

Radiation Effects on MHD Flow past an Exponentially Accelerated Infinite Isothermal Vertical Plate with Variable Temperature

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Abstract:- Radiation effects on MHD flow past an exponentially accelerated infinite isothermal vertical plate with variable temperature are considered here. The fluid considered is a gray, absorbing-emitting radiation but a non-scattering medium. An exact solution to the dimensionless governing equations is obtained by the Laplace transform method, when the plate is exponentially accelerated with a velocity $u = u_0 e^{at}$ in its own plane against gravitational field. All the numerical calculations are done with respect to air ($Pr=0.71$). The velocity and temperature are studied for different physical parameters like magnetic field parameter, radiation parameter, thermal Grashof number, time and an accelerating parameter. Expressions for velocity fields are obtained and secondary velocities are displayed graphically.

Keywords: Radiation Effects, accelerated, isothermal, vertical plate, temperature.

1. INTRODUCTION

MHD free convection flows past different types of vertical bodies are studied because of their wide applications. When free convection flows occur at high temperature, radiation effects on the flow become significant. Radiative flows are faced with in many industrial and environment processes, heating and cooling chambers, solar technology, gas turbines, for the design of fins, steel rolling, nuclear power plant and the various propulsion devices for aircraft, missiles and space vehicles are examples of such engineering areas and so on. Ahmmed, Parvin and Morshed [1] have obtained Radiation and mass transfer effects on MHD free convection flow past a vertical plate with variable temperature and concentration. Chandrakala and Narayana Bhaskar [2] have examined Radiation Effects on flow past an exponentially accelerated vertical plate with variable temperature. Ghara et al [3] discussed Effect of radiation on MHD free convection flow past an impulsively moving vertical plate with ramped wall temperature. Kishore, Rajesh and Vijayakumar Verma [4] have obtained Effects of heat transfer and viscous dissipation on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature.

Muralidharan and Muthucumaraswamy [5] have considered thermal radiation on linearly accelerated

vertical plate with variable temperature and uniform mass flux. Muthucumaraswamy and Visalakshi [6] presented MHD and thermal radiation effects on exponentially accelerated isothermal vertical plate with uniform mass diffusion. Rajesh and Vijaya Kumar Varma [7] studied Radiation and mass transfer effects on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature. Rajput and Surendra Kumar [8] presented Rotation Effect on MHD flow past an impulsively started vertical plate with variable mass diffusion. Rushi Kumar et al [9] studied Thermal diffusion and radiation effects on unsteady free convection flow in the presence of magnetic field fixed relative to the fluid or to the plate. Sami Ulhaq et al [10] investigated Radiation and Magneto hydrodynamics Effects on Unsteady Free Convection Flow in a Porous Medium. Srinivasa Sai and Jayarami Reddy [11] explains MHD free convection flow along a vertical porous plate through a porous medium, Journal for Advanced Research in Applied Sciences. Vijayalakshmi and Florence Kamalam [12] rendered Combined Effects of Thermal Radiation and MHD of Flow past an Accelerated Vertical Plate in a Rotating Fluid with Variable Temperature.

The governing equations have been transformed to a two point boundary value problem in similarity variables and the resultant problem is solved by Laplace transform method. The object of this study is to analyze the effects of radiation on infinite vertical plate with heat source by applying a simple technique. An exact solution of the partial differential equations governing the flow problem is obtained and the effects of various flow parameters on the velocity field are discussed with the help of figures.

2. BASIC EQUATIONS AND ITS SOLUTION

Magneto hydro dynamic flow past an exponentially started infinite isothermal vertical plate under the combined buoyancy force effects of thermal radiation is carried out. The flow is assumed to be in x' -direction which is taken along the vertical plate in the upward direction. The z -axis is taken to be normal to the plate. Initially at time $t' \leq 0$, both the fluid and plate are at rest and at uniform temperature T'_∞ . At time $t' > 0$, the plate starts

moving with a velocity $u = u_0 e^{at}$ in its own plane and temperature of the plate is raised or lowered to $T' = T'_w$. A magnetic field of uniform strength B_0 is applied transversely to the plate. By usual Boussinesq's approximation for unsteady flow is governed by the following set of equations:

$$\frac{\partial u}{\partial t'} = \nu \frac{\partial^2 u}{\partial z^2} + g\beta(T' - T'_\infty) - \frac{\sigma B_0 u}{\rho} \quad (1)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial z^2} - \frac{\partial q_r}{\partial z} \quad (2)$$

The initial and boundary conditions are

$$\left. \begin{aligned} u = 0, T' = T'_\infty \text{ for all } z, t' \leq 0 \\ t' > 0, u = u_0 e^{at}, T' = T'_w \text{ at } z = 0 \\ u \rightarrow 0, T' \rightarrow T'_\infty \text{ as } z \rightarrow \infty \end{aligned} \right\} \quad (3)$$

Here u is the velocity in the x -direction, ρ the density, g the acceleration due to gravity, β the coefficient of thermal expansion, T' the temperature of the fluid near the plate, C_p the specific heat at constant pressure, k the thermal conductivity, q_r the radiative flux, ν the kinematic viscosity and σ the electrical conductivity.

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial z} = -4a^* \sigma (T'^4_\infty - T'^4) \quad (4)$$

It is assumed that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher-order terms, thus

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \quad (5)$$

$$\theta(Z, t) = \frac{1}{2} \left[\exp(2\eta\sqrt{b\text{Pr}t}) \text{erfc}(\eta\sqrt{\text{Pr}} + \sqrt{bt}) + \exp(-2\eta\sqrt{b\text{Pr}t}) \text{erfc}(\eta\sqrt{\text{Pr}} - \sqrt{bt}) \right]$$

$$\begin{aligned} U(Z, t) = & f(\exp(2\eta\sqrt{(M+a)t}) \text{erfc}(\eta + \sqrt{(M+a)t}) + \exp(-2\eta\sqrt{(M+a)t}) \text{erfc}(\eta - \sqrt{(M+a)t})) \\ & - Ag(\exp(2\eta\sqrt{(M+d)t}) \text{erfc}(\eta + \sqrt{(M+d)t}) + \exp(-2\eta\sqrt{(M+d)t}) \text{erfc}(\eta - \sqrt{(M+d)t})) + \\ & \frac{A}{2} (\exp(2\eta\sqrt{Mt}) \text{erfc}(\eta + \sqrt{Mt}) + \exp(-2\eta\sqrt{Mt}) \text{erfc}(\eta - \sqrt{Mt})) + \end{aligned}$$

By using equations (6) and (7), equation (3) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial z^2} + 16a^* \sigma T'^3_\infty (T'_\infty - T') \quad (6)$$

Let us introducing the following non-dimensional quantities

$$\left. \begin{aligned} U = \frac{u}{u_0}, Z = \frac{zu_0}{\nu}, t = \frac{t'u_0^2}{\nu}, M = \frac{\sigma B_0^2 \nu}{\rho u_0^2}, \text{Pr} = \frac{\rho C_p}{k}, a = \frac{a' \nu}{u_0^2} \\ Gr = \frac{g\beta \nu (T'_w - T'_\infty)}{u_0^3}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, R = \frac{16a^* \sigma \nu^2 T'^3_\infty}{k u_0^2} \end{aligned} \right\} \quad (7)$$

where Pr the Prandtl number, R the radiation parameter, M the magnetic field parameter, Gr the Grashof number and θ the dimensionless temperature.

Using these boundary conditions in above equations, we obtain the following dimensionless form of the governing equations:

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial Z^2} + G_r \theta - MU \quad (8)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\text{Pr}} \frac{\partial^2 \theta}{\partial Z^2} - \frac{R}{\text{Pr}} \theta \quad (9)$$

The boundary conditions for corresponding order are

$$\left. \begin{aligned} U = 0, \theta = 0 \text{ for all } Z, t \leq 0 \\ U = e^{at}, \theta = 1 \text{ at } Z = 0 \\ U \rightarrow 0, \theta \rightarrow 0 \text{ as } Z \rightarrow \infty \end{aligned} \right\} t > 0 \quad (10)$$

The dimensionless governing equations (8) and (9), subject to the boundary conditions (10), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$Ag(\exp(2\eta\sqrt{\text{Pr}(b+d)t})\text{erfc}(\eta\sqrt{\text{Pr} + \sqrt{(b+d)t}}) + \exp(-2\eta\sqrt{\text{Pr}(b+d)t})\text{erfc}(\eta\sqrt{\text{Pr} - \sqrt{(b+d)t}}))$$

$$- \frac{A}{2}(\exp(2\eta\sqrt{\text{Pr}bt})\text{erfc}(\eta\sqrt{\text{Pr} + \sqrt{bt}}) + \exp(-2\eta\sqrt{\text{Pr}Mt})\text{erfc}(\eta\sqrt{\text{Pr} - \sqrt{bt}}))$$

Where $\eta = \frac{z}{2\sqrt{t}}$; $A = \frac{Gr}{(1-\text{Pr})d}$; $b = \frac{R}{\text{Pr}}$; $d = \frac{b\text{Pr} - M}{1-\text{Pr}}$; $f = \frac{\exp(at)}{2}$; $g = \frac{\exp(dt)}{2}$

3. RESULTS AND DISCUSSION

A set of numerical computations are carried out for parameters M, R, Gr, a and t. The values of Prandtl number Pr are chosen 0.71(air). The numerical values of the velocity and temperature are computed for different physical parameters are computed graphically.

The temperature profile for different values of thermal radiation parameter (R=0.2, 2.0, 2.0, 5.0) and time (t=0.2, 0.6, 0.2, 0.4) and these are shown in figure 1. It is observed that the temperature increases with decreasing radiation parameter and the temperature increases with increase of time t.

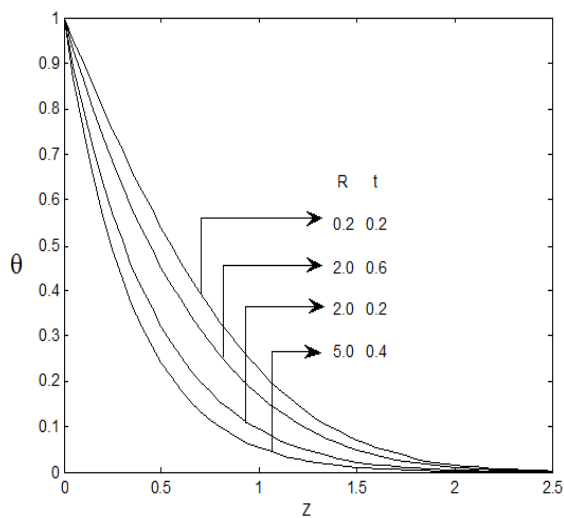


Figure1. Temperature profiles for different values of R and t

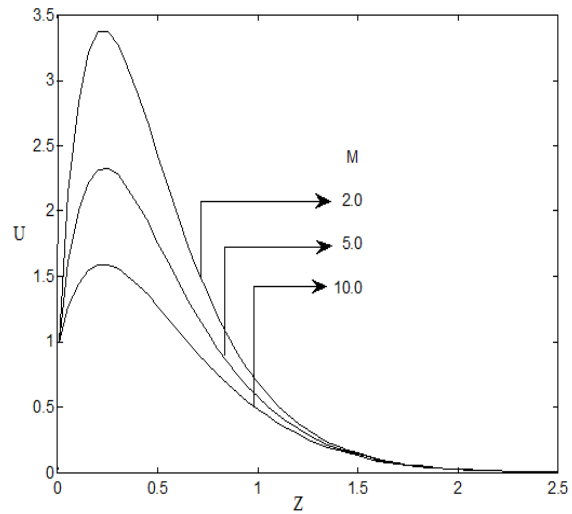


Figure2. Velocity profiles for different values of M

The velocity profiles for different values of the magnetic field parameter are shown in figure 2 when Pr=0.71, a=0.1, t=0.2, Gr=5, R=5. It is observed that the velocity decreases in the presence of magnetic field than its absence. This shows that the increase in the magnetic field parameter

leads to fall in the velocity. This agrees with expectations, since the magnetic field exerts a retarding force on the free convective flow.

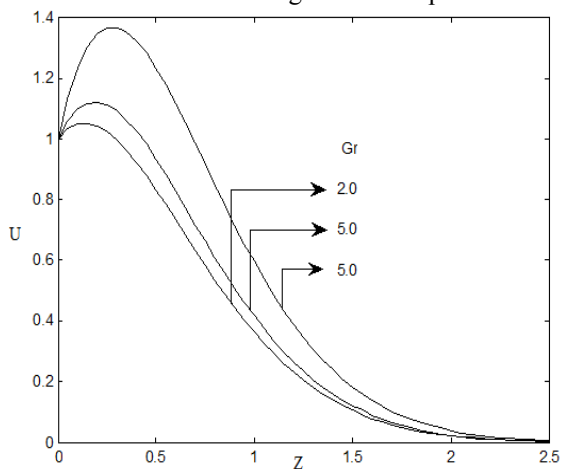


Figure3. Temperature profiles for different values of Gr

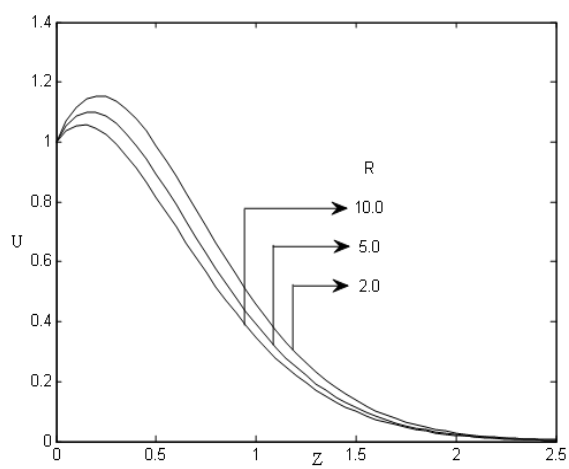


Figure4. Velocity profiles for different values of R

In the figure 3 it is observed that the velocity increases with increasing values of the thermal Grashof number. From figure 4 it is clear that the velocity increases with

decreasing values of radiation parameter. This shows that velocity decreases in the presence of high thermal radiation when $Pr=0.71$, $a=0.1$, $t=0.2$, $M=0.5$, $R=5$.

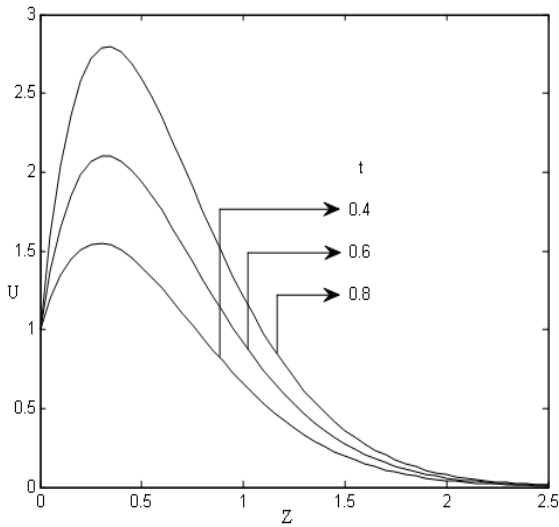


Figure5. Temperature profiles for different values of t

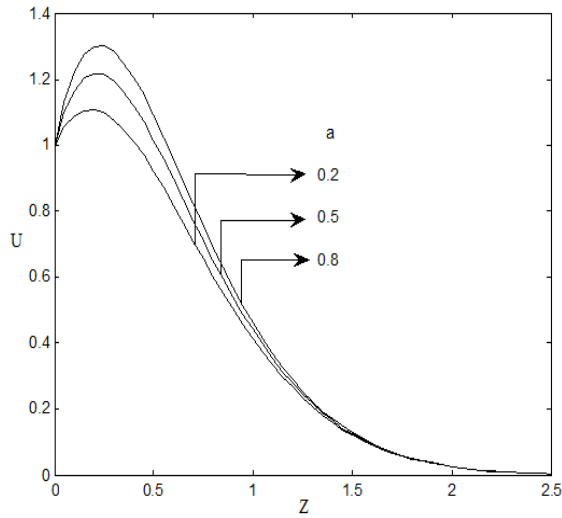


Figure6. Velocity profiles for different values of a

The velocity profiles for different values of time t when $Pr=0.71$, $R=5$, $a=0.1$, $M=0.5$, $Gr=5$, are presented in figure 5. It is observed that temperature increases with increasing values of time. The velocity profiles for different 'a' when $Pr=0.71$, $M=0.5$, $Gr=5$, $R=5$, $t=0.2$ are studied and presented in figure 6. It is evident from figures that the velocity increases with increasing values of 'a'.

4. CONCLUSION

An analytical solution for the velocity field in the presence of radiation effects on MHD flow past an exponentially accelerated infinite isothermal vertical plate with variable temperature are constructed. A uniform magnetic field is applied transversely to the flow. The dimensionless governing equations are solved by the Laplace transform technique. The effects of temperature and velocity profiles for different parameters like M , Gr , R , a and t are discussed graphically. It is observed that the velocity decreases with increasing values of magnetic field parameter and radiation parameter but increases with increasing values of Gr , a and t .

5. NOMENCLATURE

- a - Dimensionless accelerating parameter
- a' - Dimensional acceleration parameter
- a^* - Absorption coefficient
- B_0 - magnetic field strength
- C_p - specific heat at constant pressure
- g - acceleration due to gravity
- Gr - thermal Grashof number
- k - thermal conductivity of the fluid
- M - the magnetic field parameter

- Pr - Prandtl number
- qr - radiative heat flux
- R - the radiation parameter
- T' - temperature of the fluid near the plate
- T'_w - temperature of the plate
- T'_∞ - temperature of the fluid far away from the plate
- t' - time
- t - dimensionless time
- U - dimensionless velocity
- u - velocity of the fluid in the x -direction
- u_0 - velocity of the plate
- Z - coordinate axis normal to the plate
- z - dimensionless coordinate axis normal to the plate

Greek symbols

- β - the coefficient of thermal expansion
- θ - the dimensionless temperature.
- ν - the kinematic viscosity
- ρ - the density
- σ - stefan-Boltzmann constant
- $erfc$ - complementary error function

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