Q- Fuzzy Soft Ring

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***Abstract:* - In this paper, the study of Q- fuzzy soft ring by combining soft set theory. The notions of Q- fuzzy soft ring as defined and several related properties and structural characteristics are investigated some related properties., then the definition of Q- fuzzy soft ring and the theorem of homomorphic image and homomorphic pre-image are given.**

Keywords: Soft set, Fuzzy soft set, soft ring, Fuzzy soft ring, soft homomorphism, Fuzzy Soft isomorphism,Q-fuzzy set , Q-Fuzzy soft ring

1. INTRODUCTION

The concept of soft sets was introduced by Molodtsov [3] in 1999, soft sets theory has been extensively studied by many authors. It is well known that the concept of fuzzy sets, introduced by Zadeh [9], has been extensively applied to many scientific fields.

In 1971, Rosenfeld [4] applied the concept to the theory of groupoids and groups. Inan et al. have already introduced the definition of fuzzy soft rings and studied some of their basic properties. A.Solairaju and R.Nagarajan[7] have introduced and defined a new algebraic structure called Q-fuzzy subgroups. (Dr.N.sarala and B.Suganya, 2014) presented some properties of fuzzy soft groups . further (Dr.N.sarala and B.Suganya, 2014) introduced on normal fuzzy soft groups.

In this paper, we study Q-fuzzy soft ring theory by using fuzzy soft sets and studied some of algebraic properties.

2. PRELIMINARIES

In this section, we first recall the basic definitions related to fuzzy soft sets which would be used in the sequel.

Definition2.1

Suppose that U is an initial universe set and E is a set of parameters, let P(U) denotes the power set of U .A pair (F,E) is called a ***soft set*** over U where F is a mapping given by F: E→ P(U) .

Clearly, a soft set is a mapping from parameters to P(U), and it is not a set, but a parameterized family of subsets of the Universe.

Definition 2.2.

Let U be an initial Universe set and E be the set of parameters. Let $A \subset E$. A pair (F, A) is called ***fuzzy soft set*** over U where F is a mapping given by F: $A \to I^U$, where I^U denotes the collection of all fuzzy subsets of U.

Definition 2.3

Let X be a group and (F,A) be a soft set over X. Then (F,A) is said to be a ***soft group*** over X iff F(a) < X, for each $a \in A$.

Definition 2.4

Let X be a group and (f, A) be a fuzzy soft set over X. Then (f, A) is said to be a ***fuzzy soft group*** over X iff for each $a \in A$ and $x, y \in X$,

(i) $f_a(x \,.\, y) \geq T\,(f_a(x), f_a(y)$

(ii) $f_a(x^{-1}) \geq f_a(x)$

That is, for each $a \in A$, f_a is a fuzzy subgroup in Rosenfeld's sense [4]

Definition 2.5

Let (f,A) be a soft set over a ring R. Then (f, A) is said to be a ***soft ring*** over R if and only if f(a) is sub ring of R for each $a \in A$.

Definition 2.6

Let R be a soft ring. A fuzzy set 'μ' in R is called ***fuzzy soft ring*** in R if

(i) $\mu((x+y)) \geq T\{\mu(x), \mu(y)\}$

(ii) $\mu(-x) \geq \mu(x)$ and

(iii) $\mu((xy)) \geq T\{\mu(x), \mu(y)\}$, for all $x,y \in R$.

Definition 2.7

Let (φ,ψ): $X \to Y$ is a fuzzy soft function, if φ is a homomorphism from $x \to y$ then (φ,ψ) is said to be ***fuzzy soft homomorphism***. if φ is a isomorphism from $X \to Y$ and ψ is 1-1 mapping from A on to B then (φ,ψ) is said to be ***fuzzy soft isomorphism***.

3. Q-FUZZY SOFT RINGS

Definition 3.1:

Let R be a soft ring. A fuzzy set 'μ' in R is called ***Q- fuzzy soft ring*** in R if

(i) $\mu((x+y), q) \geq T\{\mu(x,q), \mu(y,q)\}$

(ii) $\mu(-x, q) \geq \mu(x,q)$ and

(iii) $\mu((xy), q) \geq T\{\mu(x,q), \mu(y,q)\}$, for all $x,y \in R$. & $q \in Q$

Proposition 3.1:

Every imaginable Q- fuzzy soft ring μ is a Q-fuzzy soft ring of R.

Proof:

Assume that μ is imaginable Q- fuzzy soft ring of R, then we have

$\mu((x+y), q) \geq T\{\mu(x,q), \mu(y,q)\}$

$\mu(-x, q) \geq \mu(x,q)$ and

$\mu((xy), q) \geq T\{\mu(x,q), \mu(y,q)\}$, for all $x,y \in R$. & $q \in Q$

Since μ is imaginable, we have

$$\min\{\mu(x,q), \mu(y,q)\} = T\{\min\{\mu(x,q), \mu(y,q)\}, \min\{x,q), \mu(y,q)\}\}$$
$$\leq T\{\mu(x,q), \mu(y,q)\}$$
$$\leq \min\{\mu(x,q), \mu(y,q)\}$$

and so

$$T\{\mu(x,q), \mu(y,q)\} = \min\{\mu(x,q), \mu(y,q)\}$$

It follows that

$$\mu((x+y), q) \geq T\{\mu(x,q), \mu(y,q)\} = \min\{\mu(x,q), \mu(y,q)\}$$ for all $x,y \in R$, $q \in Q$

Hence μ is a Q-fuzzy soft ring of R.

Proposition 3.2:

If μ is Q-fuzzy soft ring R and θ is an endomorphism of R, then μ [θ] is a Q- Fuzzy soft ring of R

Proof:

For any $x,y \in R$, we have

(FSR1)

(i) $\mu[\theta]((x+y),q)) = \mu(\theta((x+y),q))$
$= \mu(\theta(x,q), \theta(y,q))$
$\geq T\{\mu(\theta(x,q)), \mu(\theta(y,q))\}$
$\geq T\{\mu[\theta](x,q), \mu[\theta](y,q)\}$

(FSR2)

(ii) $\mu[\theta](-x,q) = \mu(\theta(-x,q))$
$\geq \mu(\theta(x,q))$
$\geq \mu[\theta](x,q)$

(FSR3)

(iii) $\mu[\theta]((xy),q)) = \mu(\theta((xy),q))$
$= \mu((\theta x,q), (\theta y,q))$
$\geq T\{\mu(\theta x,q), \mu(\theta y,q)\}$
$\geq T\{\mu\theta(x,q), \mu\theta(y,q)\}$
$\geq T\{\mu[\theta](x,q), \mu[\theta](y,q)\}$

Hence μ [θ] is a Q-fuzzy soft ring of R.

Proposition 3.3:

Let R and R' be two rings and $\theta: R \rightarrow R'$ be a soft homomorphism. If μ and f_a is a Q-fuzzy soft ring of R then the pre-image $\theta^{-1}(f_a)$ Q-fuzzy soft ring of R.

Proof:-

Assume that f_a is a Q-fuzzy soft ring of R'. Let $x, y \in R$ & $q \in Q$

(FSR1)

(i) $\mu_{\theta^{-1}[f_a]}((x+y),q)) = \mu_{f_a}(\theta(x+y),q))$
$= \mu_{f_a}((\theta x,q), (\theta y,q))$
$\geq T\{\mu_{f_a}(\theta(x,q)), \mu_{f_a}(\theta(y,q))\}$
$\geq T\{\mu_{\theta^{-1}[f_a]}(x,q), \mu_{\theta^{-1}[f_a]}(y,q)\}$

(FSR2)

(ii) $\mu_{\theta^{-1}[f_a]}(-x,q) = \mu_{f_a}(\theta(-x,q))$
$\geq \mu_{f_a}(\theta(x,q)$
$\geq \mu_{\theta^{-1}[f_a]}(x,q)$

(FSR3)

(iii) $\mu_{\theta^{-1}[f_a]}((xy),q)) = \mu_{f_a}(\theta(xy),q))$
$= \mu_{f_a}((\theta x,q), (\theta y,q))$
$\geq T\{\mu_{f_a}(\theta(x,q)), \mu_{f_a}(\theta(y,q))\}$
$\geq T\{\mu_{\theta^{-1}[f_a]}(x,q), \mu_{\theta^{-1}[f_a]}(y,q)\}$

Hence $\theta^{-1}(f_a)$ is a Q-fuzzy soft ring of R.

Proposition 3. 4

Letθ: R→ R' be an epimorphism and f_a be fuzzy soft set in R' . If $\theta[f_a]$ is q-fuzzy soft ring of R' then f_a is q-fuzzy soft ring of R.

Proof :-

Let $x,y \in R$, Then there exist $a,b \in R$ such that $\theta(a) = x$, $\theta(b) = y$. It follows that

(FSR1)

(i) $\mu_{\theta[f_a]}((x+y),q)) = \mu_{f_a}(\theta(x+y),q))$

$= \mu_{f_a}((\theta x,q), (\theta y,q))$

$\geq T\{\mu_{f_a}(\theta(x,q)), \mu_{f_a}(\theta(y,q))\}$

$\geq T\{\mu_{\theta[f_a]}(x,q), \mu_{\theta[f_a]}(y,q)\}$

(FSR2)

(ii) $\mu_{\theta[f_a]}(-x,q) = \mu_{f_a}(\theta(-x,q))$

$\geq \mu_{f_a}(\theta(x,q))$

$\geq \mu_{\theta[f_a]}(x,q)$

(FSR3)

(iii) $\mu_{\theta[f_a]}((xy),q)) = \mu_{f_a}(\theta(xy),q))$

$= \mu_{f_a}((\theta x,q), (\theta y,q))$

$\geq T\{\mu_{f_a}(\theta(x,q)), \mu_{f_a}(\theta(y,q))\}$

$\geq T\{\mu_{\theta[f_a]}(x,q), \mu_{\theta[f_a]}(y,q)\}$

Hence $\theta[f_a]$ is a Q-fuzzy soft ring of R.

Proposition 3.5:

Onto homomorphic image of a Q-fuzzy soft ring with the **sup** property is Q-fuzzy soft ring of R.

Proof:

Let f: $R \rightarrow R'$ be an onto homomorphism of Q-fuzzy soft rings and let μ be a **sup** property of Q-fuzzy soft ring of R.

Let $x^1, y^1 \in R^1$, and $x_0 \in f^{-1}(x^1)$, $y_0 \in f^{-1}(y^1)$ be such that

$\mu(x_0,q) = \sup_{(h,q) \in f^{-1}(x^1)} \mu(h,q)$,

and

$\mu(y_0,q) = \sup_{(h,q) \in f^{-1}(y^1)} \mu(h,q)$

Respectively, then we can deduce that

(FSR1)

(i) $\mu^f((x^1+y^1), q) = \sup_{(z,q) \in f^{-1}((x^1+y^1),q)} \mu(z,q)$

$\geq T\{\mu(x_0,q), \mu(y_0,q)$

$= T\{\sup_{(h,q) \in f^{-1}(x^1,q)} \mu(h,q), \sup_{(h,q) \in f^{-1}(y^1,q)} \mu(h,q)\}$

$= \min\{\mu^f(x^1,q), \mu^f(y^1,q)\}$

(FSR2)

(ii) $\mu^f(-x^1,q) = \sup_{(z,q) \in f^{-1}(-x^1,q)} \mu(z,q)$

$\geq \mu(x_0, q)$

$\geq \sup_{(h,q) \in f^{-1}(x^1,q)} \mu(h,q)$

$= \mu^f(x^1,q)$

(FSR3)

(iii) $\mu^f((x^1y^1), q) = \sup_{(z,q) \in f^{-1}((x^1y^1),q)} \mu(z,q)$

$\geq T\{\mu(x_0,q), \mu(y_0,q)\}$

$= T\{\sup_{(h,q) \in f^{-1}(x^1,q)} \mu(h,q), \sup_{(h,q) \in f^{-1}(y^1,q)} \mu(h,q)\}$

$= \min\{\mu^f(x^1,q), \mu^f(y^1,q)\}$

Hence μ^f is a Q-fuzzy soft ring of R^1.

Proposition 3. 6:

Let T be a continuous t-norm and Let f be a soft homomorphism on R. If μ is Q-fuzzy soft of R, then μ^f is Q-fuzzy soft ring of f(R).

*Proof***:**

Let $A_1 = f^{-1}(y_1,q)$, $A_2 = f^{-1}(y_2,q)$ and $A_{12} = f^{-1}((y_1+y_2), q)$ where $y_1,y_2 \in f(R)$, $q \in Q$

Consider the set

$A_1+A_2 = \{x \in R \,/\, (x,q) = (a_1,q) + (a_2,q)\}$ for some $(a_1,q) \in A_1$ and $(a_2,q) \in A_2$.

If $(x,q) \in A_1+A_2$, then $(x,q) = (x_1,q) + (x_2,q)$ for some $(x_1,q) \in A_1$ and $(x_2,q) \in A_2$ so that we have

$f(x,q) = f(x_1,q)+ f(x_2,q)$

$= y_1+ y_2$

Since $(x,q) \in f^{-1}((y_1,q)+ (y_2,q)) = A_{12}$. Thus $A_1+A_2 \in A_{12}$

It follows that

(FSR1)

(i) $\mu^f((y_1+y_2), q) = \mathbf{sup}\{\mu(x,q)/(x,q) \in f^{-1}(y_1+y_2,q)\}$

$= \mathbf{sup}\ \{\mu(x,q) / (x,q) \in A_{12}\}$

$\geq \mathbf{sup}\ \{\mu(x,q)/ (x,q) \in A_1+A_2\}$

$\geq \mathbf{sup}\ \{\mu((x_1,q)+(x_2,q))/(x_1,q)\in A_1$ and $(x_2,q) \in A_2\}$

$\geq \mathbf{sup}\ \{S(\mu(x_1,q), \mu(x_2,q))/ (x_1,q) \in A_1$ and $(x_2,q) \in A_2\}$

Since T is continuous. For every $\varepsilon > 0$, we see that if $\mathbf{sup}\ \{\mu(x_1,q) / (x_1,q)\in A1\} + (x_1^*, q)\} \leq \delta$ and $\mathbf{sup}\ \{\mu(x_2,q) / (x_2,q) \in A_2\} + (x_2^*,q)\} \leq \delta$

$T\{\mathbf{sup}\{\mu(x_1,q)/ (x_1,q)\in A_1\}, \mathbf{sup}\{\mu(x_2,q) / (x_2,q)\in A_2\} + T((x_1^*,q), (x_2^*,q) \leq \varepsilon$

Choose $(a_1,q) \in A_1$ and $(a_2,q) \in A_2$ such that $\mathbf{sup}\{\mu(x_1,q) / (x_1,q) \in A_1\} + \mu(a_1,q) \leq \delta$ and $\mathbf{sup}\{\mu(x_2,q) / (x_2,q) \in A_2\} + \mu(a_2,q) \leq \delta$.

Then we have

$T\{\mathbf{sup}\{\mu(x_1,q)/(x_1,q)\in A_1\}, \mathbf{sup}\{\mu(x_2,q)/(x_2,q) \in A_2\} + T(\mu(a_1,q),\mu(ma_2,q) \leq \varepsilon$

Consequently, we have

$\mu^f((y_1+y_2), q) \geq \mathbf{sup}\{T(\mu(x_1,q), \mu(x_2,q)) / (x_1,q) \in A_1, (x_2,q) \in A_2\}$

$\geq T(\mathbf{sup}\{\mu(x_1,q) / (x_1,q) \in A_1\}, \mathbf{sup}\{\mu(x_2,q) / (x_2,q) \in A2\}$

$\geq T\ \{(\mu^f(y_1,q), \mu^f(y_2,q)\}$

Similarly we can show $\mu^f(-x,q) \geq \mu^f(x,q)$ and $\mu^f(xy,q) \geq T\ \{(\mu^f(x,q), \mu^f(y,q)\}$

Hence μ^f is Q-fuzzy soft ring of f(R).

Proposition3.7:

Let μ be a Q-fuzzy soft ring R and let μ^* be a Q-fuzzy set in N defined by $\mu^*(x,q) = \mu(x,q) + 1 - \mu(0,q)$ for all $x\in$ N. Then μ^* is a normal Q-fuzzy subgroup of R

Proof :

For any x, $y\in R$ and $q \in Q$ we have

(FSR1)

$\mu^*((x+y),q) = \mu((x+y),q) + 1 - \mu(0,q)$

$\geq T(\mu(x,q), \mu(y,q)) + 1 - \mu(0,q))$

$\geq T(\mu(x,q)+1-\mu(0,q), (\mu(y,q)+1-\mu(0,q))$

$= T(\mu^*(mx,q), \mu^*(my,q)).$

(FSR2)

$\mu^*(-x,q) = \mu(-x,q) + 1 - \mu(0,q)$

$\geq \mu(x,q) + 1 - \mu(0,q)$

$= \mu(x,q)$

(FSR3)

$\mu^*((xy),q) = \mu((xy),q) + 1 - \mu(0,q)$

$\geq T(\mu(x,q), \mu(y,q)) + 1 - \mu(0,q))$

$\geq T(\mu(x,q)+1-\mu(0,q), (\mu(y,q)+1-\mu(0,q))$

$= T(\mu^*(mx,q), \mu^*(my,q)).$

3. CONCLUSION:

In this paper we investigate the notion of Q-fuzzy soft ring. This work focused on Q-fuzzy soft rings of fuzzy soft rings. To extend this work one could study the properties of fuzzy soft sets in other algebraic structure.

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