# Probability Analysis Of A Cold Standby Unit System With Slow Switching And Correlated Appearance And Disappearance Of Repairman

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## Abstract

*The aim of this paper is to present a reliability analysis of a cold standby unit system with the assumption that the switching is not instantaneous .There is only one repair facility. Appearance and Disappearance of repairman are assumed to be correlated. Using regenerative point technique various reliability characteristics are obtained which are useful to system designers and industrial managers. Graphical behaviors of MTSF and profit function have also been studied.*

**Keywords:** Transition Probabilities, MTSF, Availability, Busy Period, Profit Function.



## 1. INTRODUCTION

Two identical unit cold standby systems have widely studied in literature of reliability theory, repair maintenance is one of the most important measures for increasing the reliability of the system. Many authors have studied various system models under different repair policies [1-4] ,they have assumed that appearance and disappearance times of repairman are uncorrelated random variable. But in real situation rest period of the repairman depends on workload on the repairman. They have also assumed that the switching is instantaneous but in real life this is not so. Taking these facts into consideration in this paper we investigate a two unit cold standby system model assuming the possibility of slow switching and appearance and disappearance time of repairman taken as correlated random variables having their joint distribution as bivariate exponential .

## 2. SYSTEM DESCRIPTION

System consists of two identical units , initially both units are not operative but the only one of them is sufficient for operating the system, other one is in cold standby. There is single repair facility. When one unit fails another unit takes the charge but switching is not instantaneous. The joint distribution of appearance and disappearance of repairman is taken to be bivariate exponential having density function. Each repaired unit works as good as new.

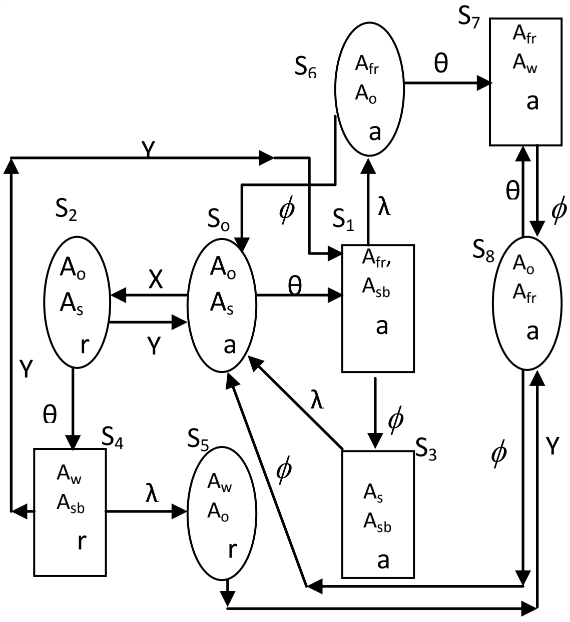


Fig.1 : Transition Diagram

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## 3. NOTATIONS

For defining the states of the system we assume the following symbols:

**$A_0$ :** Unit A is in operative mode

**$A_S$:** Unit A is in standby mode

**$A_{fr}$ :** Unit A is in failure mode

**$A_{sb}$ :** Unit A(stand by) is being switched

$\phi$ **:** Constant rate of repair of unit A

$\theta$ **:** Constant rate of failure of unit A

$\lambda$ **:** Constant rate of switching

**$A_w$:** Unit A in failure mode but in waiting for repairman

**X:** Random variables representing the disappearance of repair man

**Y:** Random variables representing the appearance of repair man

**$f_i$(x,y)** Joint pdf of $(x_i, y_i)$;i=1,2

$$= \alpha_i \beta_i (1-r_i) e^{-\alpha_i x - \beta_i y} I_0 (2\sqrt{(\alpha_i \beta_i r_i xy)}); X, Y,$$
$$0 \le r_i < 1,$$
$$where I_0(2\sqrt{\alpha_i \beta_i r_i xy}) = \sum_{j=0}^{\infty} \frac{(\alpha_i \beta_i r_i xy)^j}{(j!)^2}$$

**$k_i$(Y/X):** Conditional pdf of $Y_i$ given $X_i$=x is given by

$$= \beta_i e^{-\alpha_i r_i x - \beta_i y} I_0 (2\sqrt{(\alpha_i \beta_i r_i xy)})$$

**$g_i$(.):** Marginal pdf of $X_i = \alpha_i (1-r_i) e^{-\alpha_i (1-r_i) x}$

**$h_i$(.):** Marginal pdf of $Y_i = \beta_i (1-r_i) e^{-\beta_i (1-r_i) y}$

$q_{ij}(.)$, pdf &cdf of transition time from regenerative states pdf &cdf of transition time from regenerative state $S_i$ to $S_j$.

$\mu_i$ : Mean sojourn time in state$S_i$.

$\oplus$ : Symbol of ordinary Convolution $A(t) \oplus$

$$B(t) = \int_0^t A(t-u) B(u) du$$

$\otimes$ : symbol of stieltjes convolution

$$A(t) \otimes B(t) = \int_0^t A(t-u) dB(u)$$

### 3.1 Transition Probability and Sojourn Times

The steady state transition probability can be as follows

$P_{01} = \frac{\theta}{\theta + \alpha(1-r)}$

$P_{02} = \frac{\alpha(1-r)}{\theta + \alpha(1-r)}$

$P_{16} = \frac{\lambda}{\lambda + \phi}$

$P_{13} = \frac{\phi}{\lambda + \phi}$

$P_{20} = \frac{\beta(1-r)}{\beta(1-r) + \theta}$

$P_{24} = \frac{\theta}{\beta(1-r) + \theta}$

$P_{18.67} = \frac{\lambda\theta}{(\lambda + \phi)(\lambda + \theta)}$

$P_{10.6} = \frac{\lambda\phi}{(\lambda + \phi)(\lambda + \theta)}$

$P_{10.6} = \frac{\phi}{(\lambda + \phi)}$

$P_{32.6} = \frac{\alpha_2(1-r_2)}{\gamma_1 + \alpha_2(1-r_2)}$

$P_{40} = \frac{\gamma_2}{\gamma_2 + \alpha_1(1-r_1)}$

$P_{41.5} = \frac{\alpha_1(1-r_1)}{\gamma_2 + \alpha_1(1-r_1)}$

$P_{01}+P_{02}=1$

$P_{16}+P_{13}=1$

$P_{41}+P_{48.5}=1$

$P_{18.67}+P_{10.6}+P_{10.3}=1$

$P_{20}+P_{24}=1$

$P_{80}+P_{88.7}=1$

**(01-20)**

**Mean sojourn times:**

$$\mu_0 = \frac{1}{\alpha(1-r) + \theta}$$
$$\mu_2 = \frac{1}{\beta(1-r) + \theta}$$
$$\mu_1 = \frac{1}{\lambda + \phi}$$
$$\mu_8 = \mu_6 = \frac{1}{\theta + \phi}$$

**(21-24)**

## 4. ANALYSIS OF CHARACTERISTICS

### 4.1 MTSF (Mean Time to System Failure)

To determine the MTSF of the system, we regard the failed state of the system as absorbing state, by probabilistic arguments, we get

$$\phi_0 = Q_{01} \otimes + Q_{02} \otimes \phi_2(t)$$
$$\phi_2 = Q_{24} + Q_{20} \otimes \phi_0(t)$$

Taking Laplace Stieltjes transforms of these relations and solving for $\phi_0^{**}(s)$,

$$\phi_0^{**}(s) = \frac{N(s)}{D(s)}$$

**(25-27)**

Where

$$N = \mu_1 + \mu_2 P_{02}$$

$$D = 1 - P_{10}P_{01} - P_{20}P_{02} - P_{30}P_{03} - P_{40}P_{04}$$

**(28-29)**

### 4.2 Availability Analysis

Let $A_i(t)$ be the probability that the system is in up-state at instant t given that the system entered regenerative state i at t=0.using the arguments of the theory of a regenerative process the point wise availability $A_i(t)$ is seen to satisfy the following recursive relations

$$A_0 = M_0(t) + q_{01} \oplus A_1(t) + q_{02} \oplus A_2(t)$$

$$A_1 = q_{10.6} \oplus A_0(t) + q_{18.67} \oplus A_8(t) + q_{10.3} \oplus A_0(t)$$

$$A_2 = M_2(t) + q_{20} \oplus A_0(t) + q_{24} \oplus A_4(t)$$

$$A_4 = q_{41} \oplus A_1(t) + q_{48.5} \oplus A_8(t)$$

$$A_8 = q_{88.7} \oplus A_8(t) + q_{80} \oplus A_0(t)$$

**(30-34)**

Now taking Laplace transform of these equations and solving them for $A_0^*(s)$, We get

$$A_0^*(s) = \frac{N_1(s)}{D_1(s)}$$

**(35)**

The steady state availability is

$$A_0 = \lim_{s\to 0}(sA_0^*(s)) = \frac{N_1}{D_1}$$

Where

$$N_1 = P_{80}(\mu_0 + \mu_2 P_{02})]$$

$$D_1 = P_{80}[\mu_0 + \mu_1(P_{01} + P_{02}P_{41}P_{24}) + \mu_2 P_{02} + \mu_4 P_{02}P_{24}]$$
$$+ \mu_8(P_{01}P_{18.67} + P_{02}P_{24}P_{48.5} + P_{02}P_{24}P_{41}P_{18.67})$$

**(36-38)**

### 4.3 Busy Period Analysis of The Repairman

Let $B_i(t)$ be the probability that the repairman is busy at instant t, given that the system entered regenerative state I at t=0.By probabilistic arguments we have the following recursive relations for $B_i(t)$

$$B_0 = q_{01} \oplus B_1(t) + q_{02} \oplus B_2(t)$$

$$B_1 = W_1(t) + q_{10.6} \oplus B_0(t) + q_{18.67} \oplus B_8(t) + q_{10.3} \oplus B_0(t)$$

$$B_2 = q_{20} \oplus B_0(t) + q_{24} \oplus B_4(t)$$

$$B_4 = q_{41} \oplus B_1(t) + q_{48.5} \oplus B_8(t)$$

$$B_8 = W_8(t) + q_{88.7} \oplus B_8(t) + q_{80} \oplus B_0(t)$$

(39-43)

Taking Laplace transform of the equations of busy period analysis and solving them for $B_0^*(s)$ ,we get

$$B_0^*(s) = \frac{N_2(s)}{D_1(s)}$$

**(44)**

In the steady state

$$B_0 = \lim_{s\to 0}(sB_0^*(s)) = \frac{N_2}{D_1}$$

**(45)**

Where $N_2 = \mu_1(P_{01}P_{80} + P_{02}P_{24}P_{80})$
$+ \mu_8(P_{01}P_{18.67} + P_{02}P_{24}P_{48.5} + P_{02}P_{24}P_{41}P_{18.67})$

**(46)**

$D_1$ is already specified.

### 4.4 Expected Number of Visits by the Repairman

We defined as the expected number of visits by the repairman in (0,t],given that the system initially starts from regenerative state $S_i$

By probabilistic arguments we have the following recursive relations for $V_i(t)$

$$V_0(t) = q_{01} \otimes (1 + V_1(t)) + q_{02} \otimes V_2(t)$$

$$V_1(t) = q_{10.6} \otimes V_0(t) + q_{18.67} \otimes V_8(t) + q_{10.3} \otimes V_0(t)$$

$$V_2(t) = q_{20} \otimes V_0(t) + q_{24} \otimes V_4(t)$$

$$V_4(t) = q_{41} \otimes (1 + V_1(t)) + q_{48.5} \otimes (1 + V_8(t))$$

**(47-50)**

Taking Laplace stieltjes transform of the equations of expected number of visits

And solving them for $V_0^{**}(s)$ , we get

$$V_0^{**}(s) = \frac{N_3(s)}{D_1(s)} \quad (51)$$

In steady state

$$V_0 = \lim_{s\to 0}(sV_0^*(s)) = \frac{N_3}{D_1} \quad (52)$$

Where

$$N_3 = \mu_0(1+P_{24})P_{80} + \mu_2(P_{02} + P_{41}P_{88.7} + P_{02}P_{48.5}P_{88.7}) + (\mu_4)P_{02}P_{24}P_{80} + \mu_8(1+P_{24}P_{02}) \quad (53)$$

$D_1$ is already specified

## 5. PROFIT ANALYSIS

The expected total profit incurred to the system in steady state is given by

$$P = C_0A_0 - C_1B_0 - C_2V_0 \quad (54)$$

Where

$C_0$ =Revenue/unit uptime of the system

$C_1$ =Cost/unit time for which repairman is busy

$C_2$ =Cost/visit for the repairman

## 6. CONCLUSION

For a more clear view of the system characteristics w.r.t. the various parameters involved, we plot curves for MTSF and profit function in figure-2 and figure-3 w.r.t the failure parameter ($\theta$) of unit A for three different values of correlation coefficient ($r_1$ =0.25, $r_2$ =0.50, $r_3$ =0.75), between X and Y , while the other parameters are kept fixed as

$$\alpha = .005, \beta = .02, \lambda = 0.001, C_0 = 400,$$
$$C_1 = 200, C_2 = 40, \phi = .004$$

From the fig.-2 it is observed that MTSF decreases as failure rate increases irrespective of other parameters. the curves also indicates that for the same value of failure rate,MTSF is higher for higher values of correlation coefficient(r),so here we conclude that the high value of r between appearance and disappearance tends to increase the expected life time of the system.From the fig.-3 it is clear that profit decreases linearly as failure rate increases. Also for the fixed value of failure rate, the profit is higher for high correlation (r).From the fig.-4 it is clear that profit decreases linearly as disappearance rate of repairman increases. Also for the fixed value of disappearance rate, the profit is higher for high correlation (r).

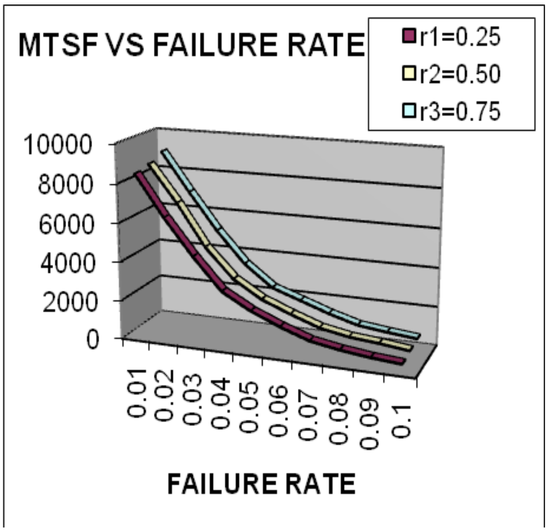


Fig. 2:MTSF vs Failure Rate

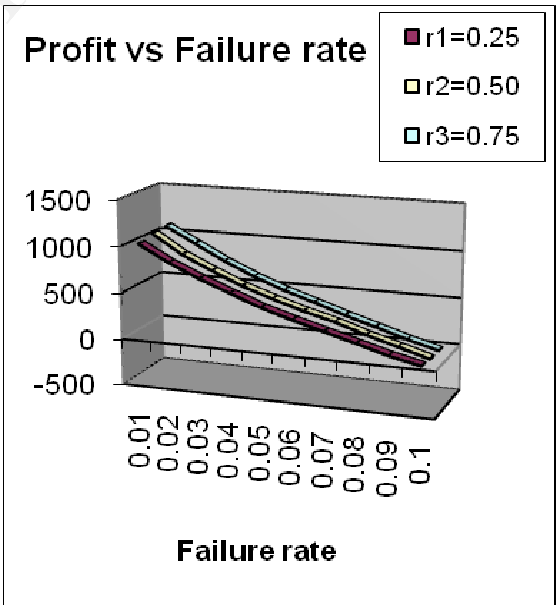


Fig. 3:Profit vs Failure Rate

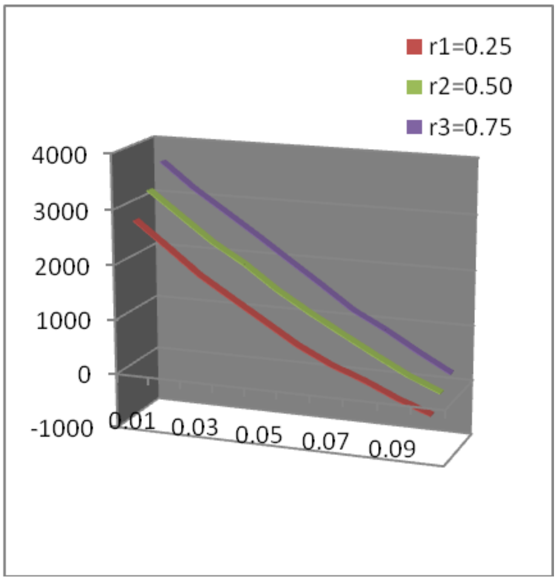


Fig. 4:Profit vs Disappear Rate

## 7. REFERENCES

[1] Goyal,V.and Murari,K."cost anlysis of a two unit standby system with regular repairman and patience time ", *microelectron.reliab*.vol.25 no.1,pp. 453-459,1984.

[2] Tuteja,R.K., Taneja, G and Vashishta, Upasana, "Cost benefit analysis of a system where operation and sometimes repair of main-unit depends on sub-unit*".Pure and Applied Mathematica Sciences,* LIII (1-2), 41-61,2001.

[3] Singh, R. P., Taneja, G. and Siwach, B. S. "Two unit warm standby system with accident and various types of repair". *Journal of Decision and Mathematical Sciences*, 5-6(1-3), pp75-86,2000.

[4] Vishal Sharma and Gaurav Varshney, *"Reliability analysis of an emergency shutdown system model in an industrial plant", RdE J. of Mathematical Sciences*, Vol.1, pp.71-82,2006.

[5] Rakesh Gupta and Gaurav Varshney, *"A two non–identical unit parallel system with Geometric failure and repair time distributions", IAPQR Trans.* Vol. 31, No. 2, pp 127–139,2007 .

[6] Pradeep Chaudhary and Dharmendra Kumar, "Stochastic analysis of a two unit cold standby system with different operative modes and different repair policies", *Int. J. Agricult.Stat. Sciences*, Vol. 3, No. 2, pp. 387-394, (2007), .

[7] Rakesh Gupta and Kailash Kumar, "Cost benefit analysis of distillery plant system", *Int. J.Agricult. Stat. Sciences*, Vol. 3, No. 2, pp. 541-554 ,2007.

[8] Amir Azaron ,Hideki Katagiri ,Kosuke Kato, Masatoshi Sakawa, "Reliability Evaluation and Optimization Of Dissimilar-Component Cold-Standby Redundant Systems*", Journal of the Operations Research Society of Japan* , Vol. 48, No. 1, 71-8, (2005)

[9] Attahiru Sule Alfa WL and Zhao, Y Q ,"Stochastic analysis of a repairable system with three units and repair facilities" *Microelectron. Reliab.* , 38(4), (1998),585-595, (1998).

[10] Chao, A. and Hwang, W. D., "Bayes estimation of reliability for special k-out-of-m:G systems*", IEEE Transactions on Reliability*, R-33, 246-249,(1983).