labeling f: $V(G_1) \rightarrow \{1,2,3 \ldots n + 2\}$ as follows

Let

f(c) = 1

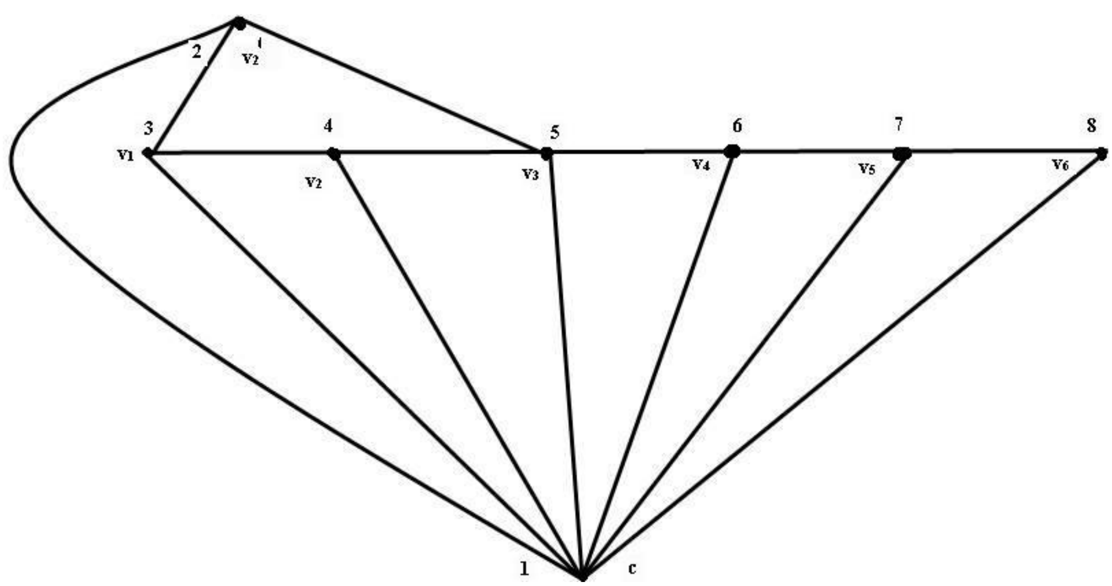
$f(v_2') = 2$

$f(v_i) = i + 2$ for $1 \leq i \leq n$.

Then f admits prime labeling.

Therefore G_1 is a prime graph

Example: Vertex duplication v_2 in a fan graph F_6



Theorem 5:

The duplication of a vertex v_3 in a fan graph F_n produces a prime graph.

Proof:

Let G=F_n be fan graph.

Let V(G) = $\{v_1, v_2, v_3 \dots v_n, C\}$ and edge of G E(G) = $\{ cv_i , 1 \leq i \leq n \}$

U $\{v_i v_i + 1 , 1 \leq i \leq n - 1 \}$

Let v_3 be the vertex duplication to v_3'.

Let the new graph be G_1

$|V(G_1)| = n + 2$

Now define a labeling f: $V(G_1) \rightarrow \{1,2,3 \dots n + 2\}$ as follows

Case (1)

For $n \neq 3k + 1$

f(c) = 1

$f(v_3') = 2$

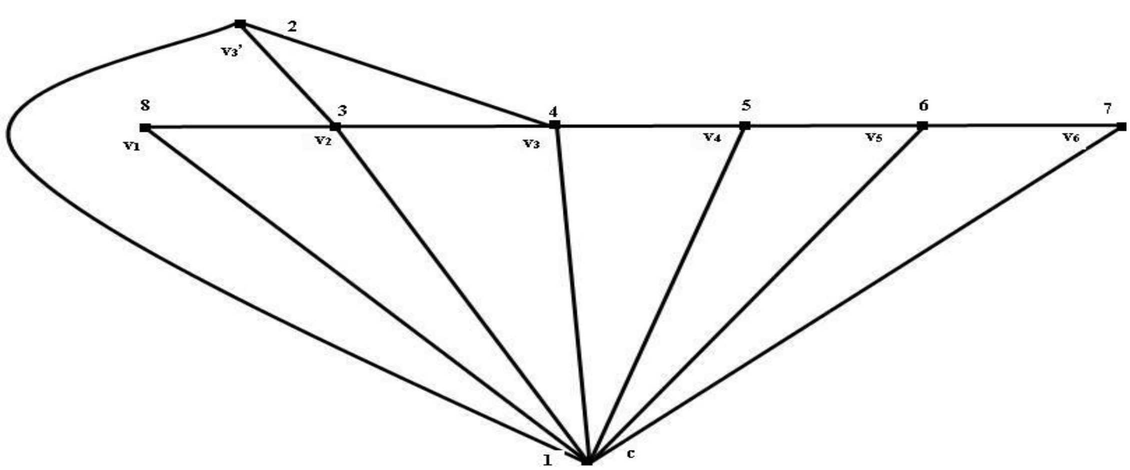
$f(v_1) = n + 2$

$f(v_i) = i + 1$, for 2$\leq i \leq n$.

Then f admits prime labeling.

For $n \neq 3k + 1$, the G_1 is a prime graph

Example: Vertex duplication of F$_6$



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Case (2)

For $n = 3k + 1$

f: $V(G_1) \rightarrow \{1,2,3 \dots n+2\}$ as follows

$f(c) = 1$

$f(v_3') = 2$

$f(v_4) = 3$

$f(v_3) = 4$

$f(v_2) = 5$

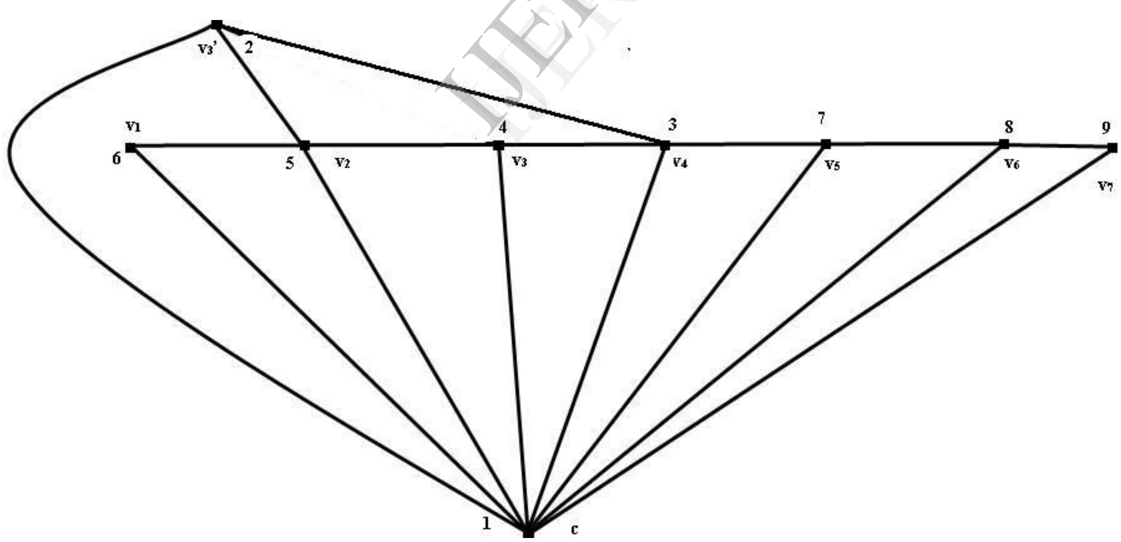
$f(v_1) = 6$

$f(v_i) = i + 2$, for $5 \leq i \leq n$.

Then f admits prime labeling.

For $n = 3k + 1$, the G_1 is a prime graph

Example: Vertex duplication of v_3 in F_7.



Theorem 6:

The duplication of the vertex v_4 in a fan graph F_n produces a prime graph.

Proof:

Let G=F_n be fan graph.

Let V(G) = $\{v_1, v_2, v_3 \dots v_n, C\}$ and edge of G E(G) = $\{ cv_i , 1 \leq i \leq n \}$

U $\{v_i v_i + 1 \text{ , } 1 \leq i \leq n - 1 \}$

Let v_4 be the vertex duplication to v_4'.

Let the new graph be G_1

$|V(G_1)| = n + 2$

Now define a labeling f: $V(G_1) \rightarrow \{1,2,3 \dots n + 2\}$ as follows

$f(c) = 1$

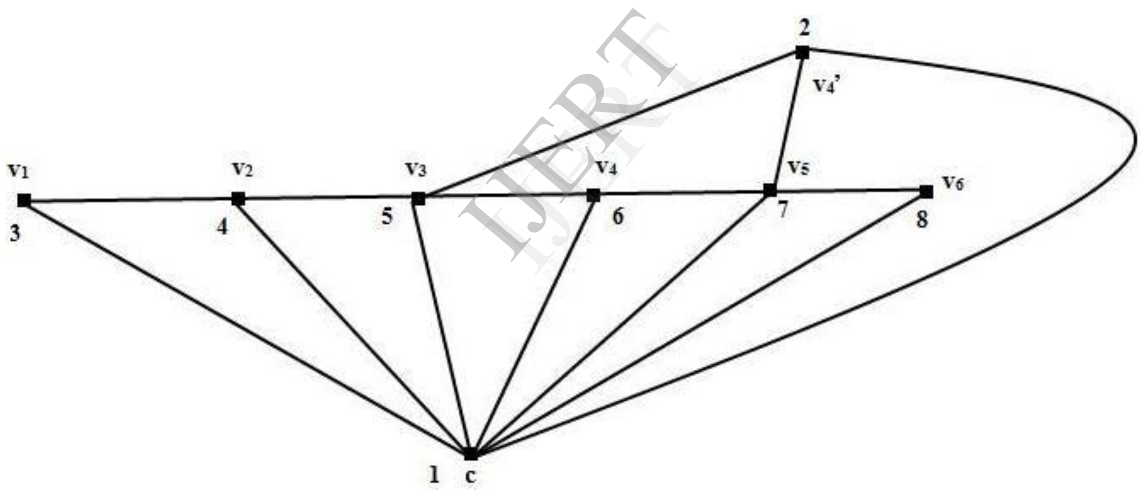
$f(v_4') = 2$

$f(v_i) = i + 2$, for $1 \leq i \leq n$.

Then f admits prime labeling.

Therefore G_1 is a prime graph

Example Duplication of v_4 in F_6



Theorem 7:

The duplication of any vertex v_k , k is even in a fan graph F_n produces a prime graph.

Proof:

Let G=F_n be fan graph.

Let V(G) = $\{v_1, v_2, v_3 \dots v_n, C\}$ and edge of G E(G) = $\{ cv_i \text{ , } 1 \leq i \leq n \}$

U $\{v_i v_i + 1 \text{ , } 1 \leq i \leq n - 1 \}$

Let $v_k (k = 2,4, \dots .)$ is even be the vertex duplicated to v_k' producing a new graph G_1.

$|V(G_1)| = n + 2$

Now define a labeling f: $V(G_1) \rightarrow \{1,2,3 \ldots n + 2\}$ as follows

$f(c) = 1$

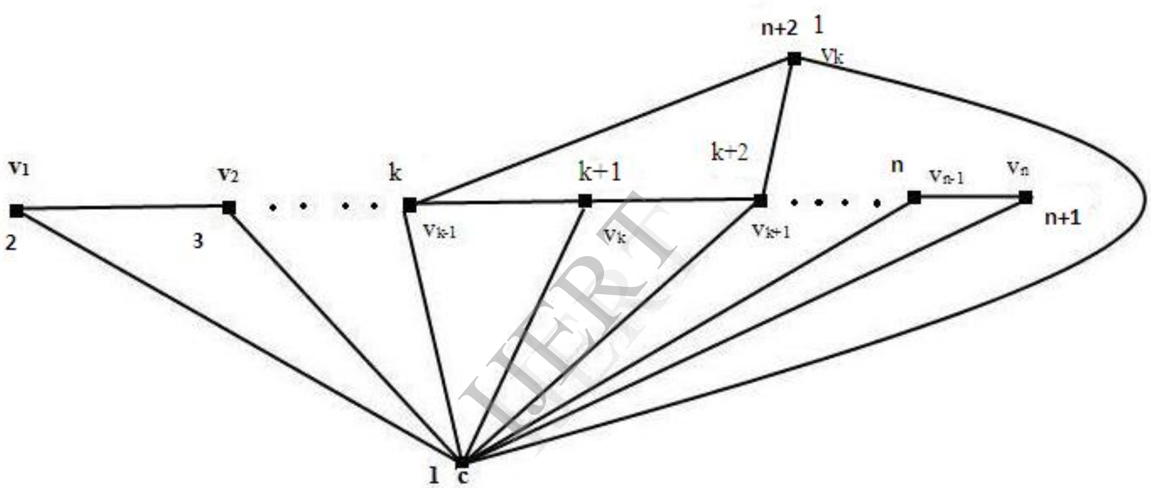
$f(v'_k) = 2$

$f(v_i) = i + 2$, for $1 \leq i \leq n$.

Then f admits prime labeling.

Therefore G_1 is a prime graph

Example



Theorem 8:

The duplication of any vertex v_k , k is odd in a fan graph F_n produces a prime graph.

Proof:

Let G=F_n be fan graph.

Let V(G) = $\{v_1, v_2, v_3 \ldots v_n, C\}$ and edge of G be E(G) = { cv_i , $1 \leq i \leq n$ }

U $\{v_i v_i + 1$, $1 \leq i \leq n - 1$ }

Let $v_k(k = odd\ vertices)$ duplicated to v'_k.

For the value $n = 6,9,11,15,17, \ldots$ define a new graph as G_1

$|V(G_1)| = n + 2$

Now define a labeling f: $V(G_1) \rightarrow \{1,2,3 \ldots n + 2\}$ as follows

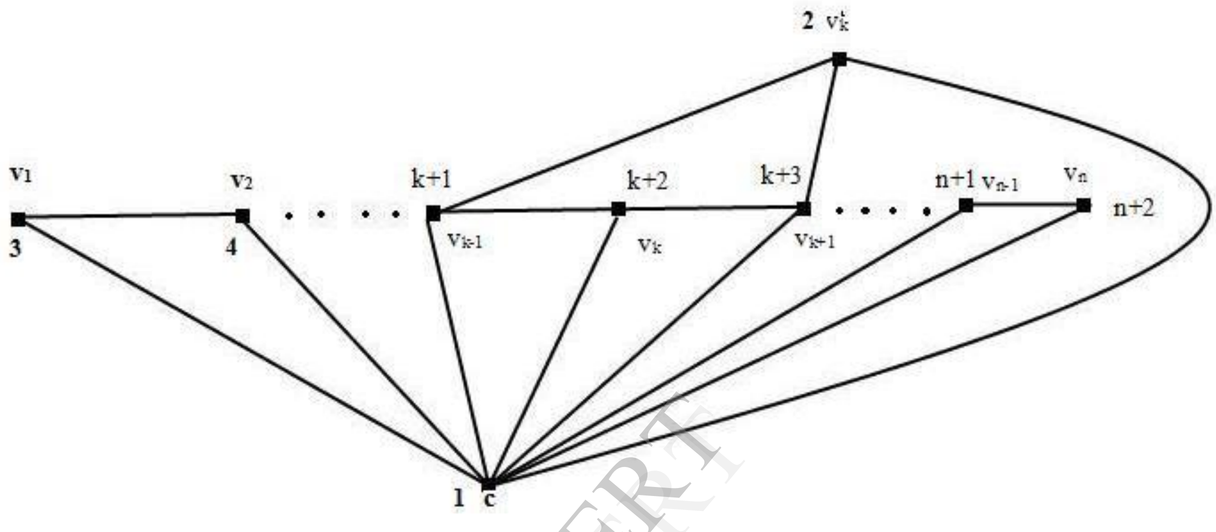
$f(c) = 1$

$f(v_i) = i + 1$, for $1 \leq i \leq n$.

$f(v'_k) = n + 2$

Then f admits prime labeling.

Therefore G_1 is a prime graph



Theorem 9:

In a fan graph $G = F_n$ fusion of v_3 with v_1 produces a prime graph.

Proof:

Let G=F_n be fan graph.

Let V(G) = $\{v_1, v_2, v_3 \ldots v_n, C\}$ and edge of G be E(G) = { cv_i , $1 \leq i \leq n$ }

U $\{v_i v_i + 1$, 1≤ $i \leq n - 1$ }

Let G_1 *be* a new graph obtained by fusion v_3 with v_1.

$|V(G_1)| = n$

Now define a labeling f: $V(G_1) \rightarrow \{1,2,3 \ldots n\}$ as follows

$f(c) = 1$

$f(v_2) = 2$

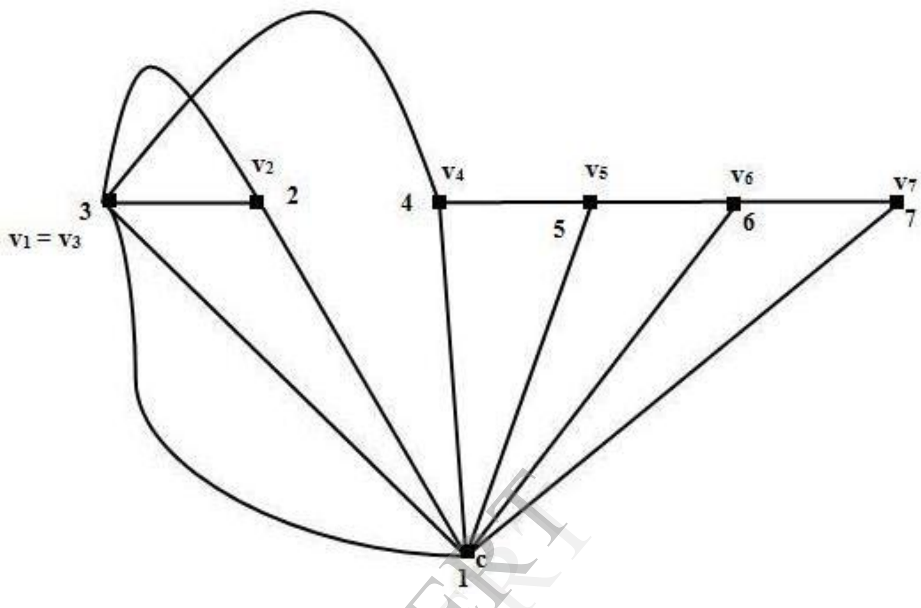
$f(v_1 = v_3) = 3$

$f(v_i) = i,$ for $4 \leq i \leq n.$

Then f admits prime labeling.

Therefore G_1 is a prime graph

Example Fusion of v_3 with v_1v1 in F7



Theorem 10:

Fusion of v_4 with v_1 in a fan graph $G = F_n$ produces a prime graph.

Proof:

Let G=F_n be fan graph.

Let $V(G) = \{v_1, v_2, v_3 \dots v_n, C\}$ and edge of G be E(G) = { cv_i , $1 \leq i \leq n$ }

U $\{v_i v_i + 1$, $1 \leq i \leq n - 1\}$

Let G_1 be a new graph obtained by fusion v_4 with v_1.

$|V(G_1)| = n$

Now define a labeling f: $V(G_1) \rightarrow \{1,2,3 \dots n\}$ as follows

For $n = 2k + 3$

$f(c) = 1$

$f(v_n) = 2$

$f(v_{n-i}) = 2 + i$ i=1,2

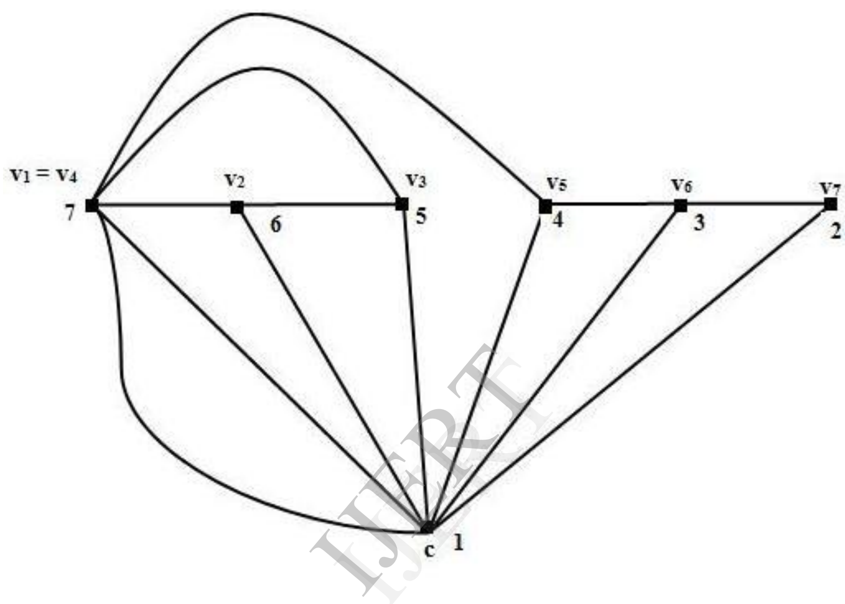
$f(v_1 = v_4) = n$

$f(v_{i+1}) = (n - i), i = 1,2.$

Then f admits prime labeling.

Therefore G_1 is a prime graph

Example



Theorem 11:

Fusion of v_5 with v_1 in a fan graph $G = F_n$ produces a prime graph.

Proof:

Let G=F_n be fan graph.

Let $V(G) = \{v_1, v_2, v_3 \dots v_n, C\}$ and edge of G be E(G) = { cv_i , $1 \leq i \leq n$ }

U $\{v_i v_i + 1$, $1 \leq i \leq n - 1$ }

$|V(G)| = n + 1$

Let G_1 *be* a new graph obtained by fusion of vertex v_5 with v_1.

$|V(G_1)| = n$

Now define a labeling f: $V(G_1) \rightarrow \{1,2,3 \dots n\}$ as follows

$f(c) = 1$

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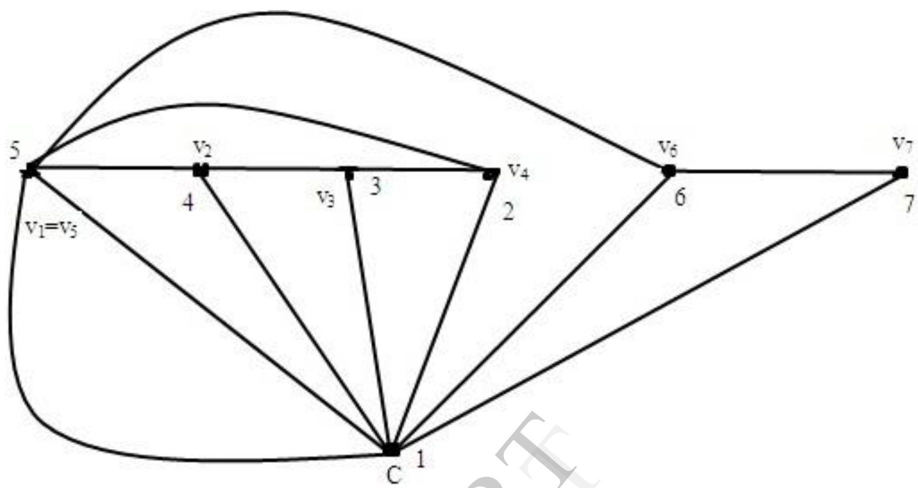
$f(v_{k-i}) = i + 1, \qquad 1 \leq i \leq k - 1$

$f(v_{k+i}) = k + i, \qquad 1 \leq i \leq n - k$

Then f admits prime labeling.

Therefore fusion of v_5 with v_1 produces prime graph.

<u>Example:</u> Fusion of v_5 with v_1 in a fan graph F_7



<u>Theorem 12:</u>

Fusion of v_6 with v_1 in a fan graph $G = F_n$ produces a prime graph for the prime n($ie., n = 7,11,13 \ldots$).

<u>Proof:</u>

Let G=F_n be fan graph.

Let $V(G) = \{v_1, v_2, v_3 \ldots v_n, C\}$ and edge of G be E(G) = { cv_i , $1 \leq i \leq n$ }

U $\{v_i v_i + 1$, $1 \leq i \leq n - 1$ }

$|V(G)| = n + 1$

Let G_1 *be* a new graph obtained by fusion of vertex v_6 with v_1.

$|V(G_1)| = n$

Now define a labeling $f: V(G_1) \rightarrow \{1,2,3 \ldots n\}$ as follows

$f(c) = 1$

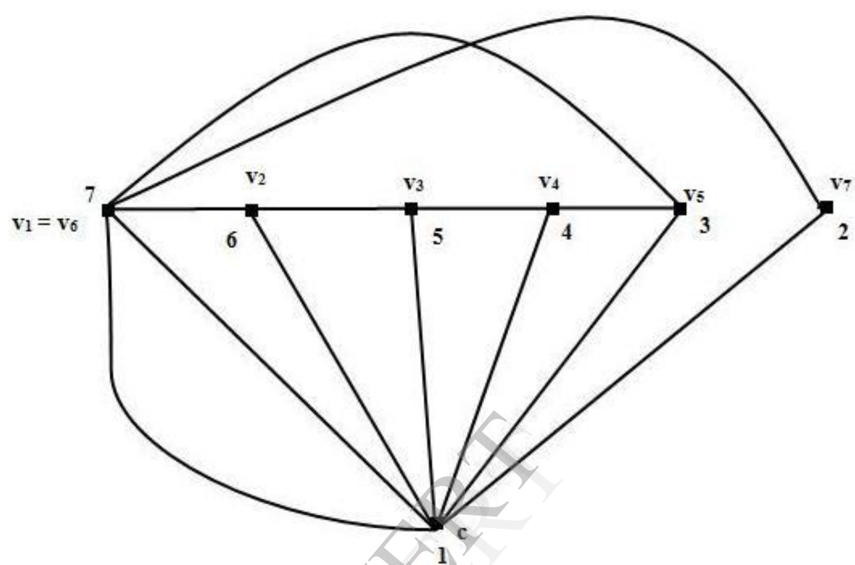
$f(v_n) = 2$

$f(v_{k-i}) = 2 + i \qquad 1 \leq i \leq k - 1$

Then f admits prime labeling.

Therefore fusion of v_6 with v_1 produces prime graph.

Example: Fusion of v_6 with v_1 in a fan graph F_7



Theorem 13:

Fusion of v_k with v_1 in a fan graph $G = F_n$ produces a prime graph for the prime k is odd($ie., k = 3,5 \ldots$).

Proof:

Let G=F_n be fan graph.

Let $V(G) = \{v_1, v_2, v_3 \ldots v_n, C\}$ and edge of G be E(G) = { cv_i , $1 \leq i \leq n$ }

U {$v_i v_i + 1$, 1≤ $i \leq n - 1$ }

Let G_1 be a new graph obtained by fusion of vertex v_k ($k = odd$).

$|V(G_1)| = n$

Now define a labeling $f: V(G_1) \rightarrow \{1,2,3 \ldots n\}$ as follows

$f(c) = 1$

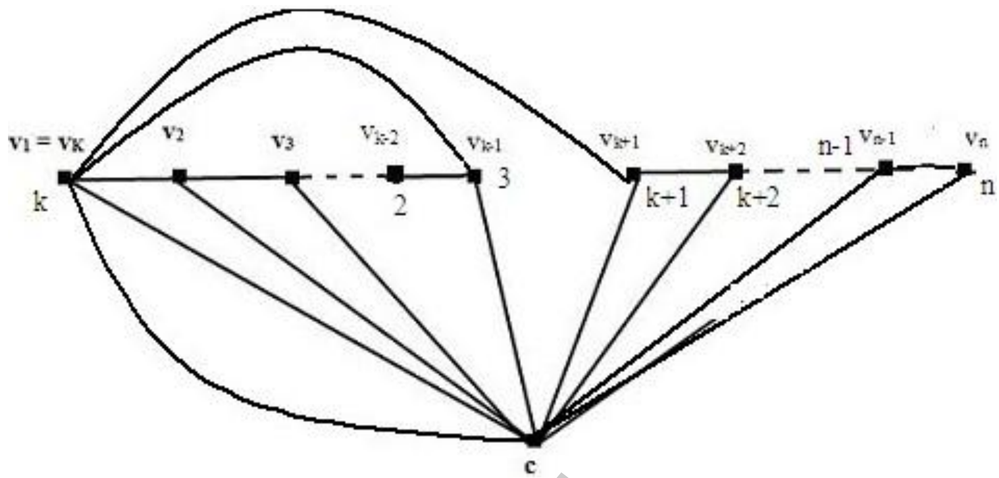
$f(v_{k-i}) = i + 1 \quad 1 \leq i \leq k - 1$

$f(v_{k+i}) = k + i \qquad\qquad 1 \leq i \leq n - k$

Then f admits prime labeling.

Therefore fusion of $v_k$with v_1 $(k\ odd)$ produces prime graph.

Example fusion of $v_k$with v_1 $(k\ odd)$



Theorem 14:

Fusion of a vertex v_k withv_1, (k even) in a fan graph $G = F_n, n$ prime produces a prime graph.

Proof:

Let G=F_n be fan graph.

Let $V(G) = \{v_1, v_2, v_3 \dots v_n, C\}$ and edge of G be E(G) = { cv_i , $1 \leq i \leq n$ }

U $\{v_i v_i + 1$, $1 \leq i \leq n - 1$ }

$|V(G)| = n + 1$

$|E(G)| = 2n - 1$

Let G_1 be a new graph obtained by fusion of vertex $v_k$with v_1.

$|V(G_1)| = n$

Now define a labeling $f\colon V(G_1) \to \{1,2,3 \dots n\}$ as follows

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Case (i)

For $n = 11, k = 6$

$f(c) = 1$

$f(v_{k+i}) = (k+1) - i \qquad 1 \leq i \leq n-k$

$f(v_{k-i}) = (k+1) + i \qquad 1 \leq i \leq k-1$

Then f admits prime labeling.

Therefore G_1 is a prime graph.

Case (ii)

For $n = 11, k = 8$

$f(c) = 1$

$f(v_{k+i}) = 5 - i \qquad 1 \leq i \leq n-k$

$f(v_{k-i}) = 4 + i \qquad 1 \leq i \leq k-1$

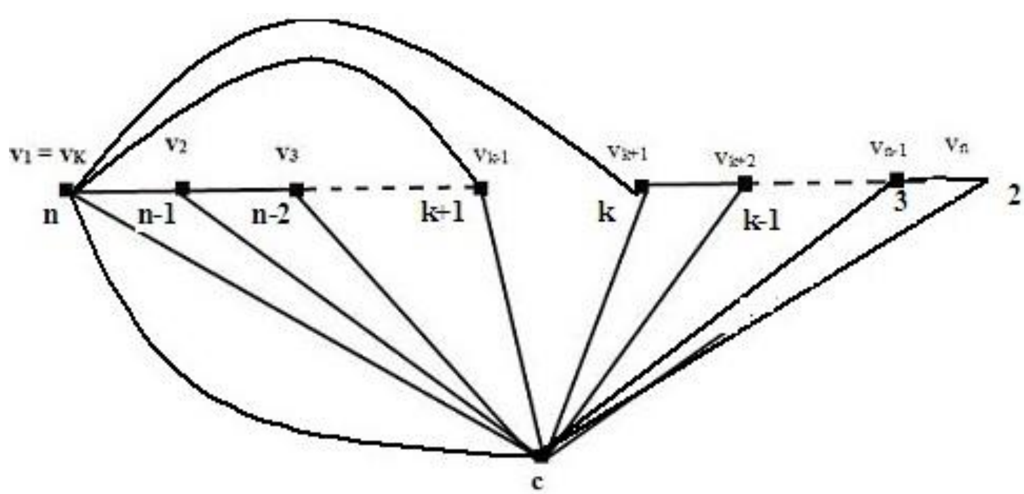
Then f admits prime labeling.

Therefore G_1 is a prime graph.

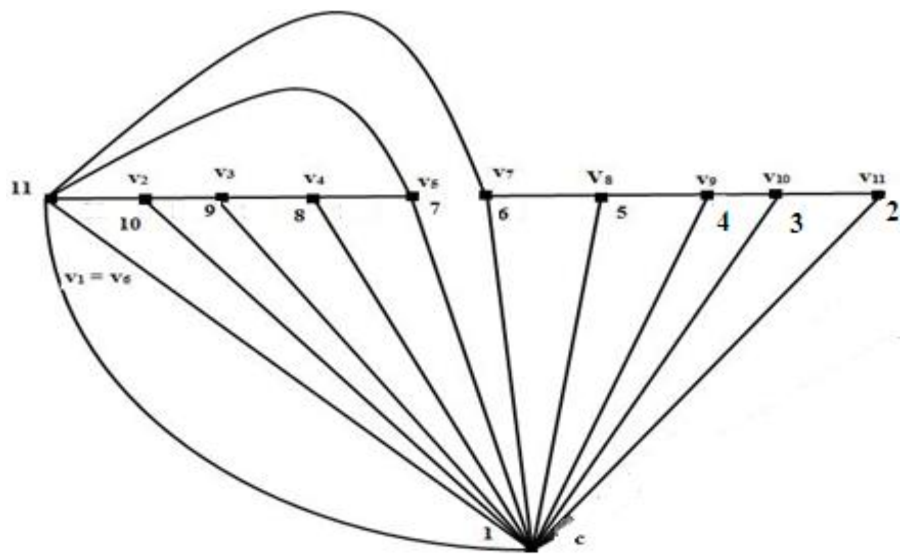
Example

Fusion in vertex v_k (k even) with v_1, n prime

General case:



Specific case:



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