Response of Beams with Various Cross-sections for Applications in Vibration Energy Harvesters

Ashwin Nitin Nayak
Student, 4th Semester, M.Tech
Bharati Vidyapeeth College of Engineering
Pune- India

Dr. Pawan Pingle
Professor, Department of Mechanical Engineering,
Bharati Vidyapeeth College of Engineering
Pune- India

***Abstract*— The objective of this project work was to study responses of beams with different cross-sections for applications in a Vibration Energy Harvesters. This analysis was primarily done to test beams with different cross sections that could be used in vibration energy harvesters.**

Modal analysis and response calculations were done using LISA™ and MATLAB™. Four beams with different cross-section were put to test viz. Rectangular, Circular, Hollow Circular and Elliptical. The boundary conditions were set as fixed-fixed. The Euler-Bernoulli beam theory was used to generate the governing equation. The response was plotted as a graph of deflection vs. time at various intervals of spatial co-ordinates. The parameters of disturbing force f and ω were changed and the results were compared.

Keywords—Euler-Bernoulli Beam; Response of beam;vibration energy harvester)

I. INTRODUCTION

In the world that we live today, a lot of wireless devices are in use. These devices follow Moore's law which states that the number of integrated circuits doubles approximately every two years. This means that the processing speed increases at a rapid rate. The use of wireless devices with this high speed processing devices requires high energy. Unfortunately the rate of development of batteries does not follow Moore's law. To add to the problem, size of wireless devices are decreasing and hence the size of batteries too. These problems have led to search of solutions to power batteries. One of the solutions is Energy Harvesting. Energy harvesting is a process of deriving energy from ambient sources like solar, thermal, wind etc and stored, in smaller amounts, so as to use in wireless devices.

Wireless Sensor network is a network of distributed autonomous wireless devices that perform certain tasks. These tasks could range from monitoring weather conditions in form of temperature, pressure or sound to capturing images or videos in form of CCTV cameras. A large number of energy harvesting devices find applications in these WSN.

According to IDTechEx Forcast for energy harvesting, approximately 1.6 million energy harvesting devices are in use in WSN in 2011 which is a $13.75 million market. The study also forecasted that wireless sensors will reach $140 million and $210 million for military and aerospace applications by 2017 [1].

Vibrations are one of the most abundantly found energy source in engineering world. Each machine has some form of vibrations. These vibrations can be sourced and converted in to electrical energy. This electrical energy can then be used to power devices that monitor any changes in the machines and taking the necessary measures and thus creating an autonomous system of machines that are capable of repairing itself. These devices can be used in machineries where the maintenance tasks are difficult or machines that are running continuously and can be shut down or the machines that work in hazardous environment.

Considering the huge market for the energy devices, an energy harvesting devices can be manufactured that can convert the ambient vibrations of the machines and use the same vibrations to repair or recondition the machine. A cantilever or a fixed beam could be fitted with a piezoelectric device so that the beam could respond to the vibrations of the machine and the piezoelectric device could convert this response to electric energy. This energy could be stored in a capacitor and could be used as and when required.

II. LITERAURE REVIEW

The analysis of a beam is a very common problem that is studied in vibration and structural dynamics. The beams various types like simply supported, cantilever and fixed-fixed beams are studied for different applications. Vibration analysis of structure became an interesting area of research since 1897, when the Chester rail bridge collapsed in England [2]. The collapse of the bridge was a problem of moving load which does not affect an energy harvester. However that was the point that triggered the general interest in the field of vibration analysis.

To study vibration analysis of beams, one has to go to the very basics of vibrations and various beam theories. The book titled 'Theory of vibrations with applications' by William. T Thomson[3] deals with the theory of vibrations and then dives into the responses of beam by developing the Euler-Bernoulli theory for beam. Various other approximation methods are discussed as well. Energy based approaches like Rayleigh Method, Virtual work and Lagrange's equations are discussed in the book. Chapter 8 of the book in the Indian edition discusses the vibration of continuous systems that starts the theory with vibrating string problem. The chapter further deals with longitudinal vibration of rods and then is followed by torsional vibration of rods. The chapter then finally ends with Euler-Bernoulli beam theory. This thesis uses a lot of its basis of beam theory from this chapter and this book. The book however fails to deal with the response of a beam with forced vibrations. The beam theory that will be required to develop a vibration energy harvester will be a continuous system with forced vibrations. The equation that will govern such a system will be of the form

$$\rho A \frac{\partial^2 u_z}{\partial t^2} + EI \frac{\partial^4 u_z}{\partial x^4} = f(x,t)$$

The force f(x, t) makes the equation difficult to solve.
Another book titled 'Principals of vibrations' by Benson H. Tongue[4] deals with the response of the beams with forced vibrations. This book also like the earlier one starts with the basics of vibrations but dives into continuous systems sooner. The book toys with the idea of the forced vibration problem but halts to first introduce the concept of eigenfunctions and their orthogonality. This book uses the concept of eigenvalues and eigenvectors to generate the differential equation of the beam and is then solved by separation of variable of beams. A lot of the theory in this thesis about solving the equation of motion for the forced vibrating beam is based upon the theory from this book.
Another big influence for this thesis is a paper by John Ellie Sader[5] that discusses the frequency response of the cantilever beams immersed in viscous fluids for applications the atomic force microscope. This paper uses the Euler-Bernoulli beam theory to generate the differential equation of motion and solves it using dirac function and green's functions. This solution showed another method to solve the differential equation that presents itself in this thesis work. A lot of effort and time was put in to understand the basics of Green's function. This paper developed the theory considering a beam of arbitrary shapes. This is very the idea of studying different shapes of the beam came about. The equation of motion for the beam in this paper is of the form that this thesis requires however the disturbing force here is a combination of hydrodynamic force due to immersion in viscous fluid and the vibrating drive force.
Another paper by Andreza Tangerino Mineto et al[6] discusses the modeling of the cantilever beam for piezoelectric energy harvesting. This paper was presented at DINCON'10, the 9th Brazilian Conference on Dynamics, Control and their Applications. This paper also uses the Euler Bernoulli beam theory to generate the governing equation. The force f(x, t) in this paper is considered to be independent of x and is just taken as f (t). Separation of variables methodology is used to generate the response equations. The paper then further adds the piezoelectric coupling to this model. This big equation is then solved to generate the response in MATLAB™. The parameters for the beams in this thesis were taken from the parameter of the beam in this paper. The properties of steel like

- Young's Modulus of Elasticity = 200E09 N/mm^2
- Density of steel = 7860 kg/m^3 are taken from this paper

Another paper titled 'Vibration Energy harvesting using single and comb-shaped piezoelectric beam structures: Modeling and Simulation' by N.H. Diyana[7] et al was a similar to the paper above. The only difference here was the shape of the piezoelectric device. Also this paper had responses calculated by two different software viz. MATLAB™ and COMSOL™
This paper presents the simulation studies of single and comb-shaped piezoelectric beam structure for energy harvesting. The derivations of the mathematical equations are based on Euler Bernoulli Beam theory. The harvester is assumed to experience lateral vibration from its base excitation.

Safa Bozkurt Coskun [8] presented the transverse vibration analysis of Euler-Bernoulli beams using analytical approximation techniques. The mode shapes of the beams with different boundary conditions are found out in this paper using approximation methods. The approximation methods used are

- Admonion decomposition method
- Variational Iteration Method
- Homotopy Perturbation Method

There are many other papers which deal with the same models but use the theory of Timoshenko beam. However this thesis has limited the beam theory to Euler-Bernoulli Beams to reduce complications.
Very little work was found on the beams of various cross sections being examined. This thesis deals with fixing this problem and works on beams of various cross sections.

III. THEORY

Consider a fixed-fixed beam as shown in the figure. Both the ends of the beam are fixed. A force is acting on the beam and is equally distributed. This force is a sinusoidal force in this case
The equation of motion of Euler-Bernoulli beam displacement is given by [1]:

$$\rho A \frac{\partial^2 u_z}{\partial t^2} + EI \frac{\partial^4 u}{\partial x^4} \qquad (1)$$

Where, $u_z = u(x, t)$ is the beam deflection for $0 \le x \le L$
Boundary conditions for the fixed-fixed beam are
Partial differential of second order with respect to time is equal to zero at x=0 and x=L
Initial conditions for fixed-fixed beam is that at time zero, displacement is zero

Assumptions taken to solve the above equations are:
1. Structural assumptions are
   - 1.1. Initially straight beam
   - 1.2. Linear elastic material
   - 1.3. Small structural deformation
2. Shear deformation and rotary inertia effects are neglected (Euler Bernoulli beam) and so height: length ratio is very small
3. Disturbing force $f(x,t)$ is a sinusoidal force and hence taken as $f(x,t) = \bar{f} \cos \omega t$

Solving this equation by separation of variable method

$$u\ (x,t) = \sum_{n=1}^{\infty} y_n(t)\ \sin(\frac{n\pi x}{L}) \qquad (2)$$

Where, $y_n(t)$ is the modal displacement and $\sin(\frac{n\pi x}{L})$ is modal function or Eigen functions.

Substituting the above equation in equation (1), we get

$$\ddot{y}(t) + \omega_n^2 y_n(t) = f_n \text{ Cos } (\omega t) \qquad (3)$$

For n= 1, 3, 5….

Where,

$\omega_n^2 = (n^2\pi^2 EI/\rho L^2)$ and

$f_n = (4f/\rho\pi n)$

And therefore complete response is given by

$$u(x,t) = \sum_{n=1,3..}^{\infty} \frac{f_n}{\omega_n^2 - \omega^2} \sin(\frac{n\pi x}{L}) \cos(\omega t)$$

IV MATHEMATICAL MODELING

The response of the beams was calculated by using the software MATLAB™. A program was written to calculate the response from the equation (_) derived in the theory. The beams of four cross-sections were analyzed.

1. Rectangular Beam
   Breadth = 0.05m, Depth =0.01m
2. Circular Beam
   Radius = 0.025m
3. Elliptical Beam
   Major axis radius =0.035m, minor axis =0.015m
4. Hollow Circular Beam
   Outer Radius = 0.05m, Inner Radius = 0.04m

These lengths were selected so that the beams would be similar in appearance. The mechanical properties of the metal were kept the same as that of the LISA™ models. Young's Modulus was 200E09 N/mm^2 and the density was 7860 kg/m^3.The calculations were made for the first mode so the value of n was taken as 1. The disturbing force was considered. This force is a time bound sinusoidal force that was highly dependent on the values of the frequency. The force f was changed multiple times so as to note of any variations that occurred in the graph. Similarly the values of were also changed to find any variations in the behavior of the beams.

The program mainly calculated the deflection of the beams at various values of spatial coordinates x and time t. The values were then used to draw the graph of the deflections of the beams versus time at three different values of space. The graphs for each of the beams were overlaid in a single graph to give a comparison. The results and the graphs are discussed ahead

In the first program the values of the various parameters were

- ω= 100 rad/s
- f = 30000 units

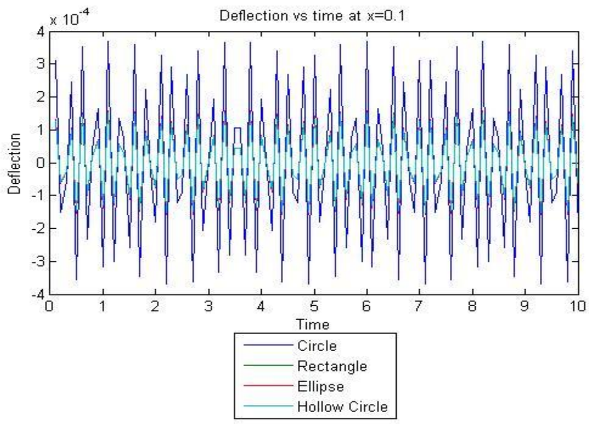


Fig. 1 vs. time at x = 0.1 (First Iteration)

The other parameters remained constant. The deflections were then calculated. The graph for each of them is shown below.

The first graph is a deflection of the beam vs. time at the spatial co-ordinate of x= 0.1.

The graph fig 1 shows deflection as a sine wave. The Y axis shows the deflection values in the range of 1E-04. The X axis signifies the time. The time for which the graph was calculated ranged from 0 to 10 seconds in the steps of 0.1 giving us about 100 deflection values to plot the graph. As we see the blue line signifies the circle. This is seen more prominently than any other beams. The hollow circle graph does not reach the heights and depths that the blue circle graph shows. This means that the circular beams gives the best response at frequency of 100 rad/sec.

The next graph fig. 4.2 shows similar trends at the midpoints of the beam. As we can see the deflections are in the range of 1E-03 which signifies that greatest deflections occur here at the centers. This is true considering that beam is vibrating in the first modal shape.

The thing to notice is that the shape of the graphs pretty much remains the same. The Circular beam shows the highest deflections. The next best deflections are shown by the rectangular beams. The elliptical and the hollow circular beams show near about the same deflections.

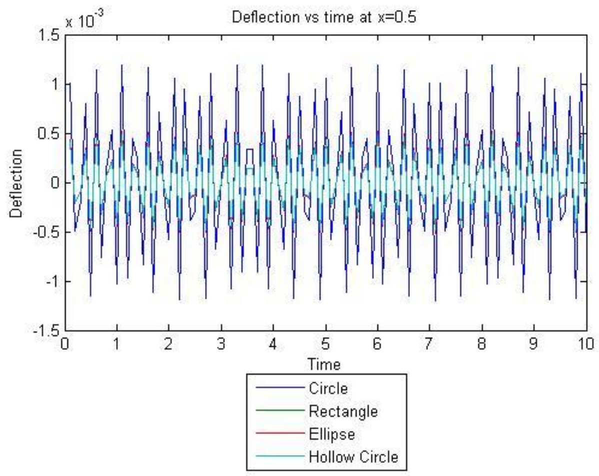
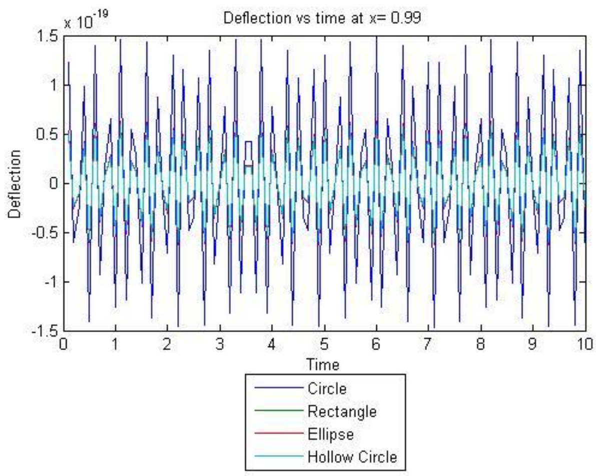


Fig 2 Deflection vs. time at x = 0.1 (First Iteration)



4.3 Deflection vs. time at x = 0.99 (First Iteration)

In the next graph fig 3, the graph shows the deflection vs. time at the spatial coordinate towards the end of the beam at 0.99 lengths of the beam. The deflection values had dropped drastically and now are in the range of 1E-24. This pretty much comes from the fact that the deflection at the end of the beam i.e. at x= 1 will be zero due to the fixed end. The graphs again show no significant changes in the shape of the curves which means that the circular beam still shows the best deflections. Considering the three graphs together, it can be seen that the circular beam shows the best response at any spatial value of x.

In the second program the values of the various parameters were

- ω= 100 rad/s
- f = 10000 units

The magnitude value of the force was lowered to notice any changes in the response. The other parameters remained constant. The deflections were then calculated. The graph for each of them is shown below.

The first graph is a deflection of the beam vs. time at the spatial co-ordinate of x= 0.1. The graph again remained the same and maintained its shape as that of the first program. The only notable difference was the reduction of the deflections proportionally. The same thing was observed in the other two graphs taken at x= 0.5 and x = 0.99. Again the circular beam showed the greatest deflections compared to any other beams.

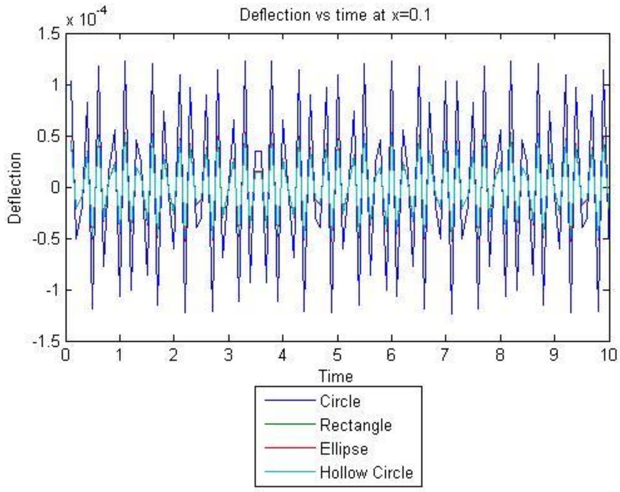
Figure 4.4 Deflection vs. time at x=0.1(Second Iteration)

In the third program the values of the various parameters were

- ω= 200 rad/s
- f = 30000 units

The force was brought back to 30000 N and the frequency of force was doubled from 100 rad/s to 200 rad/s. The other parameters remained constant. The deflections were then calculated. The graph for each of them is shown below.

The first graph is a deflection of the beam vs. time at the spatial co-ordinate of x= 0.1. This graph showed a good difference from the first graph. The different graphs showed merged on top of each other. The graph for hollow circular beam just fell short of the graph for the circular beam but did show a great rise in the deflections.



The graph at x=0.99 also showed similar trend in which the deflections for all the beams were very close leading to graphs superimposing on each other.

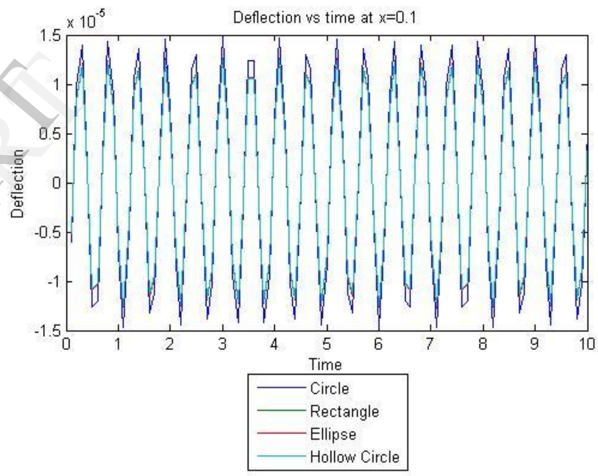


Figure 4.5 Deflection vs. Time at x = 0.1 (Third Iteration)

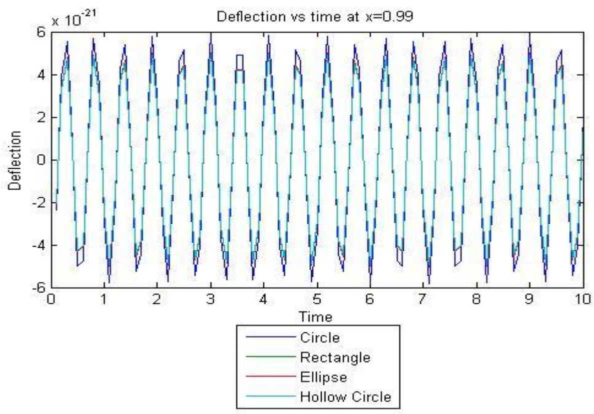


Figure 4.6 Deflection vs. Time at x = 0.99 (Third Iteration)

CONCLUSION

The displacement of the beams analyzed by the graphs showed that the circular beams responded well to lower frequencies of vibrations. When the frequency of the vibrations was increased, all the beams showed similar deflections. This means that higher the frequency of vibrations, the shape of the beam does not play a part in the response of the beams.

Hence, it can be said that if the frequency of vibration is low, then the circular beam would be the beam with greatest response while at high frequencies

A further study of all the same beams but with different boundary conditions must be studied. Energy harvesters could be very small in size and this may require the beams with different boundary conditions like cantilever beams. Cantilever beam needs to be studied with similar cross sections and shapes and then be compared to the beams in this thesis so that appropriate boundary conditions may be used if environment and design permits.

Another aspect that needs to be studied is the factor of damping. This thesis does not consider the effect of damping on the beam. A beam may be immersed in oils or coolants in various working environments and hence the damping becomes a critical factor in such cases.

REFERENCES

[1] IDTechEx (2011) Wireless Sensor Networks (WSN) 2012-2022: Forecasts, Technologies, Players. Cambridge: Dr Harrop, P. & Das,R.

[2] Willis, R. (1949) Report of the Commissioners Appointed to Inquire into the Application of Iron to Railway Structures. *London: Stationary office*.

[3] Thomson, W.T. (1998) *Theory of vibrations with applications.* United States, Pearson Education Inc.

[4] Tongue, B.H. (2002) *Principles of vibration.* New York, Oxford University Press Inc.

[5] Sader, J.E. (1998). Frequency response of cantilever beams immersed in viscous fluids with applications to atomic force microscope. *Journal of Applied Physics, Volume* 84(1),64-75

[6] Mineto, A. T., Souza Braun, M.P., Navarro, H. A., & Varoto, P. S. (2010) Modeling of a cantilever beam for piezoelectric energy harvesting. *Proceedings of the 9th Brazilian Conference on Dynamics Control and their Applications*, Serra Negra, SP

[7]] Diyana, N.H.; Asan G.A. Muthalif; Fakhzan, M.N.; & Nordin, A.N. (2012) Vibration Energy harvesting using single and comb-shaped piezoelectric beam structures: Modeling and Simulation *Procedia Engineering* 41

[8]] Safa Bozkurt Coşkun, Mehmet Tarik Atay, & Baki Öztürk (2011) *Advances in Vibration Analysis Research.*InTech publications
.