# Real-time Identification of Electromechanical Modes using Controlled Window-size Multi-Prony Analysis

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Abstract—Pronv analysis has become powerful tool in the modal contentes timation of power oscillations from measured transientdata. The accuracy of the modalestimation is limited by inherent noise present in the measured signals. Transients in the system cause oscillations in the system. The seoscillations, when estimated for the different signals in the system, having identical modal characteristic give the conflicting mode estimates. Instead of individual modal estimate for such signals, all these signals are analyzed simultaneously using multi-signal Prony analysis (MPA), resulting in one set of mode estimates. Ithasbecomepossibleto analyzeelectromechanical modeson-line by using phasor measurement unit (PMU)and globalpositioning system(GPS).Window-sizecontrolled (WSC) MPA is used to achieve the balance betweentherequirementsofquantity ofdata andtime forestimation. Same test system ifanalyzed either for fordifferent faultclearing times, behaves differently. The proposed methodsaresimulated forthe22-bus,6machinestest systemand 6machine, IEEE 30-bustestsystem.

Keywords—controlled window-size analysis; multi-Prony analysis; power system oscillations; PMU; wide area measurement system (WAMS)

# I. INTRODUCTION

Power system capacity is ever increasing. [1]has stated that modern power systems have stretched out hundreds and thousands of kilometers, it has resulted into system of very large size. In addition, [1]has proposed that, with growing generation capacity, different areas in a power system having large inertias are added. Furthermore, the power system has unbundledinto generation, transmission and supply system. It is less oriented towards the physical nature of the synchronously interconnected power systems,but it span a large area with interaction among the different sub networks and the power plants. However, with possible higher loading of the transmission system the network operators may be forced to operate the system closer to its stability limits.

Issue of small signal stability, especially inter-area oscillations, become important in large interconnected power systems [1]. Large power systems all over the world are facing the common problem of inter-area oscillations. As explained in [1], many electric systems in the world are experiencing increased loading on portions of their transmission systems, which cansometimes lead to poorly damped low frequency (0.2-0.8 Hz) inter-area oscillations. According to [2], in practical system, various modes of oscillations can be classified into three broad categories. First, intra-plant modes, in which generators in a power plant participate. The oscillation

frequencies are generally high in the range of 1.5 to 3 Hz. Second, local modes, in this case, several generators within a particular area participate. The frequencies of oscillations are in the range of 0.8 to 1.8 Hz. Third, inter-area modes, in which generators over an extensive area participate. The oscillation frequencies are low and in the range 0.2 to 0.8 Hz. Inter-area oscillations can sometimes, severely restrict system operations. It may require curtailment of electric power transfer as an operational measure in such consequence. These oscillators can also lead to widespread system blackoutsdue to oscillatory power swings[1].

In order to damp these oscillations it is needed to detect them, and control them so that, the power system is controlled. Electromechanical oscillations can be investigated by [1]simulation method in time domain. In addition to that, a more powerful approach in the frequency domain is available to enable systematic analysis of the small signal stability problem. The latter approach is known as Eigenvalue analysis or modal analysis utilized in [3] - [6]. Eigenvalue analysis investigates the dynamic behavior of a power system under different characteristic frequencies (i.e. modes). Inherent patterns behind complicated phenomena of system dynamics are indicated. Different modes, which are mixed with each other in curves of time domain simulation, are identified separately. [7] had discussed, the operation of power system by analysing the signals expressing the generator coherent groups.It explains, which one of generator coherent groups swing against each other and which generators play a significant role in maintaining the stability of the power system.

In a power system, all modes, i.e. eigenvalues, are need to be stable. Moreover, [1] has stated that, it is important, that, all electromechanical oscillations are damped out as quickly as possible. Eigenvalue analysis is given as frequency and relative damping for each oscillatory mode and results are analysed. Given an oscillatory mode,  $s = \sigma + j\omega$ , the damping ratio (or relative damping) is defined by  $\zeta = -\sigma/\sqrt{\sigma^2 + \omega^2}$ .

In order to analyze the modes of interest, WAMS and PMUs data is analyzed using different computation techniques. Fourier analysis can be used for the analysis of such signals. Fourier series computes amplitude, phase and frequency of the signal components. [8]has given some facts that, Fourier transforms distributes time-domain noise uniformly throughout the frequency domain.It can limit the certainty with which peak frequencies, widths, magnitudes and phases could be computed.Prony analysis (PA)has been discussed by [8] and

[9] in detail. If PA is used for analysis of the measured data, has the advantage of estimating damping coefficients apart from frequency, phase and amplitude. It is computationally less expensive and can accurately extract poles and residues. In addition, [8] has addressed that, due to its sensitivity to measurement noise PA yields parameter estimates with a large bias. In addition to poor fit when signal to noise ratio is small, PA is also known to be inconsistent. In this case damping and frequency terms are typically significantly miss estimated and they found to be usually much greater than the actual values.

The inherent noise in measured data can appreciably eliminated if least square Prony(LSM) analysis technique is used for the analysis. Most of the previous Prony analysis extensions assume the model to be single output [10] - [13]. This is limiting because many systems, including power systems, are multi-output. Prony analysis has been applied to several power system problems. It has proven to be a very valuable tool for transient analysis and advanced monitoring. In this paper OPA has extended to achieve multi-signal fit. A multi-signal fit, leads to quality mode estimates for the overall system because, in this case essentially richer information to the analysis has been provided.

MPA uses multiple signals for the analysis and hence, the system of equations to be solved becomes over-determined. To solve this over-determined system [14] and [15] has used well known least square method (LSM). However, an important balance between requirement of quantity of data and time for estimation must be achieved for this important task. Using current single-output analysis, individual signals are analyzed independently often resulting in conflicting mode estimates. The user is then left with the problem of determining which modal estimates are more accurate. The parameter estimation/ using PA is off-line process and oscillation parameters are time varying. OPA have been proven to be conditionally efficient in estimating parameters of exponentially damped sinusoids from a batch of measurement data. It suffer from a common disadvantage, i.e., these techniques are all processed off-line and therefore, are not suitable for detecting and tracking the time-varying parameters such as damping factors and frequencies. These shortcomings motivate the application of on-line estimation algorithms such as the Kalman filter (KF) as explained by [16], which has the ability to separate signals from noises, and to estimate latent states and cope with any variations in states. In the present analysis, identification of electromechanical modes using MPAwith window-size control (WSC) has been carried out for IEEE 30 bus test system as well as 22 bus test system. Section II is discussed the solution strategies, section III is discussed about Algorithm for on-line electromechanical mode estimation in power systems, brief discussions over the results are given in section IV; lastly, paper ends with conclusions in section V.

# II. EXTENSION OF ORIGINAL SINGLE-SIGNAL PRONY ANALYSIS TO MULTI-SIGNAL PRONY ANALYSIS

# A. Original Prony Analysis (OPA)

To derive the mathematical formulation for the OPA, let us consider a linear time-invariant (LTI) dynamic system as shown in Fig. 1.



Fig. 1. LTIsystem

The signals are referred to as: y(t): system response, x(t): state of the system, and u(t): input of the system. The evolution of the state of the system is expressed by (1)

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t) \tag{1}$$

where A and B are constant matrices. If the input is removed (u(t) = 0) and there are no subsequent inputs to the system, then (1) can be rewritten as

$$\frac{dx(t)}{dt} = Ax(t) \tag{2}$$

where A is a matrix of size n x n whose eigenvalues are  $\lambda_i$ , right eigenvectors are  $p_i$  and left eigenvectors are  $q_i$ . In (2) system order is represented by 'n'. The solution to (2) is expressed as the sum of n components as:

$$x(t) = \sum_{i=1}^{n} (q_i^T x) p_i e^{\lambda_i t}$$
(3)

As the system is a LTI system, we express y(t) in the form

$$y(t) = Cx(t) + Du(t)$$
(4)

where, *C* and *D* are constant matrices. If the input is removed (u(t) = 0) then (4) simplifies to (5)

$$y(t) = Cx(t) \tag{5}$$

The PA directly estimates the parameters of the eigen structure described in (3) by fitting a sum of complex damped sinusoids to evenly spaced samples (in time) values of the output as:

$$\hat{y}(t) = \sum_{i=1}^{n} A_i e^{(\sigma_i t)} \cos \mathbb{Z} \pi f_i t + \varphi_i$$
(6)

In (6) further notations are used,  $A_i$ : amplitude of component  $i, \sigma_i$ : damping coefficient of component  $i, \varphi_i$ : phase of component  $i, f_i$ : frequency of component i, n: total number of damped exponential components,  $\hat{y}(t)$ : estimate of observed data for y(t) consisting of N samples  $y(t_k) = y[k], k = 0, 1, 2, ..., (N-1)$  that are evenly spaced. Using Euler's theorem  $cos \mathbb{Z}2\pi f_i t + \varphi_i$ ) can be represented as a sum of exponentials:

$$\frac{\cos(2\pi f_i t + \varphi_i)}{e^{j(2\pi f_i t + \varphi_i)} + e^{-j(2\pi f_i t + \varphi_i)}}$$
(7)

Inserting (7) in (6) and letting t = kT, sampling period (*T*) less than the Nyquist period, the samples of  $\hat{y}(t)$  in (6) are rewritten as

$$y(kT) = \sum_{i=1}^{n} B_i z_i^{\ k}$$
(8)

$$B_i = \frac{A_i}{2} e^{j\varphi_i} \text{ and } z_i = e^{(\sigma_i + j2\pi f_i)T}$$
(9)

 $B_i \in C$  is the output residue for the continuous-time pole  $\lambda_i \in C$  ( $\lambda_i = \sigma_i + j2\pi f_i$ ), C indicates complex vector space.  $\lambda_i \neq \lambda_j$  for  $i \neq j$ , there must not be repeated eigen-values. The objective is to identify the residues, poles, and n that force (8) to be the least-squares fit to y(t). The OPA computes  $B_i$  and  $z_i$  in three basic steps. First, solve linear prediction (LP) model, which is constructed by the observed data set. First write (8) as a linear prediction model as:

$$y[k] = a_1 y[k-1] + a_2 y[k-2] + \dots + a_n y[k-n]$$
(10)

In (10), y[k] is computed for k = n, n + 1, ..., (N - 1),  $k^{th}$  sample is expressed in terms of previous data samples. For example, y[n] is computed at k = n

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + \dots + a_n y[0]$$
(11)

Expressing y[k] in matrix form for various values of k as

$$\begin{bmatrix} y[n] \\ y[n+1] \\ \vdots \\ y[N-1] \end{bmatrix} = \begin{bmatrix} y[n-1] & \cdots & \cdots & y[0] \\ y[n] & \cdots & \cdots & y[1] \\ \vdots & \vdots & \vdots & \vdots \\ y[N-2] & \cdots & \cdots & y[N-n-1] \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$
(12)

Assuming N > 2n the linear prediction coefficients vector a is estimated by solving over-determined least square problem. Second, find roots of characteristic polynomial (13) formed from the linear prediction coefficients.

$$z^n - a_1 z^{n-1} - a_2 z^{n-2} - \dots - a_n = 0$$
(13)

s vector **a** is known from (12), the roots  $\hat{z}_i$  of polynomial (13) can be computed. Third, solve the original set of linear equations to yield the estimates of the exponential amplitude and sinusoidal phase. Solving the (14) the residue vector **B** is calculated,

$$\begin{bmatrix} y[0]\\ y[1]\\ \vdots\\ y[N-1] \end{bmatrix} = \begin{bmatrix} 1 & 1 & \dots & 1\\ \hat{z}_1 & \hat{z}_2 & \dots & \hat{z}_n\\ \vdots & \vdots & \ddots & \vdots\\ \hat{z}_1^{N-1} & \hat{z}_2^{N-1} & \dots & \hat{z}_n^{N-1} \end{bmatrix} \begin{bmatrix} B_1\\ B_2\\ \vdots\\ B_n \end{bmatrix}$$
(14)

Compiling (8) into matrix form results in a Vandermonde matrix, this is termed the Vandermonde problem.

In most cases first and third steps involve the solution of over-determined sets of equations. Numerical procedure has been assumed that n (order of system) is known and is the order used in the LP and Vandermonde problems. Methods for choosing the model order are described in [15]. These methods involve over estimating the initial order and then selecting the

minimal subset of pole-residue pairs that best models the signal. PS is MIMO system and performing a multi-signal fit essentially improves quality of the analysis. The extension of OPA to multi-signal fit is proposed in next subsection.

# B. Multi-Prony Analysis (MPA)

The equations are an exception to the prescribed specifications of this template. You will need to determine whether or not your equation should be typed using either the Times New Roman or the Symbol font (please no other font). To create multileveled equations, it may be necessary to treat the equation as a graphic and insert it into the text after your paper is styled.

MPA is used to estimate the modes of oscillation of group of signals that are supposed to be governed by the same modes and how effectively, overall system behaviour can be analysed, has been explained here. Now consider a set of Msignals  $y_m(t), m = 1, 2, ..., M$  that share a common set of eigenvalues. An example would be the transient response measured at M locations. Linear analysis shows that the  $m^{th}$ signal is modelled as

$$\hat{y}_m(kT) = \sum_{i=1}^n B_{m_i} z_i^k, \qquad k = 0, 1, (N_m - 1)$$
(15)

 $B_{mi} \in \mathbf{C}$  is the output residue for the continuous-time pole  $\lambda_i \in \mathbf{C}$ ,  $\lambda_i \neq \lambda_j$  for  $i \neq j$ , and  $B_{mi} = \frac{A_{mi}}{2} e^{j \varphi_{mi}}$  and  $z_i = e^{(\sigma_i + 2\pi f_i)T}$ . As in the single signal case, each sampled version of  $\hat{y}_m$  satisfies (3). In the multi-output case, the LP problem is addressed by solving (16)

$$\hat{y}_m[kT] = a_1 \hat{y}_m[(k-1)T] + \dots + a_n \hat{y}_m[(k-n)T]$$
(16)

for each m = 1, 2, ..., M simultaneously for the unknown coefficients  $a_i s$ .

As discussed in [22], experiments on high-order oscillatory signals using a covariance formulation and a singular-value decomposition solution method have shown that accurate solutions are often obtained if the constraint (17) is satisfied.

$$2n < \sum_{m=1}^{M} N_m < 5n$$
 (17)

For m signals referring (12), (18) can be written as,

$$\begin{bmatrix} y_{1}[n] \\ y_{1}[n+1] \\ \vdots \\ y_{1}[N-1] \\ \vdots \\ y_{m}[n] \\ y_{m}[n+1] \\ \vdots \\ y_{m}[N-1] \end{bmatrix} = \begin{bmatrix} y_{1}[n-1] & \dots & y_{1}[0] \\ y_{1}[n] & \dots & y_{1}[1] \\ \vdots & \vdots & \vdots \\ y_{1}[N-2] & \dots & y_{1}[N-n-1] \\ \vdots & \vdots & \vdots \\ y_{m}[n-1] & \dots & y_{m}[0] \\ y_{m}[n] & \dots & y_{m}[1] \\ \vdots & \vdots & \vdots \\ y_{m}[N-2] & \dots & y_{m}[N-n-1] \end{bmatrix} \begin{bmatrix} a_{1} \\ a_{2} \\ \vdots \\ a_{n} \end{bmatrix}$$
(18)

where there are m groups of rows with (N - n) rows in each one, therefore, total number of rows is m, and the number of unknown variables remains n (from  $a_1$  to  $a_n$ ). This system of

equations is over-determined (more equations than unknowns); therefore, a least square method (LSM) of optimization must be used. The residues  $R_i^{(m)}$  are the weight of  $i^{th}$  mode (out of calculated *n* modes) on  $m^{th}$  signal. For finding them (19) needs to be solved by LSM (the super scripts *m* in brackets for  $R_i^{(m)}$  indicate the number of signals from 1 to m).

$$\begin{bmatrix} y_{1}[0] & y_{2}[0] & \dots & y_{n}[0] \\ y_{1}[1] & y_{2}[1] & \dots & y_{n}[1] \\ \vdots & \vdots & \vdots & \vdots \\ y_{1}[N-1]y_{2}[N-1] \cdots y_{n}[N-1] \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & \dots & 1 \\ \hat{z}_{1} & \hat{z}_{2} & \dots & \hat{z}_{n} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{z}_{1}^{N-1} & \hat{z}_{2}^{N-1} & \dots & \hat{z}_{n}^{N-1} \end{bmatrix} \begin{bmatrix} R_{1}^{(1)}R_{1}^{(2)} \dots R_{1}^{(m)} \\ R_{2}^{(1)}R_{2}^{(2)} \cdots R_{2}^{(m)} \\ \vdots & \vdots & \vdots \\ R_{n}^{(1)}R_{n}^{(2)} \cdots R_{n}^{(m)} \end{bmatrix}$$
(19)

The methodology of MPA can be summarized in three steps: first, solve (18) to get the coefficients  $a_i s$ . Second, obtain the roots of the polynomial (13); finally, modes  $\lambda_i = \sigma_i + j\omega_i = Ln(z_i)/T$  can be computed. The third step is to compute complex residues  $R_i$  by solving (19) by using solutions of (13).

# III. WSC ALGORITHM FOR ON-LINE MODES IDENTIFICATION IN POWER SYSTEMS

The on-line MPA can be applied to detect the modes that for a set of signals measured from PMUs. Fig. 2 shows the flowchart for the proposed methodology. In order to determine an appropriate first data window, a small-signal-stability analysis (SSSA) is first performed to identify the off-line dominant mode frequency. The size of the first window is reciprocal of the dominant mode frequency; that is,

$$T_{fw} = \frac{1}{f_{dm}} \tag{20}$$

where  $T_{fw}$  is the time of the first (fixed) data window, and  $f_{dm}$  is the minimum frequency found in the SSSA. When perturbation is detected in the power system the electromechanical modes identification is activated using  $T_{fw}$  as the first size of the passing data window. Data window to be analysed can be of fixed-size window (FSW) or controlled window-size (WSC). FSW algorithm uses fixed size of window in each successive step of analysis while WSC algorithm changes the size of window to be analysed as response to damping ratio and frequency in the previous step.

Once the number of samples of the first window is completed in the control centers, the detection of modes is computed with the M-PA during the post disturbance state. The analysis of window detects a group of oscillatory modes; the frequency and damping of the on-line dominant mode is identified  $\lambda_{dm}$ .

The dominant mode is one for which the sum of the norm of associate residues is the greatest for the groups of signals that have been analysed:

$$\lambda_{dm} = \left\{ \lambda \in \boldsymbol{C}, \max \sum_{i=1}^{m} \|R_i\| \right\}$$
(21)

 $\lambda_{dm}(t)$  is then a time-variable characteristic of the system, and it is not the same electromechanical mode every instant *t*. Therefore, as function of the frequency and damping ratio of the variable  $\lambda_{dm}$ , a new size of successive data window size is computed. Consequently, the number of samples required to wait for the formation of the new window is recalculated as the reciprocal of the dominant mode frequency multiplied by a factor K, the scale parameter.

$$T_{window} = \frac{1}{f_{dm}} \times K \tag{22}$$

where  $f_{dm}$  is the frequency in Hz of the dominant mode, and

$$K = \begin{cases} \frac{\zeta_{dm} + 1}{\zeta_{cr} + 1} & \text{if } \zeta_{dm} < \zeta_{cr} \\ 1 & \text{otherwise} \end{cases}$$
(23)

where  $0 < \zeta_{dm} < 1$ , is the damping ratio of the dominant mode, which has been detected on-line in the previous window, and  $0 < \zeta_{cr} < 1$  is constant that defines as the critical damping ratio, defined by the system operator, depending on experience and knowledge of the behaviour of the power system that is being monitored. In this analysis the value of  $\zeta_{cr}$  is used. If K = 1, the size of the data window remains constant, and this must be the default for any non-disturbed state of the system.



Fig. 2. Flow Chart for MPA using WSC and FSW analysis

The factor 0 < K < 1 is scale parameter that allows the reduction of the next window size if the damping ratio of the dominant mode is less than the defined  $\zeta_{cr}$ . The reduction is made to obtain a new estimation of the actual modes in less time. When the size of the new window is recalculated, new data from the PMUs are collected until completing the new size, and the procedure is repeated.

The identification process needs sliding data window, and, as previously stated, the window size is controlled. This analysis proposes that the identification method used is a window-size controlled (WSC) sliding window. Therefore, its length depends on the expected frequency of the critical electromechanical mode for the instant t in the next samples that are being received from the PMUs. The recalculated window size can either be greater or shorter than the previous window size, depending on dominant frequency that is being detected.

An important feature of this type of window is that the needed size of the window is measured in the number of samples from the most recently sampled one (past information). For instance, if there has been 1 sec of data, and calculated new window size is 1.01 sec, such that sampling period is 0.01 sec, just one new sample is needed from the PMU to complete the window required for making new identification of modes. This implies two aspects, the information from last 5 sec (which is the maximum window size allowed) must be stored and in size-controlled sliding window, there is no reboot of data storage each time when new window size is calculated.

#### IV. APPLICATION OF PROPOSED ALGORITHM

#### A. IEEE 30 bus test system case:

The analysis of generator angles was made for 50 Hz, 6machine, IEEE 30-bus test system shown in Fig. 3 using OP and MPA, WSC analysis. Data for the analysis is generated from the power world simulator package. Single-phase solid fault was simulated at the transmission line that joins buses 12, and 15 at t = 1 sec. Fault is cleared by opening the line after 0.3 sec. Analysis of 5 sec data (500 data samples), after the fault is cleared have been carried out. Fig. 4 shows the time series plot for p.u. relative rotor angles of generators G2 to G6 for IEEE 30-bus test system. Generators G2 to G6 are located at bus 2, bus 5, bus 8, bus 11, and bus 13 respectively. Sampling time is taken as 10 ms (i.e. 0.01 sec), to full fill the condition for the Nyquist-sampling rate for 50 Hz system frequency. Here, minimum sampling frequency is 100 Hz. SSSA for the test system determines the dominant modes as shown in TABLE I. The SSSA has done considering system as single machine infinite bus (SMIB) system.



Fig. 3. Modal damping ratio estimation using OPA and M-PA (30 bus test system case)

As the generator bus 1 is taken as slack bus, modes for the buses excluding slack bus are present in TABLE I, corresponding mode frequencies are also shown. As discussed in the WSC algorithm the initial data window has to be selected for the analysis. For this case initial window size taken was of size of 66 data samples. This is corresponding to the dominant mode -0.0167 + j9.5001. In this case fault is cleared at 1.3 sec, therefore,  $131^{th}$  to  $196^{th}$  data samples is taken as first window for analysis. Hereafter, the window is allowed to slide-over successively, so that on-line mode estimates has done for each of the next successive data window. If the size-controlled window algorithm is applied, the size of the data window may vary for each successive next window.



Fig. 4. PU relative rotor angle for generator (30 bus test system case)

 
 TABLE I.
 Opa, mpa mean modal frequencies comparitive estimation and sssa results for 30 bus test system

Gene- rator	Modes by SSSA	Dominant frequency $f_{dm}$ (Hz)	Initial window samples	Mean f <sub>dm</sub> (Hz) from OPA	Mean f <sub>dm</sub> (Hz) for system from M-PA
2	-0.0333 + <i>j</i> 14.9979	2.3882	42	2.3534	
3	-0.0375 + <i>j</i> 12.8597	2.0477	49	2.0593	
4	-0.0292 + <i>j</i> 13.6227	2.1692	46	2.1182	1.4562
5	-0.0167 + <i>j</i> 9.5001	1.5127	66	1.3646	
6	-0.0292 + <i>j</i> 12.7431	2.0291	49	1.9008	

Fig. 5 shows the on-line electromechanical modal frequency estimates, independently for the generators G2 to G6 respectively using OPA along with modal frequency estimates using multi-signal Prony analysis for overall test system. The corresponding relative rotor angle data is used for the modal estimation. These all estimates for the individual signals have done with OPA, shown by thin curves. It is radially observed that, single Prony (OPA) gives error in mode estimates. At some of the instants, modal values estimated may be near about 10 Hz, which is not expected. Estimation of modal frequencies has also done using multi-signal Prony analysis and result is shown by thick curve. In this curve it is observed that the estimated values for modal frequencies remain below 1.8 Hz,

which is expected value for the intra-area modes. Hence, MPA has edge over OPA, and the reason is that, system become over-determined for multi-signal fit. LSM algorithm has inherent property, that it eliminates error if present in the measured data. Due to multiple-signals are used for analysis, data analyzed is rich in system information content and also gives improved modal estimation. The result obtained with M-PA is single set of modes and it is close to the expected values. Fig. 6 shows on-line damping ratio (i.e. relative damping) estimation both using OPA and MPA. Thin curves show the analyses using OPA with WSC analysis for individual relative rotor angle signals while thick curve shows the analysis using MPA with WSC analysis for overall system.



Fig. 5. Modal frequency estimation using OPA and MPA (30 bus test system case)



Fig. 6. Modal damping ratio estimation using OPA and MPA (30 bus test system case)

# B. HeadingAnalysis of 22 bus system case:

The analysis of relative rotor angles was made for 50 Hz, 6machine, 22-bus test system shown in Fig. 7 using OPA and MPA, WSC analysis. In this case, generators G2 to G6 are located at buses 2 to 6 respectively. Data for the analysis is generated from the MATLAB simulation. Generator model in this case is  $10^{th}$  order model; it helps in achieving better dynamic performance than that of second order classical model. Modelling the system with higher order model always incorporates more number of dynamics, which improves the quality of analysis. Therefore, data analysed is much closer to the actual or real system values. Here, system is analysed for both stable and unstable cases.

1) Case-1, fault in the system has been cleared and stability of the system has been maintained:

Three-phase solid fault was simulated at bus number 19 at t = 0.5 sec. Fault is cleared after 0.3 sec. 333 data windows,

after the fault is cleared have been analysed. Fig. 8 shows time series plot for the relative rotor angles of generators G2 to G6 for 22 bus test system case-1. Again, sampling frequency of 100 Hz is taken. Initial dominant frequency in this case is 1.12 Hz; therefore initial data window was of 89 data samples. The analysis of electromechanical modes identification is carried out during post-fault time that is 0.8 sec  $80^{th}$  data sample) onwards.



Fig. 7. 22-Bus, 6 machine test system



Fig. 8. Relative rotor angles (22 bus test system case-1)

Fig. 9 shows the on-line electromechanical modal frequency estimates, independently for the generators G2 to G6 respectively using OPA and modal frequency estimates using multi-signal Prony analysis for overall test system. The corresponding relative rotor angle data is used for the modal estimation. These all estimates for the individual signals have done with OPA, shown by thin curves. It is radially observed that, single Prony gives error in mode estimates. At some of the

instants, modal values estimated may be near about 50 Hz, which is not expected. Estimation of modal frequencies has also done using multi-signal Prony analysis and result is shown by thick curve. In this curve it observed that the estimated values for modal frequencies remain below 1.8 Hz, which is maximum expected value for the intra-area modes.



Fig. 9. Modal frequency estimation using OPA and MPA (22 bus test system

2) Case-2, fault in the system has been cleared but stability of the system has not been maintained:

Further, three-phase solid fault was simulated at bus number 19 at t = 0.5 sec. Fault is cleared after 1 sec. It is clearly observed from the Fig. 10 that generators G2 and G3 forming the one coherent group while rest of the generators form anther coherent group. Though the curves for G2 and G3 showing the oscillations are damping out after the fault is cleared but the angle values are greater than transient stability limit of 180 degree. It means the system is no more stable and the G2 and G3 have lost synchronism. Here, in this case, 261 data windows, after the fault was cleared were analysed. The analysis of electromechanical modes identification was done during post-fault time that is 1.5 sec ( $150^{th}$  data sample) onwards.



Fig. 10. Relative rotor angles (22 bus test system case-2)

Fig. 11 shows the on-line electromechanical modal frequency estimates, independently for the generators G2 to G6 respectively using OPA along with modal frequency estimates using multi-signal Prony analysis for overall test system. The corresponding relative rotor angle data is used for the modal estimation. These all estimates for the individual signals have been carried out with OPA, shown by thin curves. Estimation of modal frequencies has also done using M-PA and result is shown by thick curve.

It is clear that system dynamics are very much sensitive not only to different fault types but also various fault clearing times. The two cases analyzed for the same test system give different electromechanical modal estimates. The results for estimation of mean modal frequencies of the individual machines using OPA and overall system using MPA are summarized in TABLE II. It clearly observed that range of mean modal frequencies obtained by using MPA remains in the range below 1.8 Hz that is specified for intra-area oscillations.



Fig. 11. Modal frequency estimation OPA and MPA (22 bus test system case-2)

TABLE II. OPA AND MPA MEAN MODAL FREQUENCIES COMPARITIVE RESULTS

Generator	Mean $f_{dm}$ (Hz) from OPA (case1)	Mean f <sub>dm</sub> (Hz) for system from MPA (case-1)	Mean $f_{dm}$ (Hz) from OPA (case-2)	Mean f <sub>dm</sub> (Hz) for system from MPA (case-2)
2	1.1908		0.9616	
3	1.4600		0.9605	
4	1.1144	0.8237	0.9338	0.9573
5	1.0346		0.9608	

#### V. CONCLUSIONS

Comparison between the OPA with multi-signal Prony analysis has been done. The multi-signal fit eliminates the inherent noise in the measured data, and improved electromechanical mode estimation has been carried out. Also the multi-signal fit gives overall system behaviour because the data analysed is rich in overall system information.

In addition, this analysis has shown that the utilisation of sliding window in modes identification method using a multi-Prony approach allows the observation of the evolution of the dynamics of electromechanical modes of the power system as response to disturbances. It allows the detection of critical damping ratios in short time associated to the period of the dominant modes obtained by off-line SSSA. In order to achieve the balance between the requirements of quantity of data and time for estimation, optimised-size window multi-signal Prony analysis is used for estimation of electromechanical modes.

The calculation of modes is performed once the on-line window recalculated size is reached. Factor K is proposed to reduce the size of the window depending on the damping level of the on-line dominant mode determined in previous window. With this proposed method, the number of calculations is

reduced. This technique ensures that a suitable window size will be formed to execute estimation modes. Using a fixed window method may cause inability to detect low-frequency responses.

It is clear that system dynamics are very much sensitive not only to different fault types but also various fault clearing times. The two cases analysed for the same test system give different electromechanical modal estimates. It is always preferable to use transient data for Prony analysis to have higher degree of accuracy and accurate fitting of data.

Further work must be oriented to the establishment of control actions using the damping behaviour of electromechanical modes and, the situational awareness assessment of the power system using phasorial information. Moreover, as an initial step after measuring signals, the application of noise/ambient on-line filters, such as Kalman, extended Kalman, and unscented Kalman must be developed to ensure that the signal is properly analysing the power signal.

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