

# Probabilistic Sensitivity Analysis of Reinforced Concrete Bridge Deck Strengthened with Carbon Fibre Reinforced Polymers (CFRP)

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**Abstract**— The strength of reinforced concrete elements retrofitted in flexure, deflection and shear by means of externally bonded Carbon Fibre Reinforced Polymers (CFRP) has attracted the attention of researchers due to many advantages highlighted by a wide set of experimental results. This research presents reliability study on Reinforced Concrete (RC) bridge deck strengthened with externally bonded CFRP subject to corrosion. The deck was subjected to reliability analysis, using First Order Reliability Method (FORM), enhanced with genetic algorithms and the inherent safety was found to yield safety index of 3.8, this agreed with the recommendation of the Joint Committee of Structural Safety Code (JCSS 2001) for structures with moderate consequence of failure. The reliability analysis for the intact and the corrosion affected deck were executed through a developed program, written using MATLAB Simulink and the result showed the detrimental effect of corrosion on reinforcement of steel in the bridge deck. Flexural capacity restoration was also undertaken for the bridge deck using Carbon Fibre Reinforced Polymer (CFRP) with the adoption of adhesive bonding technique. The results of the reliability-based analysis of the strengthened deck with Carbon Fibre Reinforced Polymer (CFRP) yielded a flexural capacity restoration as much as 100%. Sensitivity analysis for the deflection, shear and bending modes of failure were also conducted. It was observed that the stiffness of CFRP and its thickness have influence on the retrofitting capacity of the bridge deck in the deflection mode. However, the critical mode of failure to be strengthened is flexure while the shear mode of failure is not affected much by corrosion.

**Keywords**— Probabilistic, Sensitivity, Bridge, Deck, Carbon, Fibre, Polymer

## I. INTRODUCTION

Fibre Reinforced Polymer Materials (FRP) are made of fibres embedded in a polymer resin matrix [1]. The fibres are the main load-carrying element. The wide range of strengths and stiffness of the different types of fibres make them ideal for construction uses. Carbon, glass and aramid fibres are the common types used in the production of FRP composites for construction. Their stress-strain relationship up to failure is linear. The resin matrix connects the fibres together, protects them from damage and from the environment, and maintains their alignment, and allows distribution of load among them. Thermosetting resins are almost exclusively used in civil engineering [2]. Epoxy, vinyl ester and polyester are the most

common matrices of the thermosetting resins. Epoxy has a pot life around 30 min at 20 °C but can be changed with different formulations. Pot life is defined as the time available for use of the epoxy system after the resin and curing agent are mixed. The curing goes faster with increasing temperature. Epoxies have good strength, bond, creep properties and chemical resistance.

Carbon Fibre Reinforced Polymers (CFRP) are increasingly being applied for the rehabilitation and strengthening of infrastructures in lieu of traditional repair techniques such as steel plates bonding. It has proved itself cost-effective in a number of field applications strengthening concrete, masonry, steel, cast iron, and timber structures. Its use in construction industry can be either for retrofitting to strengthen an existing structure (such as bridges) that were designed to tolerate far lower service loads than they are experiencing today or as an alternative reinforcing (or pre-stressing) material instead of steel from the outset of a project. Retrofitting is popular in many instances as the cost of replacing the deficient structure can greatly exceed its strengthening using CFRP.

Fibre Reinforced Polymer plates have many advantages over steel plates in structural engineering applications, and their use can be extended to situations where it would be impossible or impractical to use steel. For example, the plates are lighter than steel plates of equivalent strength, which eliminates the need for temporary support for the plates. Also, since CFRP plates used for external bonding are relatively thin, neither the weight of the structure nor its dimensions are significantly increased. In addition, CFRP plates can easily be cut to length on site. These various factors in combination make installation much simpler and quicker than when using steel plates [3].

There were few analytical studies available for the prediction of flexural capacity of reinforced concrete deck strengthened with external laminates. Reference [4] determined the ultimate moment capacity of reinforced concrete deck externally strengthened with bonded steel plates.

The study employed rectangular stress blocks for concrete and the actual stress-strain curves of the internal steel reinforcement, and external steel plates to evaluate the internal forces and moment [3]. Several researchers have come up with techniques for attempting to predict flexural capacities and failure modes for FRP reinforced structural elements. Results of research performed by [5], indicated that the failure mode of FRP-reinforced deck was highly influenced by the reinforcement ratios of the FRP and steel. The study also offered equations for strength based on the various modes of CFRP-reinforced deck failure. The strengthening process of a typical bridge deck using CFRP is as shown in Figure 1.



Figure 1: A Typical FRP Strengthening Process of bridge deck

Generally, the current design criteria of engineering structures are based on limit state. Uncertainties in basic design variables are accommodated by set of deterministic partial safety coefficients. However, since each basic design variable is random, the best way to address the uncertainties is to use reliability method [6-10].

In this study, sensitivity analysis will be conducted in order to compute safety indices and probabilities of failure at various design scenarios in accordance with Eurocode 2.

## II. METHODOLOGY

### A. Structural Configuration and Design of the Reinforced Concrete Bridge Deck

A typical reinforced concrete bridge deck as shown in Figure 2 with 100mm depth of surfacing together with a nominal HA live load uniformly distributed load of 17.5kN/m<sup>2</sup> and knife edge load of 33kN/m. The deck was also designed to carry 30 units of HB load with a span of 12.0m center to center of bearing. The deck is simply supported, designed using a unit strip method.

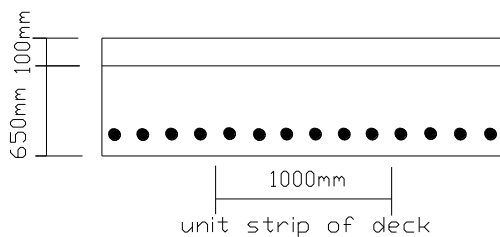


Figure 2: Unit strip of the reinforced concrete bridge deck

The unit weight of the concrete is 25kN/m<sup>3</sup>. The concrete is grade C32/40 (BS 5400) and the grade of the steel reinforcement is grade B500B (BS 4449) with nominal cover of the reinforcing steel as 60mm.

### B. Formulation of the Reliability Function

In the analyses of the deck, the effect of deterioration of reinforced steel due to corrosion was considered. The following structural reliability was carried out;

The bridge deck was designed in accordance with BS 5400 and resulted to a deck with reinforcing steel spacing 175mm and steel diameter of 32mm. It was then subjected to reliability analysis with the view to establish the inherent reliability of the designed bridge. The reliability was checked for various dead to live load ratio ranging from 0.4 to 1.6. The default dead to live load ratio obtained from the deterministic structural analysis is 1.0. The influence of the spacing of reinforcing steel was also checked.

The overall reliability analysis was undertaken using First Order Reliability Methods, developed by Hasofer. The non-normal designed variables were transformed to equivalent normal variables using Rackwitz Fiessler Algorithm. The search for the global optimum safety indices was enhanced by the application of Genetic Algorithm. Genetic Algorithm (GA), facilitate the consideration of multiple possible solution at a time rather than single possible solution when using FORM alone. The use of GA therefore yielded wider search space for the safety index. The whole process was coded using MATLAB Simulink Platform.

The program Flow chart is presented in Figure 3 where each task was implemented via specialized MATLAB subprograms.

### C. Derivation of Limit State Function for Bending, Shear and Deflection Modes of Failure

#### Bending mode of failure

The flexural reliability function for both the un-strengthened (Intact) and the strengthened cases were developed and given in Equations 1 and 2 respectively.

$$G(X) = \theta_r A_s(t) f_y \left( d - \frac{0.88 A_s(t) f_y}{b f_c} \right) - 0.125 \theta_w G_k (\alpha + 1) L^2 \quad (1)$$

Where

- $\theta_r$  is the model uncertainty for the un-strengthened deck
- $A_s(t)$  is the time dependent cross sectional area
- $f_y$  is the yield strength of the steel reinforcement
- $d$  is the concrete cover to reinforcing steel
- $b$  is the width of the deck
- $f_c$  is the concrete compressive strength
- $\theta_w$  is the model uncertainty for loading
- $G_k$  is the applied dead super-imposed loading
- $\alpha$  is the dead to live load ratio
- $L$  is the deck span

For the strengthened deck, additional flexural capacity was achieved by bond CFRP sheets at the bottom of the deck.

$$G(X) = \theta_r A_s(t) f_y \left( d - \frac{0.88 A_s(t) f_y}{b f_c} \right) + \theta_{cf} E_{cf} \epsilon_{cf} \left( h - \frac{a}{2} \right) - 0.125 \theta_w G_k (\alpha + 1) L^2 \quad (2)$$

Where

- $\theta_{cf}$  is the model uncertainty for CFRP strength
- $E_{cf}$  is the modulus of elasticity of the CFRP sheets
- $\epsilon_{cf}$  is the strain of CFRP
- $h$  is the overall depth of the un-strengthened deck

**Shear Mode of Failure**

The reliability function for the shear mode of failure is given in equation (3) as follows

$$G(x) = V_c + V_s + V_{frp} - V_a \tag{3}$$

Where  $V_{frp}$  is the shear capacity contribution from concrete,  $V_s$  is the shear capacity contribution from steel,  $V_{frp}$  is the shear capacity contribution from CFRP.

$$V_c = 0.2 \phi_c \sqrt{f'_c} b_w d \tag{4}$$

$$V_s = \frac{\phi_s f_y A_v d}{s} \tag{5}$$

$$V_{frp} = \frac{\phi_{frp} A_{frp} E_{frp} \epsilon_{frp} d_{frp} (\sin \beta + \cos \beta)}{s_{frp}} \tag{6}$$

Where all variables are as previously defined.

**Deflection Mode Failure**

The reliability function for the deflection mode of failure is given by equation (7).

$$G(x) = \frac{1000L}{250} - \frac{5Q_k(\alpha+1)L^4 \times 10^{12}}{384(EI + E_{cf} I_{cf})} \tag{7}$$

- Where L is the deck span in m\
- $Q_k$  is the live load on deck in kN/m<sup>2</sup>
- $\alpha$  is the dead to live load ration
- E is the elastic modulus of the reinforced concrete
- I is the second moment of area of the reinforced concrete
- $E_{cf}$  is the elastic modulus of the cfrp
- $I_{cf}$  is the second moment of area of the CFRP

TABLE 1: STATISTICAL MODELS OF THE BASIC DESIGN VARIABLES

S/No	Variable	Unit	Distribution Model	Mean	Coefficient of Variation
1	$\theta_u$ , model uncertainty for the un-strengthened deck	-	Lognormal	1.0	0.10
2	$A_s(t)$ , time dependent cross sectional area	mm <sup>2</sup>	Normal	Nominal	0.10
3	$f_y$ , yield strength of the steel reinforcement	N/mm <sup>2</sup>	Lognormal	500.0	0.25
4	d, concrete cover to reinforcing steel	mm	Normal	934.0	0.05
5	b, width of the deck	mm	Normal	400.0	0.05
6	$f_c$ , concrete compressive strength	N/mm <sup>2</sup>	Lognormal	30.0	0.20
7	$\theta_{cf}$ , model uncertainty for CFRP	-	Lognormal	1.0	0.10

8	strength $E_{cf}$ , modulus of elasticity of the CFRP sheets	N/mm <sup>2</sup>	Lognormal	155000	0.20
9	$\epsilon_{cf}$ , strain of CFRP	-	Normal	Nominal	0.05
10	h, overall depth of the un-strengthened deck	mm	Normal	1000.0	0.05
11	a, area	-	Normal	Nominal	0.05
12	$\theta_k$ , angle of inclination	-	Lognormal	1.0	0.10
13	$Q_k$ , live load	kN/m	Gumbel	30.0	0.40
14	$\alpha$ , dead to live load ratio	-	Normal	Nominal	0.05
15	L, deck span	m	Normal	9.0	0.05

**D. Development of the Form Program**

Robust computer program for the reliability analysis of the flexural, shear and deflection failure modes were developed using MATLAB programming language to obtain reliability safety indices.

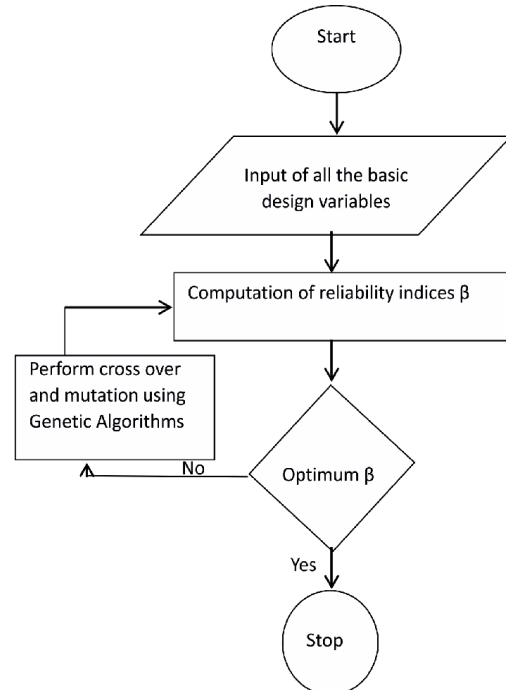


Figure 3: The Flowchart of the FORM Program

In the program, Genetic Algorithm (GA) was used to enhance the capacity of the First Order Reliability Method in the search for optimum safety indices.

In GA, a population of solution are considered at a time, and the result is updated using cross over and mutation to simulate the biological process of evolution. The flowchart of the program is as given in Figure 3. The programs consist of the main program for flexure, shear, deflection and several sub-programs.

**III. RESULTS AND DISCUSSIONS**

**A. Results of Sensitivity Analysis For Bending Mode Of Failure**

The results of the reliability and sensitivity analyses of reinforced concrete bridge deck were recorded and represented

with graphs (Figure 4 – 9) of safety indices versus load ratio for the bending, shear and deflection mode of failures.

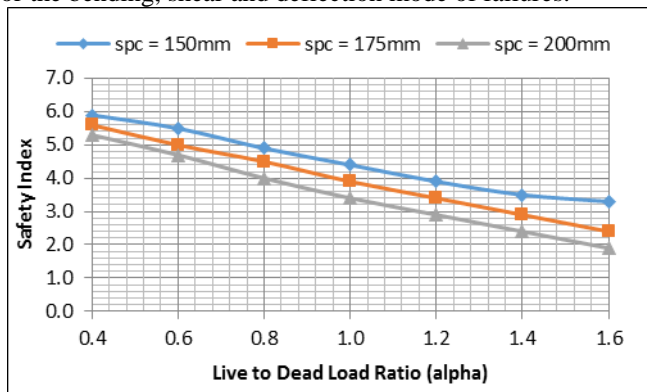


Figure 4: Relationship between safety index and live to dead load ratio for various spacing of reinforcing steel

The relationship between safety index and live to dead load ratio is presented in Figure 4. Three reinforcement spacing were considered, 150mm, 175mm and 200mm, load ratio ranging from 0.4 to 1.6 was included in the check, dead super imposed load are kept fixed and the live load comprising the HA and HB loading was varied using the load ratio. The higher the load ratio, the higher the designed load on the bridge deck.

Figure 4 shows that deviation from the design load ratio 1.0, will lead to gradual loss of safety of the deck, as the load ratio increases. For instance, changing the load ratio to 1.5 (50% increase), will lead to loss of deck safety index, from  $\beta = 3.8$  to  $\beta = 2.4$ .

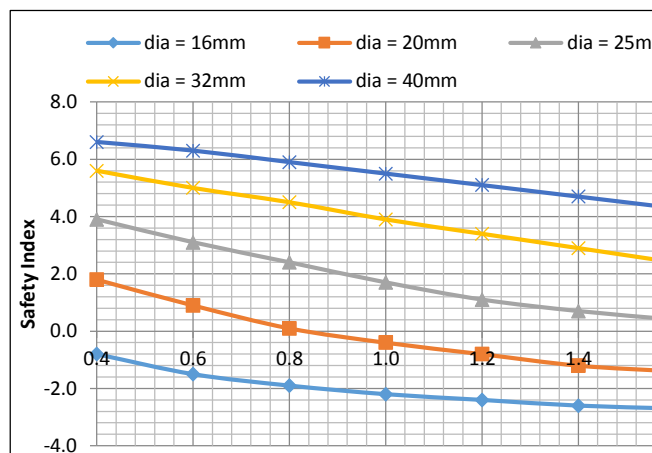


Figure 5: Relationship between safety index and live to dead load ratio for various diameter of reinforcing steel (Designed spacing of the reinforcing is 175mm)

The effect of change in the value of the diameter of the reinforcing steel is presented in Figure 5. The default (Design) diameter is 32mm. It is clear from the plot, that, the relationship between safety index and steel diameter shows non-linearity. Any decision to use steel diameter other than the one recommended in design, will lead to drastic change in structural safety. From the plot, at the default live to dead load ratio of 1.0, the inherent safety for the deck corresponds to safety index,  $\beta$  of 3.8. changing the diameter from 32mm to 25mm, 20mm and 16mm will result to drop in safety index  $\beta = 1.8, -0.2$  and  $-2.2$  respectively.

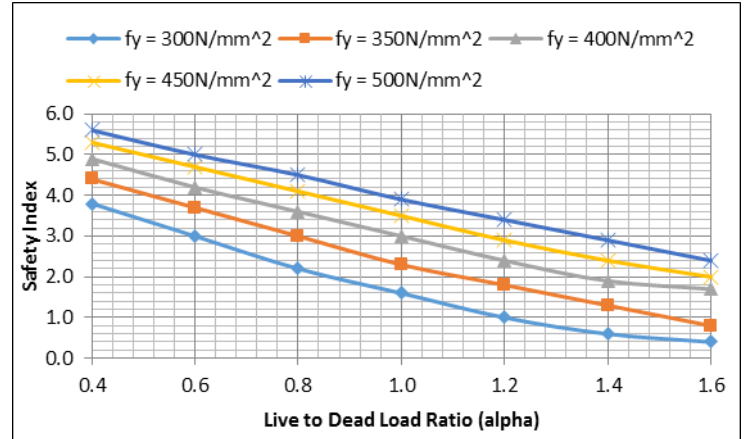


Figure 6: Relationship between safety index and live to dead load ratio for various reinforcing steel strengths (Designed spacing of the reinforcing is 175mm)

Figure 6 shows the influence of the steel strength on the overall safety of the bridge deck, and live load ratio is varied from 0.4 to 1.6 the steel strength considered in the design of the bridge deck is 500N/mm<sup>2</sup>. The inherent safety index ins.3.8 at the default live to dead load ratio. The safety index drop from 3.8 to 3.5, 3.0, 2.4 and 1.6 for steel strength 450N/mm<sup>2</sup> 400N/mm<sup>2</sup> 350N/mm<sup>2</sup> and 300N/mm<sup>2</sup> respectively. In the Eurocode 1990, and JCSS, 2001, a target safety index of 3.8 is recommended. From this investigation it is clear that, the use of substandard steel can jeopardise the safety of the bridge deck.

Generally, the result presented in Figures 4, 5 and 6 revealed that, the designed bridge deck met the target reliability, recommended in the international probabilistic model of 3.7 (JCSS, 2001). This implies that, the intact bridge deck is adequately safe. It is also clear, that so long as the design value of loading, material and geometric properties are used, the safety of the bridge is guaranteed.

#### B. Results Of Sensitivity Analysis For Shear Mode Of Failure

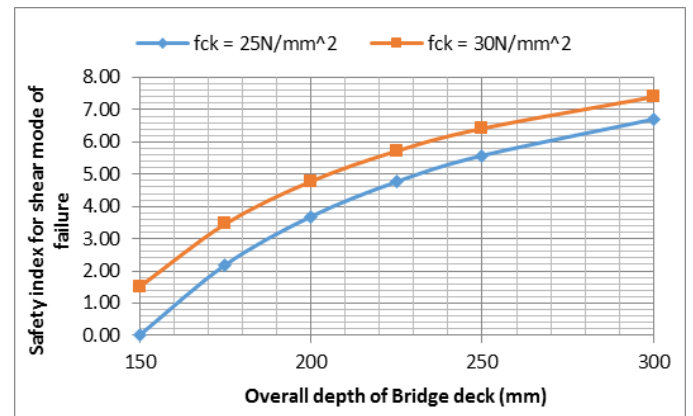


Figure 7: Relationship between shear mode safety index and depth of bridge deck for concrete grades of 25N/mm<sup>2</sup> and 30N/mm<sup>2</sup>.

Figure 7, depicts the relationship between safety index and depth of deck for the shear mode of failure considering two concrete grades 25N/mm<sup>2</sup> and 30N/mm<sup>2</sup> and it was observed that the safety index increases with increase in depth. Also, the compressive strength was observed to also influence the safety of the deck for shear mode of failure.

### C. Results Of Sensitivity Analysis For Deflection Mode Of Failure

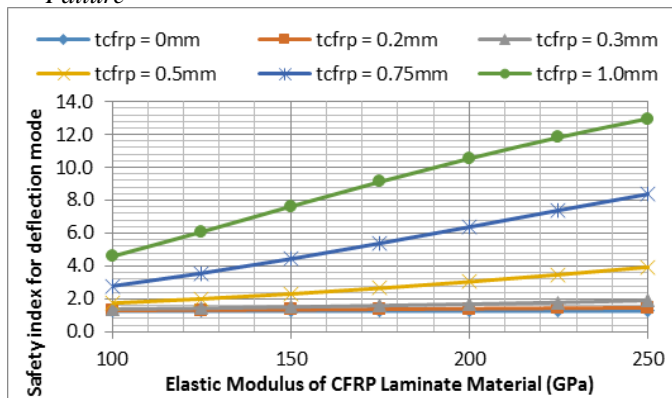


Figure 8: Relationship between flexural safety index and CFRP elastic modulus for various CFRP thicknesses

In this study, the serviceability limit state was considered for the deflection mode of failure. The stiffness of the strengthening system was considered by varying the elastic modulus of the CFRP from 100GPa to 250GPa covering the values for known CFRPs. Also, various thicknesses of the CFRP were considered ranging from 0.2mm to 1.0mm.

As observed from figure 8, the safety indices prior to application of the CFRP (thickness = 0) were very low; close to 1. As the CFRP laminate is applied with thicknesses of 0.2mm, 0.3mm, 0.5mm, 0.75mm and 1.0mm, the safety indices kept increasing. In conclusion, both the elastic modulus and thickness for CFRP have influence on the retrofitting capacity of bridge deck under corrosion. For example with CFRP thickness equal to zero and elastic modulus of 200GPa, the safety index is 1.0. However as the thickness increased from 0 to 1.0mm, the safety index for the deflection failure mode changed from 1 to 10. This result is quite interesting considering the fact that, the target safety index for serviceability limit state recommended in the Eurocode 2 (EN 1990, 2004) is 2.5.

### IV. CONCLUSIONS

1. The reliability analysis of the bridge deck using First Order Reliability Method (FORM) and enhanced with Genetic Algorithms (GAs) showed that the deck is inherently safe with safety index value of 3.8 which agreed with the recommendation of the Joint Committee of Structural Safety Code (JCSS, 2001).
2. Reliability-based analysis of the strengthened deck with Carbon Fibre Reinforced Polymer (CFRP) yielded a flexural capacity restoration of as much as 100%.
3. The sensitivity analysis for the deflection mode established that both the stiffness of CFRP and its thickness have influence on the retrofitting capacity of the CFRP for bridge deck that is about to suffer loss of serviceability due to deflection.
4. The most critical mode of failure to be strengthened is flexure.

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