Quantum Teleportation using Continuous Variables

Mahindrasingh
Department of Physics,
J N V University, Jodhpur, Rajasthan
India

***Abstract:-* Quantum teleportation for continuous variables is generally described in phase space by using Wigner function .Quantum variables states can be analyzed using the quasiprobability functions .We study problem of quantum teleportation using continuous variables in term of density operator and Wigner function. A continuous variables can be teleported with use of two mode squeezed vacuum for a quantum channel.**

key words : quantum teleportation , quasiprobability distribution functions ,

INTRODUCTION

Quantum teleportation**(4,5)** is a process of transferring quantum information between two parties commonly called Alice and Bob . Alice the sender, has two systems in her hand. One of them is system 1 , in an unknown state which is to be transmitted to Bob and system 2, which is entangled with system 3. Bob is in possession of system 3. Using local measurement and information sent by classical channel, the goal of quantum teleportation is that Bob's system acquires the unknown state of system 1 with almost 100 % fidelity. This information transfer is essential in many quantum technologies including quantum cryptography, enabling secure communication and quantum dense coding boosting the data rates with respect to classical transmission and finally quantum internet, which is a network of system using quantum information transfer protocol**.**

Quantum teleportation requires entanglement (9,6)as a resource. This weird property of quantum system was once dismissed as "spooky action at a distance" by Einstein as it implies instantaneous change in the state of system B entangled with system A when measurement is done on system A even if they are space like separated. Such EPR correlation **(1)** was considered as a potential threat to the entire edifice of quantum physics. But the developments of last 30 years have shown that quantum entanglement is experimentally verifiable and it underpins all quantum information technologies.

Quantum teleportation using optical schemes can be done with qubits as well as continuous variables(7). However ,Bell-operator measurement cannot be done with high efficiency using qubits . Recently photon added and photon subtracted states generated from classical field have attracted much attention. Such processes are useful because starting from classical field state $\hat{\rho}_{cl}$ one can obtain non classical field states $\hat{U}\,\hat{\rho}_{cl}\hat{U}^{\dagger}$**(2)** with tailor made nonclassical properties, depending on operator $\hat{U}$ which is not unitary. Wigner functions of such nonclassical states , teleportation of such states and how noise induced by environment can affected fidelity of teleportation.
Statistical properties of quantized field in term of quasi probability distribution functions. Properties of these functions are summarized below.

QUASI PROBABILITY DISTRIBUTION FUNCTIONS:

In quantum optics quasiprobabiulity distribution functions such as the Glauber- Sudarshan *P* function, the *Q* function and the Wigner function play an important role. For photon added states $\hat{a}^{\dagger m}|\alpha>$ and squeezed states, these distribution functions are given below.

(1) Photon added states ($\hat{a}^{\dagger m}|\alpha>$): These states are obtained by the application of photon creation operator $\hat{a}^{\dagger m}$ on the coherent state |α>.

$$|\alpha, m> = \sqrt{N}\hat{a}^{\dagger m}|\alpha> \qquad .....(1) \quad (\text{m=integer})$$

Where N is the normalization constant given by

$$N = [<\alpha|\hat{a}^{m}\hat{a}^{\dagger m}|\alpha>]^{-1} \qquad ..(2)$$

This can be evaluated by using normal ordering of the operator $\hat{a}^{m}\hat{a}^{\dagger m}$.

$$\hat{a}^{m}\hat{a}^{\dagger m} = \sum_{p=0}^{m}\frac{(m!)^2}{(m-p)^2!p!}\ \hat{a}^{\dagger m-p}\ \hat{a}^{m-p} \qquad ...(3)$$

Thus

$$<\alpha|\hat{a}^{m}\hat{a}^{\dagger m}|\alpha>] = \sum_{p=0}^{m}\frac{(m!)^2}{(m-p)^2!p!}\ |\alpha|^{2(m-p)}$$

$$= L_m(-|\alpha|^2)m! \qquad ..(4)$$

where $L_m(x)$ is the Laguerre polynomial of order m defined by

$$L_m(x) = \sum_{n=0}^{m}\frac{(-1)^n x^n m!}{(n!)^2(m-n)!} \qquad ..(5)$$

The state |α,m > can then be written as

$$|\alpha, m> = \frac{\hat{a}^{\dagger m}|\alpha>}{[L_m(-|\alpha|^2)m!]^{\frac{1}{2}}} . \quad ...(6)$$

In the limit $\alpha \to 0$ this state reduces to the Fock state.

$$|\alpha,m> = |0,m> = |m>$$

and in the limit $m \to 0$, it reduces to the coherent state

$$|\alpha,0> = |\alpha> .$$

Thus the state $|\alpha,m>$ is intermediate between the Fock state and the coherent state.

(a) P - function:

Glauber and Sudarshan, independently, have introduced the *P* representation for the probability density. This function is defined by

$$\hat{\rho} = \int P(\alpha)|\alpha><\alpha| d^2\alpha \quad ...(7)$$

where $|\alpha>$ is a coherent state. It can also be defined as the Fourier transform of the normally ordered characteristic function

$$P(z) = \frac{1}{\pi^2}\int \mathrm{Tr}\left[\hat{\rho} e^{\beta\hat{a}^{\dagger}} e^{-\beta^* \hat{a}}\right] \exp(z\beta^* - z^*\beta)\, d^2\beta \quad ...(8)$$

The Glauber-Sudarshan *P* function associated with the state $|\alpha,m>$ can be calculated using the inversion formula.

$$P(z) = \frac{\exp(|Z|^2)}{\pi^2}\int d^2\beta <-\beta|\alpha,m><\alpha,m|\beta> \exp[|\beta|^2-(\beta z^*-\beta^* z)] \quad ...(9)$$

$$= \frac{\exp(|Z|^2)}{\pi^2 L_m(-|\alpha|^2)m!}\int d^2\beta\, (-\beta\beta^*)^m \exp[-|\alpha|^2 + (z-\alpha)\beta^* - (z-\alpha)^*\beta] \quad ...(10)$$

$$= \frac{\exp(|Z|^2 - |\alpha|^2)}{m!\, L_m(-|\alpha|^2)} \frac{\partial^{2m}}{\partial z^{*m}\partial z^m} \delta^2(z-\alpha) \quad ...(11)$$

Thus the *P* function is highly singular which is quite typical of states exhibiting nonclassical nature.

(b) Q- function :

The *Q* representation is defined as the Fourier transform of the antinormally ordered characteristic function.

$$Q(z) = \frac{1}{\pi^2}\int \mathrm{Tr}\left[\hat{\rho} e^{-\beta^* \hat{a}} e^{\beta\hat{a}^{\dagger}}\right] \exp(z\beta^* - z^*\beta)\, d^2\beta \quad ...(12)$$

which can be shown to be the absolute magnitude squared of the projection of a state of the field on to a coherent state .

$$Q(z) = \frac{1}{\pi} < z|\hat{\rho}|z> \quad ....(13)$$

This is particularly useful in calculating the antinormally ordered expectation values. Example

$$<\hat{a}\hat{a}^{\dagger}> = \frac{1}{\pi}\int d^2\alpha\, |\alpha|^2 Q(\alpha). \quad ...(14)$$

Thus, the *Q* function for the field in the state $|\alpha,m>$ is

$$Q(z) = \frac{1}{\pi} < z|\alpha,m><\alpha,m|z> \quad ...(15)$$

$$= \frac{|Z|^{2m}}{m! L_m(-|\alpha|^2)} \exp(-|z-\alpha|^2) \quad ...(16)$$

which is no longer centered at $z = \alpha$.

(c) Wigner function :

Wigner was the first to propose the construction of a phase space(3,10) distribution function from quantum mechanical wave function . It is defined as the Fourier transform of the symmetrically ordered characteristic function.

$$W(z) = \frac{1}{\pi^2}\int \mathrm{Tr}[\hat{\rho}\, \hat{D}(\beta)] \exp(z^*\beta - z\beta^*)\, d^2\beta \quad ...(17)$$

$$= \frac{1}{\pi^2}\int \mathrm{Tr}\left[\hat{\rho} e^{(\beta\hat{a}^{\dagger} - \beta^*\hat{a})}\right] \exp(z^*\beta - z\beta^*)\, d^2\beta \quad ...(18)$$

It can also be evaluated in terms of coherent state matrix elements by using the formula.

$$W(z) = \frac{2}{\pi}\exp(2|z|^2)\int d^2\beta <-\beta|\alpha,m><\alpha,m|\beta> \exp[2(\beta^* z - \beta z^*)] \quad ...(19)$$

which on simplification reduces to

$$W(z) = \frac{2\exp(2|Z|^2 - |\alpha|^2)}{\pi^2 L_m(-|\alpha|^2)m!}\int d^2\beta (-\beta^*\beta)^m \exp(-|\beta|^2 + \beta^*(2z-\alpha) - \beta(2z-\alpha)^*) \quad ...(20)$$

the Wigner function for the state$|\alpha,m>$ is, with $\xi = 2z-\alpha$

$$W(z) = \frac{2\exp(2|Z|^2 - |\alpha|^2)}{\pi L_m(-|\alpha|^2)m!} \frac{\partial^{2m}}{\partial\xi^{*m}\partial\xi^m} \frac{1}{\pi}\int d^2\beta \exp(-|\beta|^2 + \beta^*\xi - \beta\xi^*) \quad ...(21)$$

$$W(z) = \frac{2\exp(2|Z|^2 - |\alpha|^2)}{\pi L_m(-|\alpha|^2)m!} \frac{\partial^{2m}}{\partial\xi^{*m}\partial\xi^m}\exp(-|\xi|^2) \quad ...(22)$$

$$= \frac{2(-1)^m \exp(2|Z|^2 - |\alpha|^2)}{\pi L_m(-|\alpha|^2)m!} \exp(-|\xi|^2) L_m(||\xi|^2)m! \quad ..(23)$$

$$= \frac{2(-1)^m L_m(|2z-\alpha|^2)}{\pi L_m(-|\alpha|^2)} \exp[-2(z-\alpha)^2] \quad ...(24)$$

It is obvious from Eq.(24) that the Wigner function can become negative. This crosses zero whenever $L_m(|2z-\alpha|^2) = 0$. For m = 0 (α = 0), the expression in Eq.(24) reduces to that for a coherent state (number state).

(2) Squeezed sates :

(a) Q- function

Single mode squeezed state $|\xi>$ via the application of the unitary operator

$$S(\xi) = \exp(\tfrac{1}{2}\xi\,\hat{a}^{\dagger} - \tfrac{1}{2}\xi^{*}\hat{a}^{2}) \qquad ...(25)$$

On the vacuum state

$$|\xi> = S(\xi)|0> \qquad ...(26)$$

We can write $|\xi>$ as a superposition of fock state by

decomposing the unitary operator $S(\xi)$ as

$$S(\xi)=\exp\left(e^{i\varphi}\tanh r\frac{\hat{a}^{\dagger}}{2}\right)\exp[-\left(\ln\cosh r\right)(\hat{a}^{\dagger}\hat{a}+\tfrac{1}{2})]\quad \exp(-e^{-i\varphi}\tanh r\frac{\hat{a}^{2}}{2}) \qquad ...(27)$$

The advantage of the form is that the action of the last exponential on the vacuum would yield unity as $a|0>=0$.

The middle exponential on the vacuum would lead to exp[-(ln coshr) $\frac{1}{2}|0>$] and thus

$$|\xi> = \frac{1}{\sqrt{\cosh r}}\exp\left(e^{i\varphi}\tanh r\frac{\hat{a}^{\dagger}}{2}\right)|0> \qquad ...(28)$$

(b) P- function:

The P- function for the squeezed state does not exist.

(c) Wigner function

$$W_{\xi}(\alpha)=\frac{1}{\pi^{2}}\int \mathrm{Tr}[\hat{\rho}\,\hat{D}(\beta)]\,e^{-(\beta\alpha^{*}-\beta^{*}\alpha)}\,d^{2}\beta \qquad ...(29)$$

$$\mathrm{Tr}[\hat{\rho}\,\hat{D}(\beta)] = <\xi|\hat{D}(\beta)|\xi> \qquad ...(30)$$

$$= <0|S^{\dagger}(\xi)\,\hat{D}(\beta))\,S(\xi)|0> \qquad ...(31)$$

Eq.(31) putting in eq.(29)

$$W_{\xi}(\alpha)=\frac{1}{\pi^{2}}\int d^{2}\beta\ <0|S^{\dagger}(\xi)\,\hat{D}(\beta)S(\xi)|0>e^{-(\beta\alpha^{*}-\beta^{*}\alpha)} \qquad ..... (32)$$

Using properties of displacement operator

$$S^{\dagger}(\xi)\hat{D}(\beta)S(\xi) = \exp[\beta\hat{a}^{\dagger}(\xi) - \beta^{*}\hat{a}(\xi)] \qquad ...(33)$$

But the putting value of $\hat{a}^{\dagger}(\xi)$and $\hat{a}(\xi)$

$$= \exp[(\beta(\hat{a}^{\dagger}\cosh r + \hat{a}e^{-i\varphi}\sinh r) - \beta^{*}(\hat{a}\cosh r + \hat{a}^{\dagger}e^{i\varphi}\sinh r)] \qquad ...(30)$$

$$= \exp[(\beta\hat{a}^{\dagger}\cosh r + \beta\hat{a}e^{-i\varphi}\sinh r) - \beta^{*}\hat{a}\cosh r - \beta^{*}\hat{a}^{\dagger}e^{i\varphi}\sinh r)] \qquad .(34)$$

$$=\exp[(\beta\cosh r - \beta^{*}\hat{a}\sinh r)\hat{a}^{\dagger} - \hat{a}(\beta^{*}\cosh r - \beta e^{-i\varphi}\sinh r)] \qquad ...... (35)$$

$$=\exp(\beta'\hat{a}^{\dagger} - \hat{a}\beta'^{*}) \qquad ... (36)$$

$$=\hat{D}(\beta') \qquad ...(37)$$

We can change the eq.(32)(using eq.36,37)

$$W_{\xi}(\alpha)=\frac{1}{\pi^{2}}\int d^{2}\beta'\ <0|\hat{D}(\beta')|0>e^{-(\beta'\alpha'^{*}-\beta'^{*}\alpha')} \qquad .. (38)$$

The relation (38) show that the wigner function for the squeezed vacuum can be obtained from the wigner for the vacuum state by changing α to α'

$$W_{\xi}(\alpha) = \frac{2}{\pi}\exp(-2|\cosh r - \alpha^{*}e^{i\varphi}\sinh r|^{2}) \qquad . (39)$$

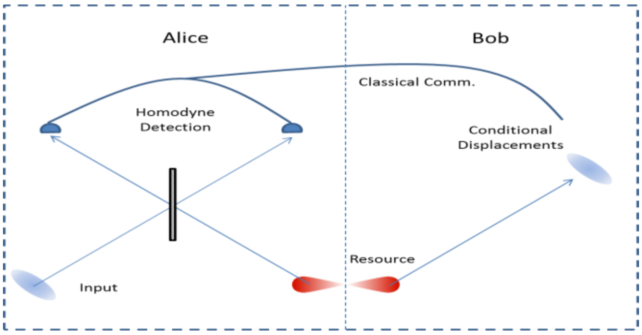
Result :

Reformulated the problem of quantum teleportation using continuous variables in terms of density operator and Wigner function .This approach will be used to study the effect of various decohrence inducing processes which are inevitable in any realistic scenario.

The initial density operator of three system state is

$$\rho^{i}(1,2,3) = \rho^{i}_{ent}(2,3) \otimes \rho^{i}(1). \qquad ...(40)$$

Here i refer to initial state and labels 1,2and 3 correspond to system 1, system 2 and system 3 respectively. The state $\rho^{i}(1)$ is unknown state of system 1 to be teleported .To start with, system 2 and system 3 are in entangled state described by density operator $\rho^{i}_{ent}(2,3)$ and combined state of three systems is outer product of density operator $\rho^{i}_{ent}(2,3)$ and $\rho^{i}(1)$.



A basic scheme of quantum teleportation is shown in fig.1

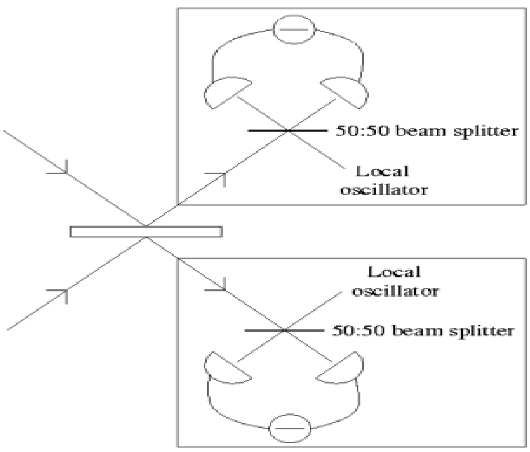


Fig 2. Configuration of the joint measurement scheme. The homodyne detectors, which are boxed in the figure, are placed after mixing two fields by a 50:50 beam splitter.

Joint measurement of commuting quadrature is made by Alice on system 1 and system 2. As a result of this measurement ρ^{i} (1,2,3) is projected to state with density operator

$$<q_1 p_2|\hat{R}(\theta)^{\dagger}\, \hat{\rho}^{i}_{ent}(2,3)\; \hat{\rho}^{i}(1)\, \hat{R}(\theta)q_1 p_2>|q_1 p_2><q_1 p_2|. \quad ..(41)$$

The result of these quadrature measurements are sent to Bob in terms of parameter μ which depend on θ. Now Bob applies unitary transformation $\hat{U}(\mu)$ on system 3 which is in his hand. Then system 3 find itself in state $\hat{\rho}(3)$ such that

$$\hat{\rho}(3)= \hat{U}(\mu) < q_1 p_2|\hat{R}(\theta)^{\dagger}\, \hat{\rho}^{i}_{ent}(2,3)\hat{\rho}^{i}(1)\, \hat{R}(\theta)|(q_1 p_2>\hat{U}(\mu)^{\dagger}$$

$$= <q_1 p_2|\hat{R}(\theta)^{\dagger}\, \hat{U}(\mu)^{\dagger}\hat{\rho}^{i}_{ent}(2,3)\, \hat{U}(\mu)\, \hat{\rho}^{i}(1)\, \hat{R}(\theta)|q_1 p_2> \quad ...(42)$$

Operations described above are made such that $\hat{\rho}(3)$ is as close to $\hat{\rho}^{i}(1)$. In other words, after these operations, state of system 1 is teleported to system 3. Efficiency of teleportation is measured by fidelity **F** defined as

$$F=Tr[\hat{\rho}^{i}(1)\, \hat{\rho}(3)]. \quad ...(43)$$

For perfect teleportation of quantum state, F**=1**.

Result of reference**(8)** is special case of approach adopted by us, when $\rho^{i}_{ent}(2,3)$ is written in coherent state representation and squeezing parameter is infinite . Moreover, our approach is easily generalized to deal with more general case including decoherences because then unitary evolution is replaced by transformation on $\hat{\rho}$ preserving its positivity . The details would depend on the model used to describe decoherence. I propose to study all these in future work.

ACKNOWLEDGMENTS:

We thank professor(Dr.) J.K.Sharma for helpful discussion . This work is financially supported by university grant commission.

REFRENCES

[1] A . Einstein , B.Podolsky and N. Rosen, Phy.Rev. Vol.47,(1935)
[2] G.S.Agrawal and E.Wolf , Phy.Rev. D2,2161(1970)
[3] G.S.Agrawal,Phy Rev.A,Vol.24,No.06,(1981)
[4] C. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, W. K. Wootters, Phys. Rev. Lett. 70, 1895 (1993)
[5] Gilles Brassaed ,"Teleportation as quantum computation" arxiv.quant.ph./9605035v1(1996)
[6] Marloan O.Scully and M.Sushail ,Zubairy, "Quantum Optics"(Cambridge University Press,1997)
[7] Samell. Braunstain and H.J.Kimble, Phy,Rev. Lett. Vol 80, NO.4(1998)
[8] Jinhyoung Lee and M.S.Kim, Journal of the Korean Phy.Socity,Vol.42,No.4 (2003)
[9] G.S.Agrawal," Quantum Optics"(Cambridge University Press,2013)
[10] H.S.Das,Arpita Chatterjee,R.Ghosh,J.Phys.B At Mol.Phy.Vol. 48,No-18(2015)