

# Prominent Ambiguity Resolution Methods For TEC Estimation Using GPS Data

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## Abstract

*The Global Positioning System (GPS) is a satellite based navigation system operated by the U.S. Department of Defense (DoD). The positional accuracy of GPS is affected by various error sources such as satellite and receiver clocks, ionosphere, troposphere, multipath, receiver measurement noise and instrumental biases. The ionospheric delay is the most predominant of all the errors and is a function of the total electron content (TEC). TEC can be estimated using the dual frequency code and/or carrier phase observations. The carrier phase observations are more precise than the code observations, but are affected by the integer cycle ambiguities. In order to take full advantage of the precision of the carrier phase data, the integer ambiguities must be resolved. Several methods for resolving integer ambiguities, based on multiple station data are reported in the literature. However, it is difficult to obtain real time TEC measurements using the multi-station techniques. In this paper, a prominent single-station technique for estimation of TEC using the carrier phase data is used. The smoothed code observables are used to reduce the effect of code measurement noise. The results of TEC estimation are validated using the Bernese GPS software.*

## 1. Introduction

The Global Positioning System (GPS) is a passive (one-way) ranging system. The satellites broadcast ranging signals and navigation data on L-band frequencies. The user receivers convert signal information into position, velocity and time estimates. GPS was designed as a “dual-use” (military and civilian) system and achieved full operational capability (FOC) on July 17, 1995 with 24 Block II/IIA satellites [1]. The basic measurement made by a GPS receiver is the time taken for a signal to propagate from the GPS satellite to the receiver. Knowing the signal transit (travel) time, the distance (range) between the satellite and the receiver can be determined, by

multiplying the transit time with the speed of light. Using a principle called trilateration, the user position in three dimensions could be computed with the knowledge of the range measurement to three satellites, with known satellite positions. As the receiver and satellite clocks are not perfectly synchronized, another range measurement from a fourth satellite is needed to estimate the receiver clock bias [2]. The measured range differs from the true range due to several errors, and hence is known as pseudorange.

The ionosphere is the largest source of error in GPS positioning and navigation. It is a region of ionized particles, with equal number of free electrons and positive ions that control the behaviour of radio waves. The ionosphere extends from about 50 km to more than 1000 km above the earth's surface. In the ionosphere, the principal source of ionization is the electromagnetic radiation from the sun. As the GPS signal travels through the ionosphere, it is delayed by the effect of free electrons. This delay due to the ionosphere changes the transit time, and therefore, the apparent range to the satellite. The ionospheric delay error depends on the density of free electrons along the propagation path from the satellite to the receiver, and can vary from a few metres to more than fifteen metres within a day. A single frequency GPS receiver can be used to estimate the ionospheric delay, but it can remove only 60% of the error [3]. The dual frequency GPS observables, viz. code and carrier phase can be used to estimate the ionospheric delay more accurately. The carrier phase measurements can precisely track TEC. In order to fully exploit the high accuracy of the carrier phase data, the integer ambiguities present in them must be resolved to their correct integer value since one cycle on L1 carrier may translate, to a position error of about 19 cm.

## 2. Review of ambiguity resolution techniques

Carrier phase ambiguity resolution is the key element for high precision positioning. Integer ambiguity refers to the unknown number of integer carrier cycles lapsed before the GPS receiver locks into the incoming carrier. This phase ambiguity remains

constant as long as no loss of signal occurs and also if there is no relative motion between the satellite and the receiver [4].

The basic approaches for resolving integer ambiguity include:

- i) Resolving ambiguity with single frequency phase data,
- ii) Resolving ambiguity with dual frequency phase data,
- iii) Resolving ambiguity by combining the dual frequency carrier phase and code data,
- iv) Resolving ambiguity by combining the triple frequency carrier phase and code data.

Ambiguity resolution (AR) techniques based on measurement information utilization can be classified as [5],

- a) Ambiguity resolution in the measurement domain,
- b) Search technique in the coordinate domain, and
- c) Search technique in the ambiguity domain.

The first category of AR techniques uses the pseudoranges to find out the ambiguities of the corresponding carrier phase observables. The basic approaches (i-iv), fall in this category. The second category of AR algorithms is the one developed initially, viz. Ambiguity Function Method (AFM). This method uses fractional value of instantaneous carrier-phase measurements [6]. The third category of AR techniques, are based on the theory of integer least squares. Here, parameter estimation is carried in three steps, viz. float solution, integer ambiguity estimation, and fixed solution. Each technique uses diverse ambiguity search processes at the integer estimation step [5].

The following are some of the AR techniques in the third category:

- a) Least Squares Ambiguity Search Technique
- b) Fast Ambiguity Resolution Approach
- c) Least-Squares AMBiguity Decorrelation Adjustment
- d) Fast Ambiguity Search Filter
- e) Optimal Method for Estimating GPS Ambiguities

In this investigation, a single-station technique for resolving ambiguities using dual frequency GPS data is used. This technique falls under the first category, i.e. AR in measurement domain. Both dual frequency code and carrier phase observations are used to determine the ambiguities of the corresponding carrier phase observations.

### 3. TEC estimation algorithm using single station carrier phase observations

TEC can be estimated using the dual frequency code and/or carrier phase data taking advantage of the dispersive nature of the ionosphere. The TEC estimates obtained from the code observables are unambiguous, but coarse in nature. On the other hand, that obtained from the carrier phase observables are more precise, but ambiguous. A single-station method based on carrier phase ambiguity resolution is adopted for estimating the line-of-sight TEC from dual frequency GPS data.

The dual frequency code and carrier phase observables on  $f_1$  (1575.42 MHz) and  $f_2$  (1227.60 MHz) can be expressed as [7],

$$P_1 = \rho + \frac{40.3TEC}{f_1^2} + \varepsilon_1 \quad (1)$$

$$P_2 = \rho + \frac{40.3TEC}{f_2^2} + \varepsilon_2 \quad (2)$$

$$L_1 = \rho - \frac{40.3TEC}{f_1^2} - \lambda_1 N_1 - \zeta_1 \quad (3)$$

$$L_2 = \rho - \frac{40.3TEC}{f_2^2} - \lambda_2 N_2 - \zeta_2 \quad (4)$$

where,  $P_1, P_2$ : measured pseudoranges (m),

$\rho$ : sum of geometric range, tropospheric error, and clock error (m),

$TEC$ : total electron content in the path of observation (electrons/m<sup>2</sup>),

$\varepsilon_1, \varepsilon_2$ : sum of all errors due to instrumental delays, multipath, and random noise for  $f_1$  and  $f_2$  code measurements (m),

$\zeta_1, \zeta_2$ : sum of all errors due to instrumental delays, multipath, and random noise for  $f_1$  and  $f_2$  phase measurements (m),

$N_1, N_2$ : phase ambiguities for  $f_1$  and  $f_2$  signal (cycles),

$f_1, f_2, \lambda_1, \lambda_2$ : frequency and wavelength of  $f_1$  and  $f_2$  GPS signals.

Differencing the code and carrier phase measurement equations, frequency independent terms such as tropospheric delay, satellite and receiver clock errors are eliminated. We define four quantities namely  $A_p, B_p, C_L$  and  $D_L$ , which are linear combination of dual frequency code and carrier phase observables as [7],

$$A_p = \frac{f_1^2 P_1 - f_2^2 P_2}{f_1^2 - f_2^2} \quad (5)$$

$$B_p = \frac{P_2 - P_1}{CT} \quad (6)$$

$$C_L = \frac{f_1^2 L_1 - f_2^2 L_2}{f_1^2 - f_2^2} \quad (7)$$

$$D_L = \frac{L_1 - L_2}{CT} \quad (8)$$

where

$$CT = 40.3 \frac{f_1^2 - f_2^2}{f_1^2 f_2^2} \quad (9)$$

By inserting Eqs. (1), (2), (3) and (4) into Eqs. (5), (6), (7) and (8) and solving, we get the two equations with two unknowns  $N_1, N_2$ .

$$A_p - C_L = \frac{f_1^2 \lambda_1 N_1 - f_2^2 \lambda_2 N_2}{f_1^2 - f_2^2} + \frac{f_1^2 b_1 - f_2^2 b_2}{f_1^2 - f_2^2} + \frac{f_1^2 m_1 - f_2^2 m_2}{f_1^2 - f_2^2} \quad (10)$$

$$B_p - D_L = \frac{\lambda_1 N_1 - \lambda_2 N_2 + b_2 - b_1 + m_2 - m_1}{CT} \quad (11)$$

where notation  $b$  and  $m$  is used to designate the instrumental delays and multipath respectively.

Solving equations (10), (11), for  $N_1, N_2$  we get

$$N_1 = \frac{X - f_2^2 Y}{\lambda_1 (f_1^2 - f_2^2)} \quad (12)$$

$$N_2 = \frac{X - f_1^2 Y}{\lambda_2 (f_1^2 - f_2^2)} \quad (13)$$

where

$$X = [(A_p - C_L)(f_1^2 - f_2^2)] - (f_1^2 b_1 - f_2^2 b_2) - (f_1^2 m_1 - f_2^2 m_2)$$

$$Y = [(B_p - D_L)CT] - (b_2 - b_1) - (m_2 - m_1)$$

Knowing  $N_1$  and  $N_2$ , one can estimate line of sight TEC as,

$$TEC = \frac{(L_1 - L_2) + (\lambda_1 N_1 - \lambda_2 N_2)}{CT} \quad (14)$$

The ambiguity free  $L_1$  is  $L_1 + \lambda_1 N_1$ , and ambiguity free  $L_2$  is  $L_2 + \lambda_2 N_2$ . Despite continuing improvements in the GPS receivers, multipath signal propagation has remained a dominant cause of error in positioning. Multipath is mainly caused due to the reflecting surfaces near the receiver. Multipath error on both  $f_1$  and  $f_2$  signals has been found using the Translations, Editing Quality Check (TEQC) software available in public domain [8]. The satellite instrumental biases determined by the Centre for Orbit Determination (CODE), Europe are considered. These are computed using data from several receivers in the International GNSS (IGS) network [9].

#### 4. Significance of TEC estimation for SBAS applications

The GPS Standard Positioning Service (SPS) can provide a horizontal accuracy of about 13 m and vertical accuracy of about 22 m with 95% probability level [10]. These estimates are for the signal-in-space (SIS) only and the contribution of the various error sources is not included. The current level of accuracy, integrity and availability provided by the standalone GPS does not meet the more stringent air navigation requirements, particularly during the critical phases of flight like non-precision and precision approaches. To overcome these deficiencies of GPS and to use it for all phases of flight, augmentation systems have been proposed [11]. A Satellite Based Augmentation System (SBAS) is based on a technique known as Differential GPS and is intended to serve a large country or a continent. It uses data collected by a number of widely separated ground reference stations to compute error corrections that are broadcasted to users via geostationary (GEO) satellites.

An SBAS system supplements GPS with three services: additional ranging signals through the use of geostationary satellites, a differential corrections service that can enable more accurate positioning, and integrity alerts to protect users from the effects of erroneous GPS signals. The differential corrections include satellite clock corrections, a correction for the three-dimensional position of the satellite, and a set of corrections for the ionospheric delay. The SBAS system will enable GPS to be used as the primary navigational aid in civil aviation for all phases of the flight from takeoff through Category-I precision approach. In addition, SBAS can also provide benefits beyond aviation to all modes of transportation including maritime, highways, and railroads [12]. Around the world, different countries including USA, Europe, Japan and India have planned to develop SBASs to meet the navigation accuracy requirements of civil aviation [13-15]. The U.S. SBAS known as

Wide Area Augmentation System (WAAS) has matured through development stage and is progressing through operational implementation. In India, the Airports Authority of India (AAI) and Indian Space Research Organisation (ISRO) are jointly implementing a SBAS named as GPS Aided Geo Augmented Navigation (GAGAN), to meet civil aviation requirements for various phases of a flight, over the Indian airspace. An important component in the GAGAN implementation programme is the generation and transmission of accurate ionospheric corrections to the users via a geostationary satellite. To develop suitable TEC models, dual frequency receivers are installed at various airports over the Indian region [15]. The dual frequency GPS data of Hyderabad station (78.47°E, 17.45°N) is used in this analysis.

## 5. Results and Discussion

The results of TEC obtained from the algorithm, considering the Hyderabad station (78.47°E, 17.45°N) data of GAGAN network, are presented. The dual frequency GPS observation data in the Receiver Independent Exchange (RINEX) format is used [16]. This provides the dual frequency GPS code and carrier phase observables. Figure 1 shows the slant TEC obtained using the code observables. The line-of-sight code TEC is found to vary between 35.56 TECU and -6.911 TECU. The corresponding TEC obtained using the raw carrier phase observables are shown in Figure 2. Due to the inherent integer ambiguities in the carrier phase data, these provide only a relative estimate of TEC. The TEC due to carrier phase measurements are found to vary between -5.183 TECU and -45.83 TECU.

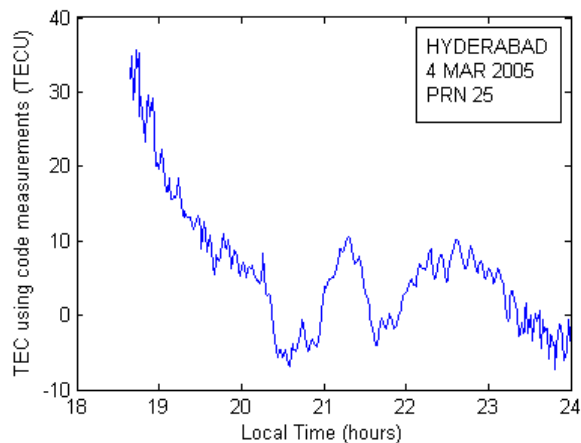


Figure 1. Slant TEC using code observations

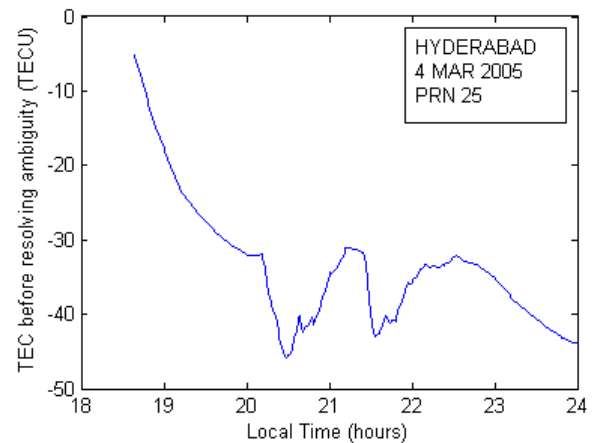


Figure 2. Slant TEC using carrier phase observations before ambiguity resolution

In order to determine the absolute TEC, the integer ambiguities,  $N_1$ ,  $N_2$  (on  $f_1$  and  $f_2$  signals) are computed. In order to reduce the effect of receiver measurement noise, the code measurements are smoothed using a Hatch filter [17] and used for the estimation of integer ambiguities. The mean value of the multipath error on the two frequencies ( $f_1$ ,  $f_2$ ) are found to be  $m_1 = 0.177706$  m and  $m_2 = 0.251150$  m, respectively using the TEQC software. The satellite differential instrumental bias for the satellite (PRN 25) considered is -2.932 ns. Figure 3 shows the variation of slant TEC after resolving the integer ambiguity using Eq.(14). After resolving integer ambiguity, TEC is found to vary between 34.72 TECU and -5.923 TECU.

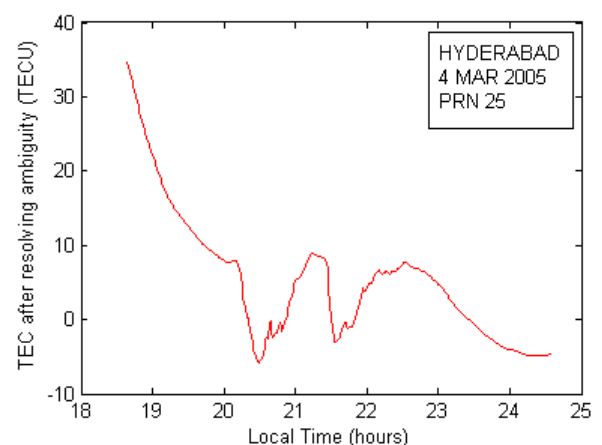


Figure 3. Absolute slant TEC after resolving integer ambiguities

**Table 1. Comparison of results due to TEC algorithm and Bernese software (PRN2)**

Inputs (RINEX observation data)	Outputs		
	Parameter	TEC algorithm	Bernese software
$P_1 = 23822957.75800$ m	Ambiguity free $L_1$ (cycles)	125190418.072	125190416.888
$P_2 = 23822962.07740$ m	Ambiguity free $L_2$ (cycles)	97550957.734	97550956.376
$\phi_1 = -5352732.11200$ cycles	Slant TEC (TECU)	40.41	41.43
$\phi_2 = -4170965.62440$ cycles			

**Table 2. Comparison of results due to TEC algorithm and Bernese software (PRN9)**

Inputs (RINEX observation data)	Outputs		
	Parameter	TEC algorithm	Bernese software
$P_1 = 21639441.20300$ m	Ambiguity free $L_1$ (cycles)	113715979.795	113715980.074
$P_2 = 21639444.37340$ m	Ambiguity free $L_2$ (cycles)	88609841.704	88609841.547
$\phi_1 = -18561100.92600$ cycles	Slant TEC (TECU)	29.48	30.35
$\phi_2 = -14463178.45340$ cycles			

In order to validate the performance of the TEC algorithm, the Bernese GPS software (version 4.2) is used [18]. Table 1 and 2 compare the ambiguity-free slant TEC obtained from the TEC algorithm with that obtained from Bernese software, due to two satellites, PRN2 and PRN9 respectively, for a particular epoch. It can be observed that the difference in TEC due to the two methods is around 1 TECU.  $P_1$  and  $P_2$  are the code measurements on the GPS  $L_1$  and  $L_2$  frequencies.  $\phi_1$  and  $\phi_2$  are the corresponding carrier phase measurements.

## 6. Conclusions

In order to take full advantage of the carrier phase observables in the estimation of TEC, resolution of integer ambiguities plays a very significant role. Here, ambiguity has been resolved using one of the prominent measurement domain techniques. The TEC estimates are validated using the Bernese GPS software. The results of carrier phase derived TEC due the single-station ambiguity resolution method show good correlation with the TEC obtained due to code observations. The significance of this technique is that it is based on single station data, and can be used for real time high precision navigation applications.

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