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Regular Generalized Semipreopen Sets in Intuitionistic Fuzzy Topological Spaces

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***Abstract*— In this paper, we introduce the notion of intuitionistic fuzzy regular generalized semipreopen sets. Furthermore, we investigate some of the properties and theoretical applications of the intuitionistic fuzzy regular generalized semipreopen sets.**

***Keywords*— Intuitionistic fuzzy topology, Intuitionistic fuzzy semipreopen sets, Intuitionistic fuzzy regular generalized semipreclosed sets, Intuitionistic fuzzy regular generalized semipreopen sets.**

I. INTRODUCTION

The establishment of fuzzy sets was made by Zadeh [12] in 1965. Later the introduction of fuzzy topology was given by Chang [2] in 1967. This was followed by the introduction of intuitionistic fuzzy sets by Atanassov [1]. Using this notion, Coker [3] constructed the basic concepts of intuitionistic fuzzy topological spaces. Subsequently this was followed by the introduction of intuitionistic fuzzy regular generalized semipreclosed sets by Vaishnavy V and Jayanthi D [10] in 2015. We now extend our idea towards intuitionistic fuzzy regular generalized semipreopen sets and study some of their properties and applications.

II. PRELIMINARIES

Definition 2.1 [1]: An *intuitionistic fuzzy set* (IFS in short) A is an object having the form

$$A=\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the function $\mu_A : X \longrightarrow [0,1]$ and $\nu_A : X \longrightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

An intuitionistic fuzzy set A in X is simply denoted by $A=\langle x, \mu_A, \nu_A\rangle$ instead of denoting $A=\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$.

Definition 2.2 [1] : Let A and B be two IFSs of the form

$$A=\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$
and
$$B=\{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}.$$

Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$;

(b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$;

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$;

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$;

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$.

The intuitionistic fuzzy sets $0\sim = \langle x, 0, 1\rangle$ and $1\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 2.3 [3] : An *intuitionistic fuzzy topology* (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0\sim , 1\sim \in \tau$

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1 ,G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called the *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The compliment A^c of an IFOS A in IFTS (X,τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4 [3] : Let (X, τ) be an IFTS and $A= \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the *intuitionistic fuzzy interior* and *intuitionistic fuzzy closure* are defined by

$$\text{int}(A) = \cup\{G \mid G \text{ is an IFOS in X and } G \subseteq A\}$$
$$\text{cl}(A) = \cap\{K \mid K \text{ is an IFCS in X and } A \subseteq K\}.$$

Note that for any IFS A in (X, τ), we have $\text{cl}(A^c) = (\text{int}(A))^c$ and $\text{int}(A^c) = (\text{cl}(A))^c$.

Definition 2.5 [4] : An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS (X,τ) is said to be an

(i) *intuitionistic fuzzy semi open set* (IFSOS in short) if $A \subseteq \text{cl}(\text{int}(A))$

(ii) *intuitionistic fuzzy pre open set* (IFPOS in short) if $A \subseteq \text{int}(\text{cl}(A))$

(iii) *intuitionistic fuzzy α open set* (IFαOS in short) if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$

(iv) *intuitionistic fuzzy β open set* (IFβOS in short) if $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$.

Definition 2.6 [11] : An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS (X,τ) is said to be an *intuitionistic fuzzy semi-pre open set* (IFSPOS in short) if there exists an IFPOS B such that $B \subseteq A \subseteq \text{cl}(B)$.

Definition 2.7 [9] : An IFS A is an

(i) *intuitionistic fuzzy regular closed set* (IFRCS in short) if A = cl(int(A))
(ii) *intuitionistic fuzzy regular open set* (IFROS in short) if A= int(cl(A))
(iii) *intuitionistic fuzzy generalized closed set* (IFGCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFOS
(iv) *intuitionistic fuzzy regular generalized closed set* (IFRGCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is IFROS.

Definition 2.8 [5] Let *A* be an IFS in an IFTS (*X*, *τ*). Then the *semi-pre interior* and the *semi-pre closure* of *A* are defined as

spint (*A*) = ∪ {*G* | *G* is an IFSPOS in *X* and *G* ⊆ *A*},
spcl (*A*) = ∩ {*K* | *K* is an IFSPCS in *X* and *A* ⊆ *K*}.

Definition 2.9 [7] An IFTS (X,τ) is said to be an $IFT_{1/2}$ space if every IFGCS in (X,τ) is an IFCS in (X,τ).

Definition 2.10 [10] An IFS A in an IFTS (X,τ) is said to be an *intuitionistic fuzzy regular generalized semipreclosed set* (IFRGSPCS in short) if spcl(A) ⊆ U whenever A ⊆ U and U is an IFROS in (X,τ).

III. INTUITIONISTIC FUZZY REGULAR GENERALIZED SEMIPREOPEN SETS

In this section we introduce the notion of intuitionistic fuzzy regular generalized semipreopen sets and study some of their properties.

Definition 3.1 The complement A^c of an IFRGSPCS A in an IFTS (X,τ) is called an *Intuitionistic fuzzy regular generalized semipreopen set* (IFRGSPOS in short) in X.

The family of all IFRGSPOSs of an IFTS (X,τ) is denoted by IFRGSPO(X).

Theorem 3.2 Every IFOS, IFGOS, IFSOS, IFPOS, IFSPOS, IFαOS, IFβOS, IFROS is an IFRGSPOS but the converses are not true in general.
Proof: Straightforward.

Example 3.3 Let X = {a, b} and G = ⟨x, (0.5, 0.4), (0.5, 0.6)⟩ where $\mu_G(a) = 0.5$, $\mu_G(b) = 0.4$, $\nu_G(a) = 0.5$, $\nu_G(b) = 0.6$. Then τ = {0~, G, 1~} is an IFT on X. LET A = ⟨x, (0.6, 0.7), (0.4, 0.2)⟩ be an IFS in X. Then, IFPC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \geq 0.5$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1$}. Therefore, IFSPC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1$}. As spcl(A^c) = A^c, we have $A^c \subseteq G^c$ implies spcl(A^c) ⊆ G^c, where G is an IFROS in X. This implies that A^c is an IFRGSPCS in X and hence A is an IFRGSPOS. Now since int(A) = G ≠ A, A is not an IFOS in X. Also G^c ⊆ A but G^c ⊈ int(A). Therefore A is not an IFGOS in X. Now int(cl(A)) = int(1~) = 1~ ≠ A. Therefore A is not an IFROS in X. Hence A is an IFRGSPOS but not IFOS, IFGOS, IFROS.

Example 3.4 Let X = {a, b} and G = ⟨x, (0.5, 0.6), (0.5, 0.4)⟩ where $\mu_G(a) = 0.5$, $\mu_G(b) = 0.6$, $\nu_G(a) = 0.5$, $\nu_G(b) = 0.4$. Then τ = {0~, G, 1~} is an IFT on X. Let A = ⟨x, (0.5, 0.3), (0.5, 0.7)⟩ be an IFS in X. Then, IFPC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $\mu_b < 0.6$ whenever $\mu_a \geq 0.5$, $\mu_a < 0.5$ whenever $\mu_b \geq 0.6$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1$}. Therefore, IFSPC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $\mu_b < 0.6$ whenever $\mu_a \geq 0.5$, $\mu_a < 0.5$ whenever $\mu_b \geq 0.6$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1$}. As spcl(A^c) = 1~, we have A^c ⊆ 1~ implies spcl(A^c) ⊆ 1~, where 1~ is an IFRCS. This implies that A^c is an IFRGSPCS in X. Hence A is an IFRGSPOS in X. Now since A ⊈ int(cl(A)) = int(G^c) = 0~, we get A is not an IFPOS in X. Further A ⊈ cl(int(cl(A))) = cl(int(G^c)) = cl(0~) = 0~. Hence A is not an IFβOS in X. Also A ⊈ cl(int(A)) = cl(0~) = 0~. Thus A is not an IFSOS in X. Now since A ⊈ int(cl(int(A))) = int(cl(0~)) = int(0~) = 0~, A is not an IFαOS in X. Further there exists no IFPOS B such that A ⊆ B ⊆ cl(A). Therefore A is not an IFSPOS in X. Hence A is an IFRGSPOS but not IFPOS, IFβOS, IFSOS, IFαOS, IFSPOS.

Theorem 3.5 Let (X, τ) be an IFTS. Then for every A ∈ IFRGSPO(X) and for every B ∈ IFS(X), spint(A) ⊆ B ⊆ A ⇒ B ∈ IFRGSPO(X).
Proof: Let A be any IFRGSPOS of X and B be any IFS of X. Let spint(A) ⊆ B ⊆ A. Then A^c is an IFRGSPCS and $A^c \subseteq B^c \subseteq$ spcl(A^c). Therefore B^c is an IFRGSPCS[10] which implies B is an IFRGSPOS in X. Hence B ∈ IFRGSPO(X).

Theorem 3.6 Let (X, τ) be an IFTS. Then for every A ∈ IFS(X) and for every B ∈ IFPO(X), B ⊆ A ⊆ cl(int(B)) ⇒ A ∈ IFRGSPO(X).
Proof: Let B be an IFPOS. Then B ⊆ int(cl(B)). By hypothesis, A ⊆ cl(int(B)) ⊆ cl(int(int(cl(B)))) = cl(int(cl(B))) ⊆ cl(int(cl(A))) as B ⊆ A. Therefore A is an IFβOS and by Theorem 3.2, A is an IFRGSPOS. Hence A ∈ IFRGSPO(X).

Theorem 3.7 An IFS A of an IFTS (X, τ) is an IFRGSPOS if and only if F ⊆ spint(A) whenever F is an IFRCS and F ⊆ A.
Proof: Necessity: Suppose A is an IFRGSPOS. Let F be an IFRCS such that F ⊆ A. Then F^c is an IFROS and $A^c \subseteq F^c$. By hypothesis A^c is an IFRGSPCS, we have spcl(A^c) ⊆ F^c. Therefore F ⊆ spint(A).
Sufficiency: Let F be an IFRCS such that F ⊆ A, then F ⊆ spint(A). That is $(\text{spint}(A))^c \subseteq F^c$. This implies spcl($A^c$) ⊆ F^c where F^c is an IFROS. Therefore A^c is an IFRGSPCS. Hence A is an IFRGSPOS.

Theorem 3.8 Let (X, τ) be an IFTS then for every A ∈ IFSPO(X) and for every IFS B in X, A ⊆ B ⊆ cl(A) ⇒ B ∈ IFRGSPO(X).
Proof: Let A be an IFSPOS in X. Then by Definition 2.6, there exists an IFPOS, say C such that C ⊆ A ⊆ cl(A). By hypothesis, A ⊆ B. Therefore C ⊆ B. Since A ⊆ cl(C), cl(A) ⊆ cl(C) and B ⊆ cl(C). Thus C ⊆ B ⊆ cl(C). This implies B is an IFSPOS in X. Then by Theorem 3.2, B is an IFRGSPOS. That is B ∈ IFRGSPO(X).

IV. APPLICATIONS

The concept of intuitionistic fuzzy semipre $T_{1/2}$ space was introduced by Santhi, R. and Jayanthi, D [7] in 2009. In this section we have discussed some applications of intuitionistic fuzzy regular generalized semipreclosed sets.

Definition 4.1 If every IFRGSPCS in (X, τ) is an IFSPCS in (X, τ), then the space can be called as an *intuitionistic fuzzy regular semipre $T_{1/2}$ space* (IFRSPT$_{1/2}$ in short).

Theorem 4.2 An IFTS (X, τ) is an IFRSPT$_{1/2}$ space if and only if IFSPO(X) = IFRGSPO(X).

Proof: Necessity: Let A be an IFRGSPOS in (X, τ), then A^c is an IFRGSPCS in (X, τ). By hypothesis, A^c is an IFSPCS in (X, τ) and therefore A is an IFSPOS in (X, τ). Hence IFSPO(X) = IFRGSPO(X).

Sufficiency: Let A be an IFRGSPCS in (X, τ). Then A^c is an IFRGSPOS in (X, τ). By hypothesis A^c is an IFSPOS in (X, τ) and therefore A is an IFSPCS in (X, τ). Hence (X, τ) is an IFRSPT$_{1/2}$ space.

Remark 4.3 Not every IFRSPT$_{1/2}$ space is an IFT$_{1/2}$ space. This can be seen easily by the following example.

Example 4.4 Let X = {a, b} and G = ⟨x, (0.5, 0.4), (0.5, 0.6)⟩ where $\mu_G(a) = 0.5$, $\mu_G(b) = 0.4$, $\nu_G(a) = 0.5$, $\nu_G(b) = 0.6$. Then τ = {0~, G, 1~} is an IFT on X. Then, IFPC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \geq 0.5$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1$}. Therefore, IFSPC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1$}. Since all IFRGSPCS in X are IFSPCS in X, (X, τ) is an IFRSPT$_{1/2}$ space. But it is not a IFT$_{1/2}$ space since if A = ⟨x, (0.5, 0.7), (0.5, 0.3)⟩, then cl(A) = 1~ ⊆ 1~ whenever A ⊆ 1~. Hence A is an IFGCS in X but cl(A) = 1~ ≠ A, A is not an IFCS in X. Therefore (X, τ) is not an IFT$_{1/2}$ space.

Theorem 4.5 Let (X, τ) be an IFTS and X is an IFRSPT$_{1/2}$ space, then the following conditions are equivalent:

(i) A ∈ IFRGSPO(X)
(ii) A ⊆ cl(int(cl(A)))
(iii) cl(A) ∈ IFRC(X).

Proof: (i) ⇒ (ii) Let A be an IFRGSPOS. Then since X is an IFRSPT$_{1/2}$ space, A is an IFSPOS. Since every IFSPOS is an IFβOS [5] we get, A ⊆ cl(int(cl(A))).

(ii) ⇒ (iii) Let A ⊆ cl(int(cl(A))). Then cl(A) ⊆ cl(cl(int(cl(A)))) = cl(int(cl(A))) ⊆ cl(cl(A)) = cl(A). Therefore cl(A) = cl(int(cl(A))). Hence cl(A) ∈ IFRC(X).

(iii) ⇒ (i) Since cl(A) is an IFRCS, cl(A) = cl(int(cl(A))) and since A ⊆ cl(A), A ⊆ cl(int(cl(A))). Therefore A is an IFβOS. Hence by Theorem 3.2, A ∈ IFRGSPO(X).

Theorem 4.6 Let (X, τ) be an IFTS and X is an IFRSPT$_{1/2}$ space, then the following conditions are equivalent:

(i) A ∈ IFRGSPC(X)
(ii) int(cl(int(A))) ⊆ A
(iii) int(A) ∈ IFRO(X).

Proof: (i) ⇒ (ii) Let A be an IFRGSPCS. Then since X is an IFRSPT$_{1/2}$ space, A is an IFSPCS. Since every IFSPCS is an IFβCS [5] we get, int(cl(int(A))) ⊆ A.

(ii) ⇒ (iii) Let int(cl(int(A))) ⊆ A. Then int(A) ⊇ int(int(cl(int(A)))) = int(cl(int(A))) ⊇ int(int(A)) = int(A). Therefore int(cl(int(A))) = int(A). Hence int(A) ∈ IFRO(X).

(iii) ⇒ (i) Since int(A) is an IFROS, int(A) = (int(cl(int(A))) and since int(A) ⊆ (A), int(cl(int(A))) ⊆ A. Therefore A is an IFβCS which implies A^c is an IFβOS. Hence by Theorem 3.2, A^c is an IFRGSPOS. Therefore A ∈ IFRGSPC(X).

Definition 4.7 An IFTS (X, τ) is said to be an *intuitionistic fuzzy regular semipre $T^*_{1/2}$ space* (IFRSPT$^*_{1/2}$ in short) if every IFRGSPCS is an IFRCS in (X, τ).

Remark 4.8 Every IFRSPT$^*_{1/2}$ space is an IFRSPT$_{1/2}$ space but not conversely.

Proof: Let (X, τ) be an IFRSPT$^*_{1/2}$ space. Let A be an IFRGSPCS in (X, τ). By hypothesis, A is an IFRCS. Since every IFRCS is an IFSPCS, A is an IFSPCS in (X, τ). Hence (X, τ) is an IFRSPT$_{1/2}$ space.

Example 4.9 Let X = {a, b} and G = ⟨x, (0.5, 0.4), (0.5, 0.6)⟩ where $\mu_G(a) = 0.5$, $\mu_G(b) = 0.4$, $\nu_G(a) = 0.5$, $\nu_G(b) = 0.6$. Then τ = {0~, G, 1~} is an IFT on X. Then, IFPC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \geq 0.5$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1$}. Therefore, IFSPC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1$}. Since all IFRGSPCS in X are IFSPCS, (X, τ) is an IFRSPT$^*_{1/2}$ space since if A = ⟨x, (0.5, 0.7), (0.5, 0.3)⟩, then spcl(A) = A ⊆ 1~ whenever A ⊆ 1~. Hence A is an IFRGSPCS in X but since cl(int(A)) = cl(G) = G^c ≠ A, A is not an IFRCS in X. Therefore (X, τ) is not an IFRSPT$^*_{1/2}$ space.

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