Radiation Effects On Heat And Mass Transfer Over A Vertical Platewith Newtonian Fluid.

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Abstract
The radiation effect on heat and mass transfer over a vertical plate with Newtonian fluid is studied. The fluid is gray, absorbing –emitting but non scattering medium and Rosseland approximation is used to describe the radiative heat flux in energy equation. The dimensionless governing equations are solved analytically using the Laplace transform technique. The temperature,velocity and concentration fields are studied for different physical parameters like thermal Grashof number, mass Grashof number, Schmidt number and time. Temperature decreases with increased strength of radiation and Prandtl number.It is also observed that the velocity increases with increasing values thermal Grashof or mass Grashofnumber . Similarly, velocity increases with decreasing values of the Schmidt numbers.

Keywords; Radiation, Newtonian fluid and Heat and Mass transfer.

1.0 Introduction
Natural flows are attributed to either temperature differences or concentration differences or both, and the flows are accomplished withthe heat and mass transfer. These heat and mass transfer differences have been the subjects of many investigations globally. This is due to the fact that these flows have many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids in textiles industries, thermal insulation, nuclear power plant, gas turbines and various propulsion devices for aircraft, satellites and space vehicles. If the temperature of the surrounding fluid is rather high, radiation effects play an important role and these situations exist in space technology and many other related fields.

Singh and Kumar (1984) studied flow of a viscous incompressible fluid past an impulsively started infinite vertical plate in the presence of foreign mass. Basant and Ravindra (1990) analysed mass transfer effects on the flow past an accelerated infinite vertical plate with heat sources.Seddeck (2001) investigated the thermal radiation and buoyancy effects on MHD free convective heat generating flow over an accelerating permeable surface with temperature dependent viscosity. Also analysis of thermal radiation on unsteady free convection and mass transfer of an optically thin gray gas was done by Raptis and Perdikis (2003). Makinde and Mhone (2005), descrived the effects of heat transfer to MHD oscillatory flow in a channel filled with porous medium. Radiative heat and mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction was studied by Muthucumaraswamy and Raj (2006). Analysis of mass transfer effects on exponentially accelerated isothermal vertical plate was made byMuthucumaraswamyand Natarajon(2008). Rajesh and Varma (2009) analysed the radiation and mass transfer effects on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature. Also Narahari and Ishak (2011), studied radiation effects on free convection flow near a moving vertical plate with newtonian heating. Chamkha (2007) studied heat and mass transfer for a non-newtonian fluid flow along a surface embedded in a porous medium.Muthucumaraswamy and Subramanian (2010), studied the heat transfer effects on accelerated vertical plate with variable temperature and mass flux. Das and Jana (2010), analysed heat and mass transfer effects on unsteady MHD free convection. Patrick and Jane(2010), observed the effect of natural convective heat transfer from a narrow vertical flat plate with a uniform surface heat flux and with different plate edge condition. Experimental studies of Radiation effects on MHD free convection flow over a vertical plate has been described by Abdussamad and Rahman (2006),Sivaiah and Nagarajan(2010).

2.0 Mathematical Formulation.

Consider the unsteady flow of a viscous incompressible fluid past a moving vertical plate. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical radiation between the diffusing species and the fluid. The x'-axis is taken along the plate in vertically upward direction and they'-axis is taken normal to the plate. Initially the plate and the fluid are at the same temperature and concentration. At time t'>0, the plate is given an impulsive motion in the vertical direction against gravitational field with constant velocity u_0 in a fluid, in the presence of thermal radiation. At the same time, the plate temperature is raised to T_W. As the level of concentration is raised to C'_w, the fluid is considered grey, absorbing-emitting radiation but a non-scattering medium. It is also assumed that there exists a homogeneous first order chemical reaction between the fluid and species concentration. By usual Boussinesq's approximation, the unsteady flow is governed by the following equations:Muthucumaraswamy and Raj (2006).

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + v\frac{\partial^2 u}{\partial y^2} \quad (1)$$

$$\rho Cp\frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \quad (2)$$

$$\frac{\partial C'}{\partial t} = D\frac{\partial^2 C'}{\partial y^2} - K_l C. \quad (3)$$

with the following initial and boundary conditions;

$t' \le 0: \; U = 0 \;\; T = T_\infty \;\; C' = C'_\infty$ for all y,

$t' > 0: \; U = u_0 \;\; \frac{dT}{dy} = -\frac{h}{k}T \; , C' = C'_w$, at y =0(4)

$U = 0 \;\; T \to T_\infty \;\; C' = C'_\infty$ as $y \to \infty$

Where, β volumetric coefficient of the thermal expansion, v the kinematic viscosity, ρ is density, μ the coefficient of viscosity, β^* is volumetric coefficient of expansion with concentration, u_0 the velocity of the plate, y the coordinate axis normal to the plate, g acceleration due to gravity, q_r the radiation heat flux in the y direction, Cp is specific heat at constant pressure, C' is specific concentration in the fluid, C'_∞ the concentration in the fluid far away from the plate, C'_W the concentration on the plate, D the mass diffusion coefficient, t' is time, T the temperature of the fluid near the plate, T_W the temperature of the plate, T_∞ the temperature of the fluid far away from the plate.

The local radiant for the case of an optically thin grey gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^*\sigma(T_\infty^4 - T^4) \quad (5)$$

where a^* is absorption coefficient, σ the Stefan Boltzmann constant, K_l chemical reaction parameter, k the thermal conductivity of the fluid.

We assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressedas a linear function of the temperature.This is accomplished by expanding T^4 in Taylor series about T_∞ and neglecting higher order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \qquad (6)$$

By using equations (5) and (6), equation (2) reduces to

$$\rho Cp \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} + 16a^* \sigma T_\infty^3 (T_\infty - T) \qquad (7)$$

Introducing the following non-dimensional quantities,

$$U = \frac{u}{u_0}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad t = \frac{t' u_0^2}{V}, Y = \frac{y u_0}{V}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad Gr = \frac{g\beta V\left(T_w - T_\infty\right)}{u_0^3}$$

$$Gc = \frac{Vg\beta^*\left(C'_w - C'_\infty\right)}{u_0^3}, \quad \Pr = \frac{\mu Cp}{k}, \quad R = \frac{16a^* \partial v^2 T_\infty^3}{k u_0^2}, Sc = \frac{v}{D}, \quad K = \frac{vK_l}{u_o^2}$$

Equation (1),(3) and (7) respectively become

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} \qquad (8)$$

$$\Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial Y^2} - R\theta \qquad (9)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} - KC \qquad (10)$$

The initial and boundary conditions in non-dimensional quantities are

$U = 0$, $\theta = 0$ $C = 0$ for all Y $t \le 0$

$t^{3}\ \ 0\ U=1\ \frac{\partial\theta}{\partial Y}=-(1+\theta)\ C=1$ at Y=0 (11)

$U=0\,,\quad \theta\to 0\quad C\to 0\ ,\qquad Y\to\infty$

Where Gcthe mass Grashof number, Gr the thermal Grashof number, C the dimensionless concentration, Kthe dimensionless chemical reaction parameter,Pr the Prandtl number, R the radiation parameter, Sc the Schmidt number, t the dimensionless time, U the dimensionless velocity, Y the dimensionless coordinate axis normal to the plate, θ the dimensionless temperature.

According to above non dimensional process, $\frac{¶ q}{¶ Y} = -\ B_i\{g + q\}$

Where $B_i = \frac{vh}{u_0 k}$ calledBiot number , $g = \frac{T_¥}{T_w -\ T_¥}$ .For simplicity we take $B_i = 1 = g$.

The dimensionless governing equations (8) to (10), subject to the boundary conditions (11), are solved by the usual Laplace transform techniques and the solutions are derived as follows;

$$C=\frac{1}{2}\left\{\exp(2\eta\sqrt{ScKt})\,erfc\left(\eta\sqrt{Sc}+\sqrt{Kt}\right)+\exp(-2\eta\sqrt{ScKt})erfc\left(\eta\sqrt{Sc}+\sqrt{Kt}\right)\right\} \quad (12)$$

$$\theta=\frac{1}{2\left(\sqrt{R}-1\right)}\left\{\exp(2\eta\sqrt{\Pr ct})\,erfc\left(\eta\sqrt{\Pr}+\sqrt{ct}\right)+\exp(-2\eta\sqrt{\Pr ct})erfc\left(\eta\sqrt{\Pr}-\sqrt{ct}\right)\right\}$$
$$-\frac{\Pr}{1-R}\left\{\frac{\exp(\eta^{2}\Pr-ct)}{\sqrt{\Pr\pi t}}erfc\left(\eta\sqrt{\Pr}+\sqrt{ct}\right)\right\} \quad (13)$$

$$
\begin{aligned}
U = & \left[1-\frac{Gr}{a(1-\Pr)(1-\sqrt{R})}+\frac{Gc}{b(1-Sc)}\right] erfc(\eta) \\
& +\frac{Gr\ \exp(at)}{2a(1-\Pr)(1-\sqrt{R+\Pr a})}\left\{\begin{array}{l} \exp(2\eta\sqrt{at})erfc\left(\eta+\sqrt{at}\right)+\left(\exp(-2\sqrt{at})erfc\left(\eta-\sqrt{at}\right)\right) \\ -\exp(2\eta\sqrt{\Pr(a+c)t})erfc\left(\eta\sqrt{\Pr}+\sqrt{(a+c)t}\right)+\exp(-2\eta\sqrt{\Pr(a+c)t})erfc\left(\eta\sqrt{\Pr}-\sqrt{(a+c)t}\right) \end{array}\right\} \\
& +\frac{Gr(\Pr)^2}{(1-\Pr)(1-R)(1-R-\Pr a)}\left\{\left(\frac{\exp(-\eta^2)}{\sqrt{\pi t}}\ erfc(\eta\sqrt{\Pr}+\sqrt{ct})-\left(\frac{\exp(\eta^2\Pr-ct)}{\sqrt{\Pr\pi t}}erfc(\eta\sqrt{\Pr}+\sqrt{ct})\right)\right)\right\} \\
& +\frac{Gr}{2a(1-\Pr)(1-\sqrt{R})}\left\{\exp(2\sqrt{\Pr ct})erfc\left(\eta\sqrt{\Pr}+\sqrt{ct}\right)+\exp(-2\eta\sqrt{\Pr ct})erfc\left(\eta\sqrt{\Pr}-\sqrt{ct}\right)\right\} \\
& +\frac{Gc\exp(bt)}{2b(1-Sc)}\left\{\begin{array}{l} \left(\exp(2\eta\sqrt{Sc(K+b)t})erfc\left(\eta\sqrt{Sc}+\sqrt{(K+b)t}\right)+\exp(-2\eta\sqrt{Sc(K+b)t})erfc\left(\eta\sqrt{Sc}-\sqrt{(K+b)t}\right)\right) \\ -\left(\exp(2\eta\sqrt{bt})erfc\left(\eta+\sqrt{bt}\right)+\left(\exp(-2\eta\sqrt{bt})erfc\left(\eta-\sqrt{bt}\right)\right)\right) \end{array}\right\} \\
& -\frac{Gc}{2b(1-Sc)}\left(\exp(2\eta\sqrt{ScKt})erfc\left(\eta\sqrt{Sc}+\sqrt{Kt}\right)+\exp(-2\eta\sqrt{ScKt})erfc\left(\eta\sqrt{Sc}-\sqrt{Kt}\right)\right)
\end{aligned}
$$

(14)

3.0 Results and Discussion

For physical insight of the problem,numericalcomputations are carried out for different physical parameters such as Gr, Gc, Sc, Pr, K,and t to access the nature of the flow and transport. The value of Schmidt numberSc is taken to be (0.6)which corresponds to water vapour, and the value of Prandlt number Pr is chosen to represent air (Pr=0.71).The numerical values of the temperature, velocity and concentration are computed for different physical parameters like Prandtl number , Schmidt number ,thermal Grashof number, mass Grashof number and time.

3.1 Temperature **profiles.**

The temperature profiles ofdifferentvalues of Radiationparameter (R = 5, 10, 15, 20), Pr = 0.71, t = 0.2, is shown in figure1.It is observed that the temperature decreases with increase in values of R.When radiation is present, the thermal boundary layer always found to thicken,this shows that the radiationprovidesadditionalmeanstodiffuse energy.The effect of temperature for different values of Prandtl number (Pr = 0.71, 0.85, 1.0), R = 7, t = 0.2, are studied and presented in figure 2. It shows that there is slight decrease in temperature with increase in Pr.

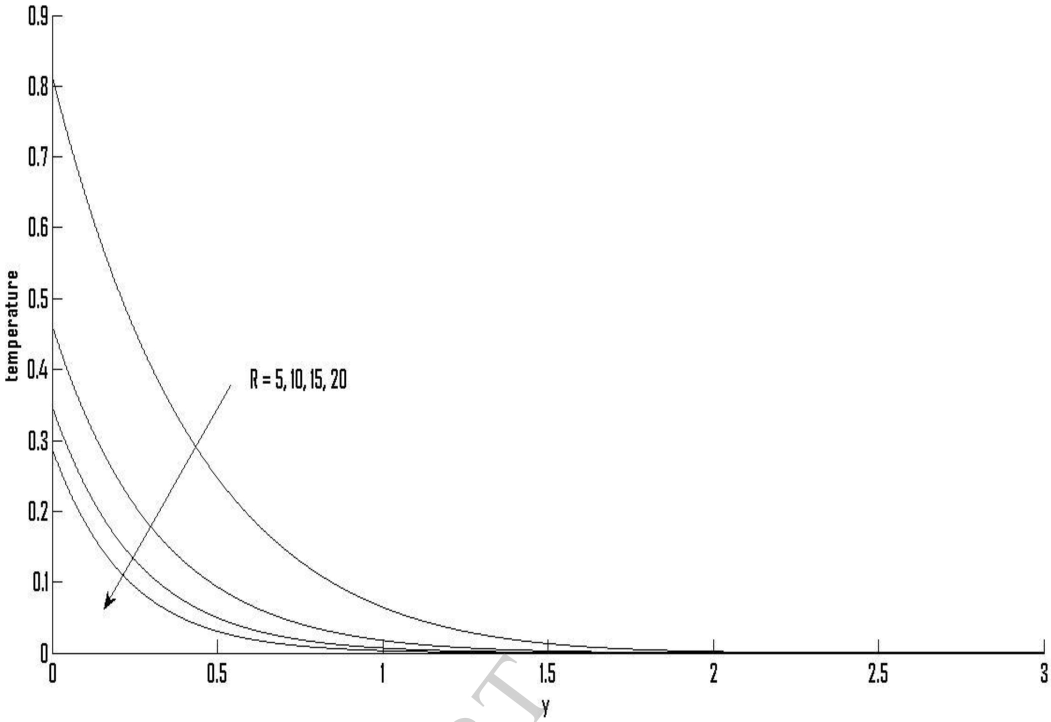

Figure1.Temperature profile for different values of R

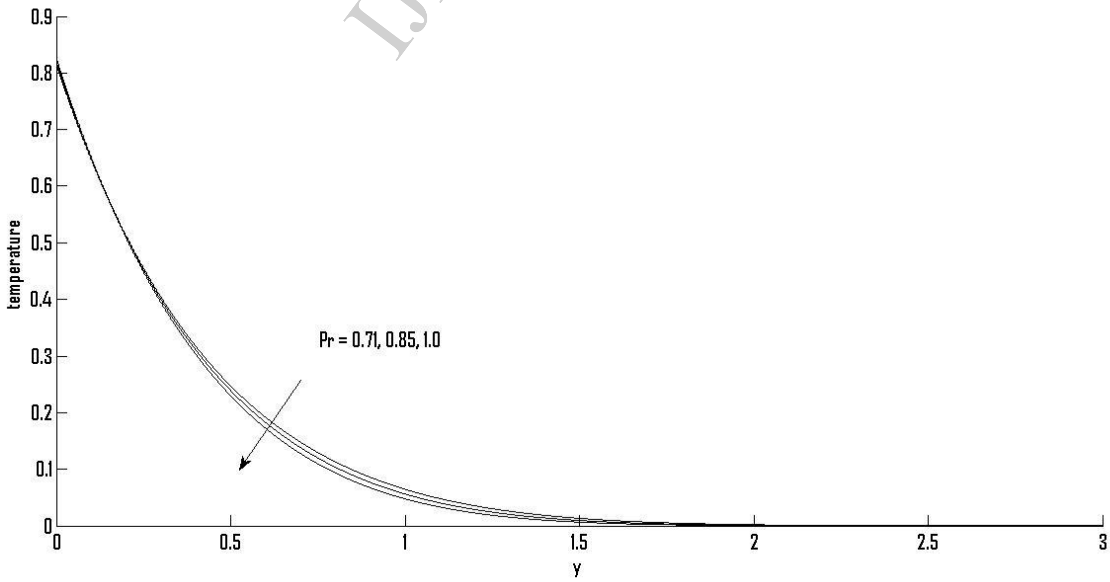

Figure 2 .Temperature profile for different values of Pr.

3.2 Velocity profiles.

From figure 3,the velocity profile for different values of mass Grashof number (Gc = 1, 5, 10, 15, 20, 25), Gr = 5, R = 7, Pr = 0.71, Sc = 0.6, K = 0.22, t = 0.2, is shown . The velocity profile increases with increase in the value of Gc, the increase is relatively higher. In figure 4, the velocity profiles for different values of Gr are studied,Gc = 5, R = 7, Pr = 0.71, Sc = 0.6, K = 0.22 and t = 0.2. It is observed that an increase Gr leads to increase in the velocity, but the increase is significantly small compared to increase in Gc.In figure5, the velocity increases with increase in the values of t, when Gc = Gr = 5, Pr = 0.71, Sc = 0.6 and K= 0.22. Demonstration in figure 6, indicates that velocity decreases with increase in the values of Radiation parameter (R = 0.16, 0.3, 0.6), when Gr = Gc = 5, K = 0.22, at t = 0.2.In figure 7 above, the velocity profile decreases with increase in the values of Sc. This means that the larger the values of Sc the thinner the momentum boundary layer size, hence decrease in the velocity.

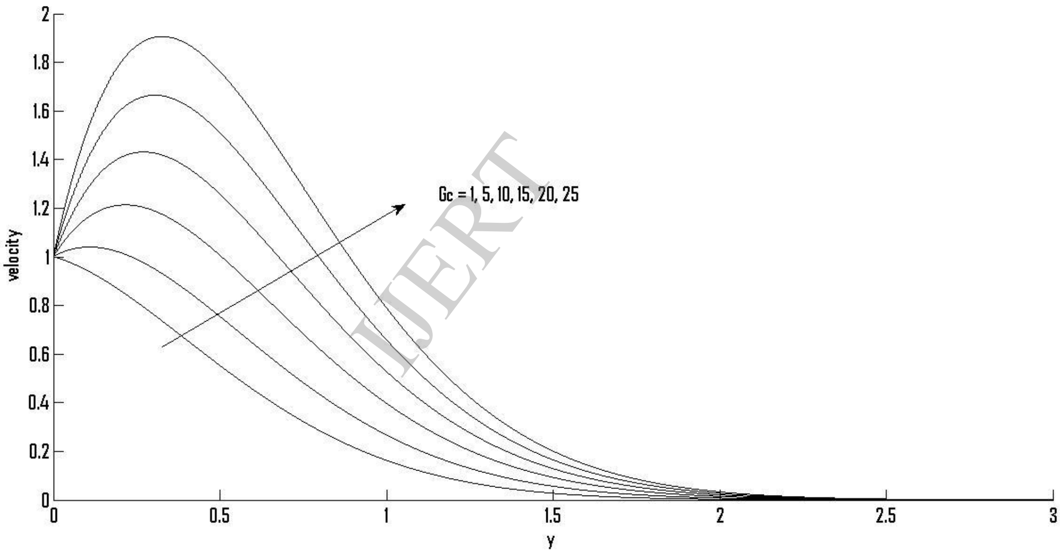

Figure 3.Velocity profile for different values of Gc.

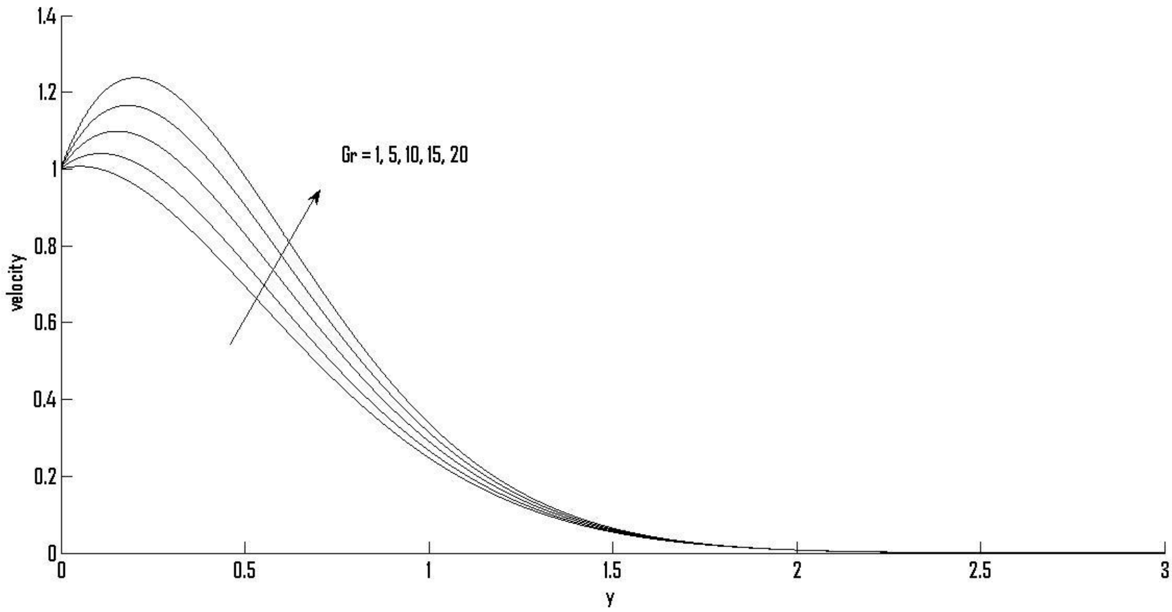

Figure 4.Velocity profile for different values of Gr.

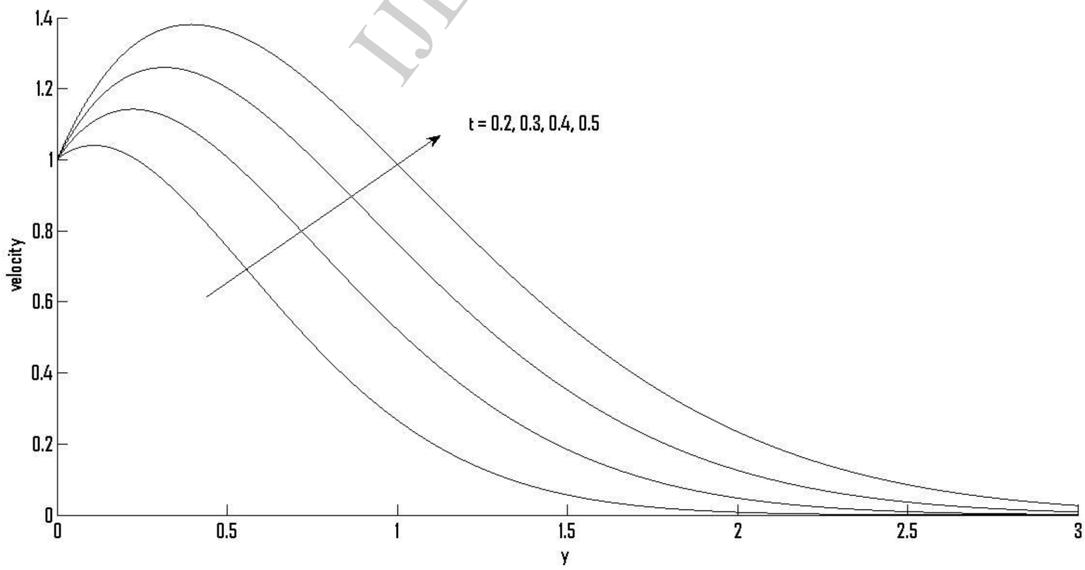

Figure5.Velocityprofile for different values of t.

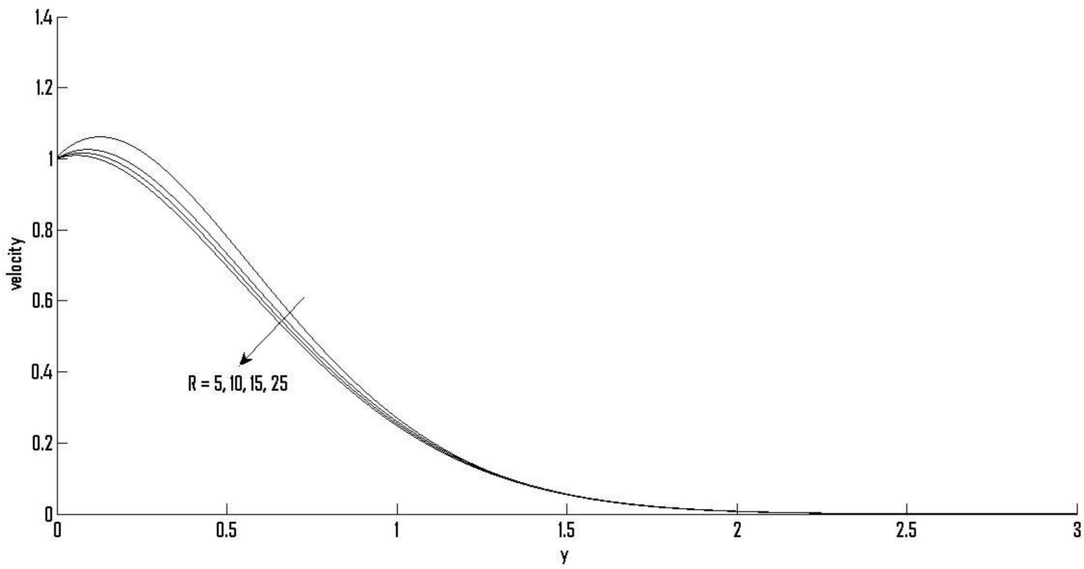

Figure6.Velocity profile for different values of R.

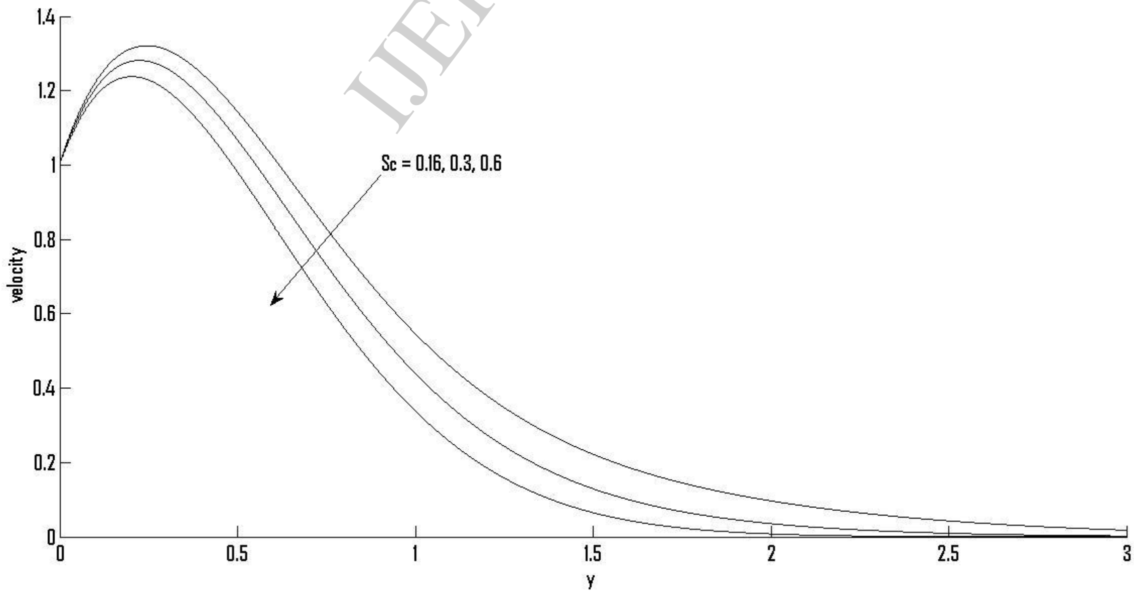

Figure7.Velocity profile for different values of Sc.

3.3 Concentration profiles

The effects of various parameters had been demonstrated in figures 8, 9, and 10. In figure 8, the concentration profile decreases with increase in the values of Sc. Also in figure 9, various values of K decreases the concentration profile. Furthermore, figure 10 shows that increase in values t results in decrease in concentration profile.

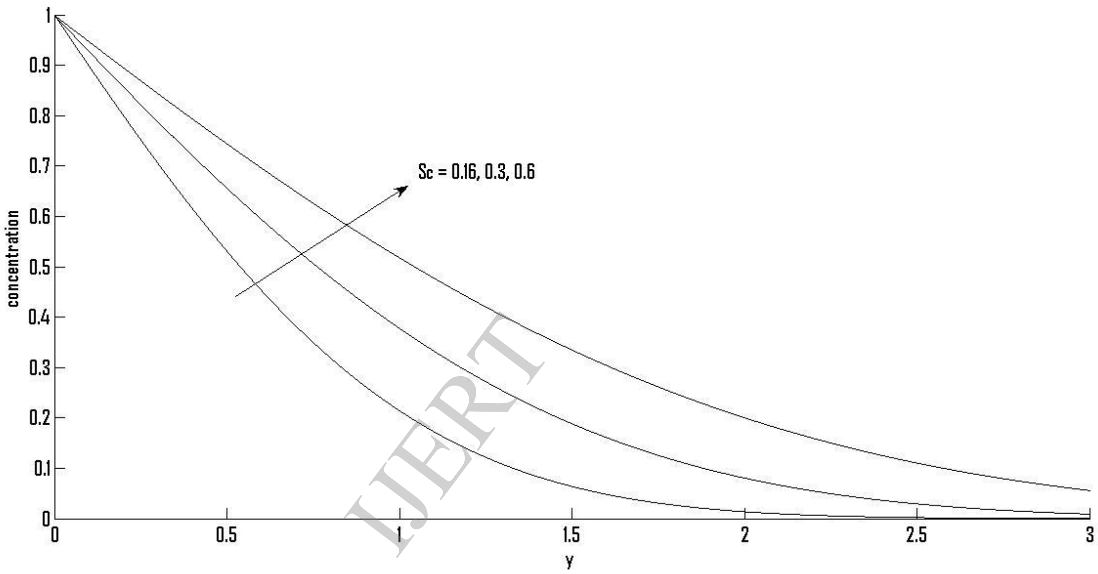

Figure 8.Concentrationprofile for different values of Sc.

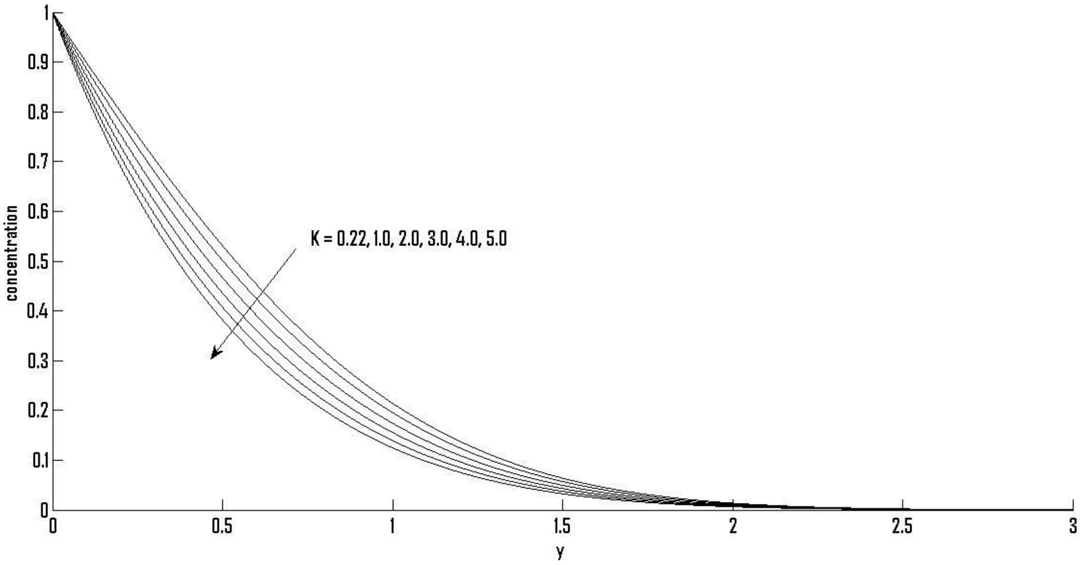

Figure 9.Concentration profile for different values of K.

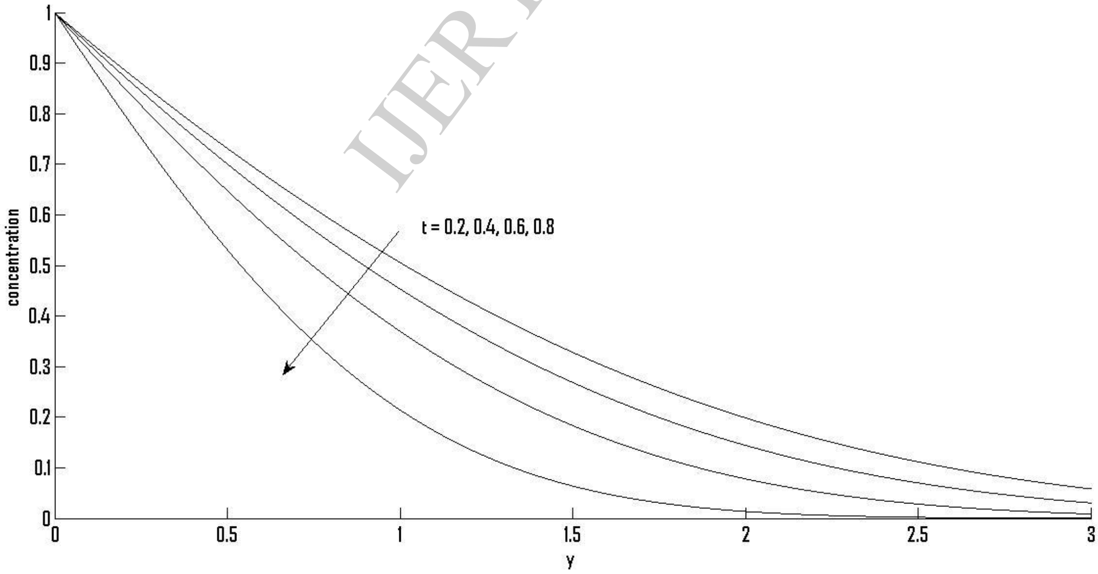

Figure 10.Concentration profile for different values of t.

4.0 Conclusions

The radiation effects on heat and mass transfer over a vertical plate with a Newtonian fluid is analysed. The dimensionless governing equations are solved by usual Laplace-transform method. The effect of different parameters like thermal Grashof number, mass Gashof number,Schmidt number and time are studied and discussed.The conclusions of the study are as follows;

The temperature increases with decreasing radiation and Prandtl number.The velocity increases with increasing values of Gr, Gc and t. The velocity decreases with increase in Sc and R.The concentration increases with decreasing Schmidt number and time.

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