Replenishment Policy for EMQ Model with Rework, Multiple Shipments, Switching and Packaging

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Abstract.

Presently the advancement of technology is mind-blowing .The manufacturers employ various tactics to make their production the best among their opponents. The products that are made are expected to be free from defects, but that is not possible always. The manufactured products become defective either due to technical faults or due to recklessness of the labourers. Thus the presence of defective items in a manufactured lot is quite common. To rectify the defects, the firm owners focus on rework and waste disposal of scrap items. As the processes involved in manufacturing and reworking are distinct, switching cost is incurred when the production process is shifted to the process of remanufacture and vice versa. This paper presents a mathematical model which determines optimal inventory replenishment policy for the Economic Manufacturing Quantity (EMQ) with rework and multiple shipments along with the inclusion of switching and packaging cost.

Key words : rework, switching, packaging

1 . Introduction

Today the EMQ model is broadly applied industry wide. The classic economic manufacturing quantity model assumes all items produced are of perfect quality. However, in a real-life manufacturing system, due to process of deterioration or various other factors, generation of defective items seems inevitable. The defective items produced sometimes can be reworked and repaired. The items that are not fit for rework are expelled as waste. Thus rework helps in reducing the total costs of production–inventory. Many researchers namely Liu., Tang., Chiu et al., Jamal et al. developed EMQ models with rework. The classic EMQ model also assumes a continuous issuing policy for satisfying product demand. However, in real life vendor–buyer integrated supply chain environment, a multiple deliveries policy is commonly used in dealing with customer's demands. Many studies have since been carried out to address various aspects of vendor–buyer supply chain optimization issues. To bridge the gap between the joint effect of rework and multi-shipment policy Yuan.et.al proposed an EMQ model to determine the optimal replenishment policy.The model developed by Yuan.et.al is modified by the inclusion of switching costs, which is the change over costs that is incurred at times of shifting from production to repair and vice versa. A variety of switching costs like additional setup cost to switch between workstations, jobs, or products, Switching the production rate, Machine start-up/shutdown, Machine cleaning, Tooling, Machine adjustment, Additional labor and so on.[1]. Packaging is yet another significant gaining phenomena whose usage is mandatory these days because of its role in safeguarding the quality of the products.Among the various types of packaging tertiary packaging is employed for the shipment of large quantities of goods [5].To comprise all the concepts such as rework, multiple shipment, switching and packaging along with its associated costs this model is proposed. The intention of this proposal is to determine the effect of including switching cost and packaging cost to EMQ model with rework and multiple replenishment. In this paper the EMQ model is formulated along with the discussion of 2 special cases. A numerical example is also presented which validate the model.

2 Model Formulation

This paper incorporates rework process, multiple shipments along with switching and packaging costs into an imperfect EMQ model with random defective rate. Consider a manufactured product has a flat annual demand rate λ and this item can be produced at a constant production rate P. The process may randomly generate a portion x of defective items at a production rate d. All items produced are screened and inspection cost is included in the unit production cost C. All defective items produced are reworked at a rate of P_1, immediately after the regular process ends. A portion θ_1 (where $0 \leq \theta_1 \leq 1$) of reworked items fails and becomes scrap. To prevent a shortage from occurring, the production rate P is assumed to be larger than the sum of demand rate λ and production rate of defective items d. That is: $(P - d - \lambda) > 0$ or $(1 - x - \lambda/P) > 0$; where d can be expressed as $d = Px$. Let d_1 denote the production rate of scrap items during the rework process, then d_1 can be expressed as: $d_1 = P_1 \theta_1$. It is further assumed that a multiple shipment policy is employed and the finished items can only be delivered to customers at the end of the rework when the whole lot is quality assured. Fixed quantity n instalments of a finished batch are delivered to customers at a fixed interval of time during production downtime t_3. In addition to the above cost repair setup cost r is incurred before the commencement of the rework process. Also the cost of switching from production to rework (r_1) and rework to delivery (d_1) are included.

2.1 Assumptions

The following assumptions are used throughout this paper

1. Shortages are not allowed.
2. Remanufactured products are considered as good ones.
3. The production process is shifted to remanufacturing process only once during the cycle length T.

2.2 Notations

The following notations as in [11] including the additional notations are as follows

H_1 = Maximum level of on- hand inventory in units when regular production process ends.

H = the maximum level of on- hand inventory in units when the rework process finishes

t_1 = the production uptime for the proposed EMQ model

t_2 = time required for reworking of defective items

t_3 = time required for delivering all quality assured finished products

n = number of fixed quantity instalments of the finished batch to be delivered by request to customers

t_n = a fixed interval of time between each instalment of finished products delivered during production downtime t_3

Q = manufacturing batch size, to be determined for each cycle

I(t) = on-hand inventory of perfect quality items at time t

I_d (t) =on-hand inventory of defective items at time t

TC(Q) = total production-inventory-delivery costs per cycle for the proposed model

TC_1 (Q) = total production-inventory-delivery costs per cycle when all defective items are reworked and repaired (i.e. special case 1: $\theta_1 = 0$)

TC_2 (Q) = total production-inventory-delivery costs per cycle when no defective items are produced (i.e. special case 2: x =0)

E[TCU(Q)] = the long – run average costs per unit time for the proposed model

E[TC_1U(Q)] = the long run average costs per unit time for the model in special case 1

E[TC_2U(Q)] = the long run average costs per unit time for the model in special case 2

x = defective rate ; λ = demand rate; C = variable production cost;K = production set up cost ;C_k = variable rework cost; C_s = disposal cost; K_1 = fixed delivery cost per shipment;C_T = variable delivery cost per shipment; h = variable holding cost for time reworked;h_1 = holding cost during

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uptime t_1; h_2 = holding cost during the rework time t_2; h_3 = holding time during the delivery time t_3;T = cycle length

The additional notations that are introduced newly are as follows

r = repair set up cost; r_1 = switching cost from production to repair; d_1 = switching cost from repair to delivery; p = packaging cost per unit that is to be delivered

The production cycle length is $T = t_1 + t_2 + t_3$ and $T = \frac{Q}{\lambda}\left(1 - \theta_1\, x\right)$

For any given production cycle the total production–inventory–delivery costs are TC(Q).

$$\mathbf{TC(Q)} = \mathbf{CQ} + K + C_R[xQ] + C_s[xQ\,\theta_1] + n\,K_1 + C_T[Q(1-\theta_1 x)] + h.\frac{P_1 t_2}{2}\,t_2$$
$$+ h_1[(H_1 + dt_1)/2\ \ t_1] + h_2\,[(H_1 + H)/2\ \ t_2\,] + h_3\left((n-1)/2n\right)\,H t_3 + r_1 + d_1$$
$$+ r + p[Q(1-\theta_1 x)] \quad (1)$$

Where $H_1 = (1-x)Q$, $H = Q(1 - x\theta_1)$, $t_1 = \frac{H_1}{P-d}$, $t_2 = \frac{xQ}{P_1}$, $t_3 = Q\left[\frac{(1-x\theta_1)}{\lambda} - \frac{1}{P} - \frac{x}{P_1}\right]$

Eq. (1) consists of variable production costs, the production setup cost, variable rework costs, disposal costs, fixed and variable delivery costs, variable holding cost (h) for items reworked, holding cost (h_1) during uptime t_1, holding cost (h_2) during reworking time t_2, and holding cost (h_3) for finished goods during the delivery time t_3 where n fixed-quantity instalments of the finished batch are delivered to customers at a fixed interval of time, repair setup cost, delivery setup cost ,switching cost and packaging cost.Defective rate x is assumed to be a random variable with a known probability density function. To take this randomness into account, one can use the expected value of x in the cost analysis. Substituting all related parameters in TC(Q), the expected production–inventory cost per unit time E[TCU(Q)] can be obtained.

$$E[TCU(Q)] = \frac{E[TC(Q)]}{E[T]} = \frac{C\lambda}{1-\theta_1 E(x)} + \frac{K\lambda}{Q\left(1-\theta_1 E(x)\right)} + \frac{\lambda E(x) C_R}{1-\theta_1 E(x)} + \frac{E(x)\lambda C_s \theta_1}{1-\theta_1 E(x)}$$

$$= \frac{n\lambda K_1}{Q\left(1-\theta_1 E(x)\right)} + \frac{\lambda(r_1 + d_1 + r)}{Q\left(1-\theta_1 E(x)\right)} + C_T\lambda + p\lambda +$$

$$([[E(x)]]^2 hQ\lambda)/(2p_1 1-\theta_1 E(x)) \quad (Q\lambda h_1)/(2p1-\theta_1 E(x)) + (n-1)/2n[(h_3\, Q\lambda)/(1-\theta_1 E(x))\{([[1-\theta_1 E(x)]]^2)/\lambda - (1-\theta_1 E(x))/p - \frac{E(x)[1-\theta_1 E(x)]}{p_1}]$$

$=$

$$\frac{C\lambda}{1-\theta_1 E(x)}+\frac{K\lambda}{Q\left(1-\theta_1 E(x)\right)}+\frac{\lambda E(x)C_R}{1-\theta_1 E(x)}+\frac{E(x)\lambda C_s\theta_1}{1-\theta_1 E(x)}+\frac{n\lambda K_1}{Q\left(1-\theta_1 E(x)\right)}+\frac{\lambda(\mathbf{r_1}+\mathbf{d_1}+\mathbf{r})}{Q\left(1-\theta_1 E(x)\right)}+C_T$$

$+p\lambda$ +

$(h[[E(x)]]^2 Q\lambda)/(2p_1 1-\theta_1 E(x))+(Q\lambda h_1)/(2p1-\theta_1 E(x))+(Q\lambda h_2)/(2p_1 1-\theta_1 E(x))\ [2E(x)-(E(x))^2 \theta_1\{[E(x)]\}^2$
$+(h_3 Q(n-1))/2n[\{1-\theta_1 E(x)-\lambda/p-E(x)\lambda/p_1\ ]$

Determining the optimal replenishment policy

The optimal inventory replenishment lot size can be obtained by minimizing the expected cost function E[TCU(Q)].Differentiating E[TCU(Q)] with respect to Q, the first and second derivatives of E[TCU(Q)] are shown in Eqs. (2) and (3).

$$\frac{dE(TCU(Q))}{dQ}=\frac{-K\lambda}{Q^2\left(1-\theta_1 E(x)\right)}-\frac{n\lambda K_1}{Q^2\left(1-\theta_1 E(x)\right)}-\frac{\lambda(\mathbf{r_1}+\mathbf{d_1}+\mathbf{r})}{Q^2\left(1-\theta_1 E(x)\right)}+\frac{\lambda \mathbf{h_1}}{2p1-\theta_1 E(x)}$$

$$+\frac{\lambda\mathbf{h_2}}{\mathbf{2p_1 1-\theta_1 E(x)}}\left[2\mathbf{E(x)}-(\mathbf{E(x)})^2-\theta_1\{\mathbf{E(x)}\}^2\right]+$$

$$(h_3\ (n-1))/2n[\{1-[\theta]_1(1)\ E(x)-\lambda/p-E(x)\lambda/p_1\ ] \quad (2)$$

$$\frac{d^2E(TCU(Q))}{d^2Q}=\frac{2\lambda(K+nK_1+\mathbf{r_1}+\mathbf{d_1}+\mathbf{r})}{Q^3\left(1-\theta_1 E(x)\right)} \quad (3)$$

Eq. (3) is resulting positive, because variables K, n, K_1, λ, Q, and (1 – θ_1 E[x]) are all positive. The second derivative of E[TCU(Q)] with respect to Q is greater than zero. Therefore, E[TCU(Q)] is a convex function for all Q different from zero. Then, the optimal replenishment lot size Q^* can be obtained by setting the first derivative of E[TCU(Q)] equal to zero

$$\frac{dE(TCU(Q))}{dQ}=\frac{-K\lambda}{Q^2\left(1-\theta_1 E(x)\right)}-\frac{n\lambda K_1}{Q^2\left(1-\theta_1 E(x)\right)}-\frac{\lambda(\mathbf{r_1}+\mathbf{d_1}+\mathbf{r})}{Q^2\left(1-\theta_1 E(x)\right)}$$

$$+\frac{h\left[[E(x)]\right]^2\lambda}{2p_1 1-\theta_1 E(x)}+\frac{\lambda h_1}{2p1-\theta_1 E(x)}+\frac{\lambda\mathbf{h_2}}{2p_1 1-\theta_1 E(x)}\left[2E(x)-\left(E(x)\right)^2-\theta_1\{E(x)\}^2\right]+$$

$$(h_3\ (n-1))/2n[\{1-\theta_1(1)\ E(x)-\lambda/p-E(x)\lambda/p_1\ ]=0$$

And the optimal order quantity is (4)

Special cases to the proposed model

Case 1. When $\theta_1 = 0$.

Suppose that the rework process is perfect, i.e. all reworked items are repaired.

The expected production–inventory–delivery cost per unit time for this specific model becomes

$$E[TCU_1(Q)] = \frac{E[TC_1(Q)]}{E[T]} = C\lambda + \frac{K\lambda}{Q} + E(x)C_R + \frac{n\lambda K_1}{Q} + \frac{\lambda(\mathbf{r_1} + \mathbf{d_1} + \mathbf{r})}{Q} + C_T\lambda + p\lambda +$$

$$\frac{h[[E(x)]]^2 Q\lambda}{2p_1} + \frac{Q\lambda h_1}{2p} + \frac{Q\lambda h_2}{2p_1}\left[2E(x) - (E(x))^2\right] +$$

$$(n-1)/2n[h_{\downarrow}3\ Q\lambda\{1/\lambda - 1/p - E(x)/p_{\downarrow}1\}]$$

By following the above procedure we obtain

$$Q^{\uparrow}* = \sqrt{}((\lambda(K + nK_{\downarrow}1 + r_{\downarrow}1 + d_{\downarrow}1 + r))/((h[[E(x)]]^{\uparrow}2\,\lambda)/(2p_{\downarrow}1) + (\lambda h_{\downarrow}1)/2p + (\lambda h_{\downarrow}2)/(2p_{\downarrow}1)\,[2E - (E(x))^{\uparrow}(2)] + (h_{\downarrow}3\,(n-1))/2n[\{1 - \lambda/p - E(x)\lambda/p_{\downarrow}1\}])) \quad (7)$$

Case 2. When x = 0.

Suppose all items produced are of perfect quality, i.e. x = 0.

The expected production–inventory–delivery cost per unit time for this specific model becomes

$$E[TCU_2(Q)] = \frac{E[TC_2(Q)]}{E[T]} = C\lambda + \frac{K\lambda}{Q} + \frac{n\lambda K_1}{Q} + \frac{\lambda(\mathbf{r_1} + \mathbf{d_1} + \mathbf{r})}{Q} + C_T\lambda + p\lambda + \frac{Q\lambda h_1}{2p} +$$

$(n-1)/2n[\mathbf{h}_{\downarrow}3\ Q\lambda\{1/\lambda - 1/p\}$ .In this case the optimal order quantity is

$$Q^* = \sqrt{\frac{\lambda(K + nK_1 + \mathbf{r_1} + \mathbf{d_1} + \mathbf{r})}{\frac{\lambda h_1}{2p} + \frac{\mathbf{h_3}(n-1)}{2n\left[\left\{1 - \frac{\lambda}{p}\right\}\right]}}} \quad (10)$$

4 Numerical Example

Assume a manufactured item can be produced at a rate of 60,000 units per year and has a flat demand rate of 3400 units per year. A random defective rate x is assumed during the production uptime, where x follows a uniform distribution over the interval [0, 0.3]. All defective items are reworked at a rate of P_1 = 2200 units per year. A portion θ_1 = 0.1 of reworked items fails during the reworking and becomes scrap. The following are values of other variables considered in this example: C = 100 per item, K = 20,000 per production run,K_1 = 4350 per shipment, a fixed cost. C_T = 0.1 per item delivered, C_R = 60, repaired cost for each item reworked,C_s = 20, disposal cost for each scrap item, n = 4 installments of the finished batch are delivered per cycle, h = 20 per item per year,h_1 = 40 per item per year.h_2 = 30 per item per yearh_3 = 35 per item per year = 80 , a fixed cost,r_1 = 50, a fixed cost,d_1 = 45, a fixed cost. By using Eq. (4) the optimal replenishment policy Q^*= 2723 is obtained.

The optimal replenishment policy Q* = 2721 for special case 1 (i.e. situation when all reworked items are 100% repaired) is obtained by using Eq.(7)

For special case 2 (i.e. a situation when all items produced are of perfect quality) the optimal lot size $Q^* = 3075$ is obtained by using Eq.(10).

Conclusion

This paper studies the optimal inventory replenishment policy for the economic manufacturing quantity model with rework, multiple shipments, switching and packaging cost. It also comprises of a model with two special cases .This model is very helpful to the inventory managers for maintaining balance between the process of production and rework also it facilitates the firm owners to allocate the setup cost for rework to make it desirable

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