Radio D-Distance Number of Some Basic Graph

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Abstract – A Radio d-distance labeling of a connected graph G is an injective map f from the vertex set V(G) to $\mathbb{N}$ **such that for two distinct vertices u and v of G,** $d^d(u,v) + |f(u) - f(v)| \geq 1 + diam^d(G)$, **where** $d^d(u,v)$ **denotes the d-distance between u and v and** $diam^d(G)$ **denotes the d-diameter of G. The Radio d-distance number of f,** $rn^d(f)$ **is the maximum label assigned to any vertex of G. The Radio d-distance number of G,** $rn^d(G)$ **is the minimum value f of G. In this paper we find the radio d-distance number of some basic graphs.**

Keywords–d-distance, Radio d-distance, Radio d-distance number.

I.   INTRODUCTION

By a graph $G = (V(G), E(G))$ we mean a finite undirected graph without loops or multiple edges. Let (G) and (G) denotes the vertex set and edge set of G. The order and size of G are denoted by p and q respectively.T. Jackuline, J. Golden Ebenezer Jebamani and D. Premalatha introduced the concept of d^d-distance by considering the degrees of various vertices presented in the path, in addition to the length of the path.

Let u, v be two vertices of a connected graph G. Then the d-length of a u-v path defined as d^d(u,v) = d(u, v) + deg(u) + deg(v) + deg(u) deg(v), where d(u, v) is the shortest distance between the vertices u and v

In this paper, we introduced the concept of radio d-distance labeling of a graph G. Radio   d-distance labeling is a function f from V(G) to $\mathbb{N}$ satisfying the condition $d^d(u,v) + |f(u) - f(v)| \geq 1 + diam^d(G)$, where $diam^d(G)$ is the d-distance diameter of G. A d-distance radio labeling number of G is the maximum label assigned to any vertex of G. It is denoted by $rn^{Gd}(G)$. Let G be a connected graph of diameter d and let k an integer such that $1 \leq k \leq d$. A radio k-coloring of G is an assignment f of colors (positive integers) to the vertices of G such that $d(u,v) + |f(u) - f(v)| \geq 1 + k$ for every two distinct vertices u, v of G. The radio k-coloring number $rc_k(f)$ of a radio k-coloring f of G is the maximum color assigned to any vertex of G. The radio k-chromatic number $rc_k(G)$ is min$\{rc_k(f) \}$ over all radio k-colorings f of G. A radio k-coloring f of G is a minimal radio k-coloring if $rc_k(f) = rc_k(G$. When k = Diam(G), the resulting radio k-coloring is called radio coloring of G. The radio number of

G is defined as the minimum span of a radio coloring of G and is denoted as rn(G).

Radio labeling can be regarded as an extension of distance-two labeling which is motivated by the channel assignment problem introduced by W. K. Hale [6]. G. Chartrand et al.[2] introduced the concept of radio labeling of graph. Also G. Chartrand et al.[3] gave the upper bound for the radio number of path. The exact value for the radio number of path and cycle was given by Liu and Zhu [10]. However G. Chartrand et al.[2] obtained different values for them. They found the lower and upper bound for the radio number of cycle. Liu [9] gave the lower bound for the radio number of Tree. The exact value for the radio number of Hypercube was given by R. Khennoufa and O. Togni [8]. In [4] C. Fernandez et al. found the radio number for complete graph, Star graph, Complete Bipartite graph, Wheel graph and Gear graph. In this paper, we fined the radio d-distance labeling of some basic graphs.

II.   MAIN RESULTS

Theorem 2.1

The radio d-distance number of the complete graph, $rn^d(K_n) = n \; \forall \; n$

Proof

Let, V(K_n) = $\{v_1, v_2, \dots, v_n\}$ be the vertex set

$then, d^d(v_i, v_j) = n^2 \; for \; 1 \leq i, j \leq n$

It is obvious that the $diam^d(K_n) = n^2$

The radio d-distance condition is
$$d^d(u,v) + |f(u) - f(v)| \geq 1 + diam^d(G) = n^2 + 1$$

Now, fix $f(v_1) = 1$
$$d^d(v_{1,} v_2) + |f(v_1) - f(v_2)| \geq n^2 + |1 - f(v_2)|$$
$$\geq n^2 + 1$$

$|1 - f(v_2)| \geq 1,$ which implies $f(v_2) = 2$

$\therefore f(v_i) = i, \ 1 \leq i \leq n$

Hence, $rn^d(K_n) = n, \forall n$

Theorem 2.2

The radio d-distance number of a path
$rn^d(P_n) \leq n^2 - 4n + 10, \ n \geq 4$

Proof.

Let $V(P_n) = \{v_1, v_2, \ldots, v_n\}$ be the vertex set and $E(P_n) = \{v_i v_{i+1} ; 1 \leq i \leq n-1\}$ be the edge set

Then, $d^d(v_1, v_n) = d^d(v_2, v_n) = n + 2,$
$d^d(v_1, v_2) = d^d(v_{n-1}, v_n) = 6, d^d(v_i, v_{i+1}) = 9;$
$2 \leq i \leq n - 2, d^d(v_2, v_{n-1}) = n + 5$

It is clear that $diam^d(P_n) = n + 5$
Without loss of generality $f(v_1) < f(v_2) < \cdots < f(v_n)$

We shall check the radio d-distance condition
$d^d(u, v) + |f(u) - f(v)| \geq 1 + diam^d(G) = n + 6$

Fix $f(v_1) = 1$ for (v_1, v_2)
$d^d(v_1, v_2) + |f(v_1) - f(v_2)| \geq 6 + |1 - f(v_2)| \geq n + 6$
$|1 - f(v_2)| \geq n$, which implies $f(v_2) = n + 1$

For (v_2, v_3)
$d^d(v_2, v_3) + |f(v_2) - f(v_3)| \geq 9 + |n + 1 - f(v_3)| \geq n + 6$
$|n + 1 - f(v_3)| \geq n - 3$, which implies $f(v_3) = 2n - 2$

$\therefore f(v_i) = n(i - 1) - 3i + 7, \ 2 \leq i \leq n - 1$

Hence, $rn^d(P_n) \leq n^2 - 4n + 10, \ n \geq 4$

Note. $rn^d(P_n) = n$ if n = 2,3

Theorem 2.3

The radio d-distance number of a star graph,
$rn^d(K_{1,n}) = 2n^2 - 4n + 3, \ n \geq 3$

Proof.

Let $V(K_{1,n}) = \{v_0, v_1, v_2, \ldots, v_n\}$ be the vertex set, where v_0 be the central vertex and
$E(K_{1,n}) = \{v_0 v_i ; 1 \leq i \leq n\}$ be the edge set

Then, $d^d(v_0, v_i) = 2n + 2; 1 \leq i \leq n, d^d(v_i, v_j) = 5;$
$1 \leq i, j \leq n; i \neq j$

So, $diam^d(K_{1,n}) = 2n + 2$
Without loss of generality,
$f(v_1) < f(v_0) < f(v_2) < \cdots < f(v_n)$

We shall check the radio Gd-distance condition
$d^d(u, v) + |f(u) - f(v)| \geq 1 + diam^d(G) = 2n + 3$

Fix $f(v_1) = 1$, for (v_1, v_0)
$d^d(v_1, v_0) + |f(v_1) - f(v_2)| \geq 2n + 2 + |1 - f(v_0)| \geq 2n + 3$
$|1 - f(v_0)| \geq 1$, which implies $f(v_0) = 2$

For (v_1, v_2)
$d^d(v_1, v_2) + |f(v_1) - f(v_2)| \geq 5 + |1 - f(v_2)| \geq 2n + 3$
$|1 - f(v_2)| \geq 2n - 2$, which implies $f(v_2) = 2n - 1$

For (v_2, v_3)
$d^d(v_2, v_3) + |f(v_2) - f(v_3)| \geq 5 + |2n - 1 - f(v_3)| \geq 2n + 3$
$|2n - 1 - f(v_3)| \geq 2n - 2$, which implies $f(v_3) = 4n - 3$

$\therefore f(v_i) = (2i - 2)n - 2i + 3, \ 1 \leq i \leq n$

Hence, $rn^d(K_{1,n}) = 2n^2 - 4n + 3, \ n \geq 3$

Theore. 2.4

The radio d-distance number of bistar graph,
$rn^d(B_{n,n}) = 2n^3 + 8n^2 - 4n + 2, \ n \geq 2$

Proof.

Let $V(B_{n,n}) = \{v_1, v_2, \ldots, v_n, x_1, x_2, u_1, u_2, \ldots, u_n\}$ be the vertex set
and $E(B_{n,n}) = \{x_1 v_i, x_2 u_i, x_1 x_2 ; 1 \leq i \leq n\}$ be the edge set

Then, $d^d(x_1, v_i) = d^d(x_2, u_i) = 2n + 4; 1 \leq i \leq n,$
$d^d(x_1, x_2) = (n + 2)^2, d^d(u_i, v_j) = 6; 1 \leq i, j \leq n,$
$i \neq j, d^d(v_i, v_j) = d^d(u_i, u_j) = 5; 1 \leq i, j \leq n$

It is clear that $diam^d(B_{n,n}) = (n + 2)^2 = n^2 + 4n + 4$

Without loss of generality, $f(u_1) < \cdots < f(u_n) < f(x_2) < f(x_1) < f(v_1) < \ldots < f(v_n)$

We shall check the radio d-distance condition
$d^d(u, v) + |f(u) - f(v)| \geq 1 + diam^d(G) = n^2 + 4n + 5$

Fix, $f(u_1) = 1$, For (u_1, u_2)
$d^d(u_1, u_2) + |f(u_1) - f(u_2)| \geq 5 + |1 - f(u_2)| \geq n^2 + 4n + 5$
$|1 - f(u_2)| \geq n^2 + 4n$, which implies
$f(u_2) = n^2 + 4n + 1$

For (u_2, u_3)
$d^d(u_2, u_3) + |f(u_2) - f(u_3)| \geq 5 + |n^2 + 4n + 1 - f(u_3)| \geq n^2 + 4n + 5$
$|n^2 + 4n + 1 - f(u_3)| \geq n^2 + 4n$ which implies

$f(u_3) = 2n^2 + 8n + 1$

$\therefore f(u_i) = n^2(i - 1) + n(4i - 4) + 1, \ 1 \leq i \leq n$

Therefore, $f(u_n) = n^3 + 3n^2 - 4n + 1$

For (u_n, x_2)

$d^d(u_n, x_2) + |f(u_n) - f(x_2)|$

$\geq 2n + 4 + |n^3 + 3n^2 - 4n + 1 - f(x_2)|$

$$\geq n^2 + 4n + 5$$

$| n^3 + 3n^2 - 4n + 1 - f(x_2)| \geq n^2 + 2n + 1$, which implies $f(x_2) = n^3 + 4n^2 - 2n + 2$

For (x_2, x_1)
$$d^d(x_2, x_1) + |f(x_2) - f(x_1)|$$
$$\geq n^2 + 4n + 4 + | n^3 + 4n^2 - 2n + 2 - f(x_1)|$$
$$\geq n^2 + 4n + 5$$

$| n^3 + 4n^2 - 2n + 2 - f(x_1)| \geq 1$, which implies
$f(x_1) = n^3 + 4n^2 - 2n + 1$

For (x_1, v_1)
$$d^d(x_1, v_1) + |f(x_1) - f(v_1)|$$
$\geq 2n + 4 + | n^3 + 4n^2 - 2n + 1 - f(v_1)| \geq n^2 + 4n + 5$
$| n^3 + 4n^2 - 2n + 1 - f(v_1)| \geq n^2 + 2n + 1$, which implies $f(v_1) = n^3 + 5n^2 + 2$

For (v_1, v_2)
$$d^d(v_1, v_2) + |f(v_1) - f(v_2)|$$
$\geq 5 + |n^3 + 5n^2 + 2 - f(v_2)| \geq n^2 + 4n + 5$
$|n^3 + 5n^2 + 2 - f(v_2)| \geq n^2 + 4n$, which implies $f(v_2) = n^3 + 6n^2 + 4n + 2$

For (v_2, v_3)
$$d^d(v_2, v_3) + |f(v_2) - f(v_3)|$$
$\geq 5 + |n^3 + 6n^2 + 4n + 2 - f(v_3)| \geq n^2 + 4n + 5$
$|n^3 + 6n^2 + 4n + 2 - f(v_2)| \geq n^2 + 4n$, which implies $f(v_2) = n^3 + 7n^2 + 8n + 2$

$\therefore f(v_i) = n^3 + (i + 4)n^2 + (4i - 4)n + 2, \ 1 \leq i \leq n$
Hence, $rn^d(B_{n,n}) = 2n^3 + 8n^2 - 4n + 2, \ n \geq 2$

Theorem 2.5

The radio d-distance number of a subdivision of a star, $rn^d S(K_{1,n}) = 6n^2 - 12n + 6, \ n \geq 3$

Proof

Let $V(S(K_{1,n})) = \{v_0, v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n,\}$ be the vertex set, where v_0 is the central vertex and $E(S(K_{1,n})) = \{v_0 u_i, v_i u_i; \ 1 \leq i \leq n\}$ be the edge set

Then, $d^{Gd}(v_0, u_i) = 3n + 3; 1 \leq i \leq n$,
$d^{Gd}(v_i, v_j) = 7, d^{Gd}(u_i, u_j) = 10; 1 \leq i, j \leq n$
$d^{Gd}(v_i, u_i) = 6; \ 1 \leq i \leq n$

It is clear that $diam^d(S(K_{1,n})) = 3n + 3$
Without loss of generality $f(u_1) < f(v_0) < f(u_2) < \cdots < f(u_n) < f(v_1) < \cdots < f(v_n)$

We shall check the radio d-distance condition
$$d^d(u, v) + |f(u) - f(v)| \geq 1 + diam^d(G) = 3n + 4$$

Fix $f(u_1) = 1$, for $(u_1, u_0) \ 1 \leq i \leq n$
$$d^d(u_1, v_0) + |f(u_1) - f(v_0)| \geq 3n + 3 + |1 - f(v_0)|$$
$$\geq 3n + 4$$

$|1 - f(v_0)| \geq 1$ which implies $f(v_0) = 2$

For $(u_i, u_{i+1}), \ 1 \leq i \leq n - 1$
$$d^d(u_1, u_2) + |f(u_1) - f(u_2)| \geq 10 + |1 - f(u_2)|$$
$$\geq 3n + 4$$
$|2 - f(u_2)| \geq 3n - 6$, which implies $f(u_2) = 3n - 5$

For $(u_2, u_3), \ 1 \leq i \leq n$
$$d^d(u_2, u_3) + |f(u_2) - f(u_3)| \geq 10 + |3n - 5 - f(u_3)|$$
$$\geq 3n + 4$$
$|3n - 5 - f(u_3)| \geq 3n - 6$, which implies
$f(u_3) = 6n - 11$

$\therefore f(u_i) = n(3i - 3) - 6i + 7, \ 1 \leq i \leq n$

Therefore, $f(u_n) = 3n^2 - 9n + 7$

For $(u_n, v_i), \ 1 \leq i \leq n$
$$d^d(u_n, v_1) + |f(u_n) - f(v_1)|$$
$$\geq 8 + |3n^2 - 9n + 7 - f(v_1)| \geq 3n + 4$$

$|3n^2 - 9n + 7 - f(v_1)| \geq 3n - 4$ which implies
$$f(v_1) = 3n^2 - 6n + 3$$

For $(v_i, v_{i+1}), \ 1 \leq i \leq n - 1$
$$d^d(v_1, v_2) + |f(v_1) - f(v_2)|$$
$$\geq 7 + |3n^2 - 6n + 3 - f(v_2)| \geq 3n + 4$$
$|3n^2 - 6n + 3 - f(v_2)| \geq 3n - 3$, which implies
$f(v_2) = 3n^2 - 3n$

For (v_2, v_3)
$d^d(v_2, v_3) + |f(v_2) - f(v_3)| \geq 7 + |3n^2 - 3n - f(v_3)|$
$$\geq 3n + 4$$
$|3n^2 - 3n - f(v_3)| \geq 3n - 3$, which implies
$$f(v_3) = 3n^2 - 3$$

$\therefore f(v_i) = 3n^2 + n(3i - 9) - 3i + 6, \ 1 \leq i \leq n$

Hence, $rn^d S(K_{1,n}) \leq 6n^2 - 12n + 6, \ n \geq 3$

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