Robust Deadbeat Control of Twin Rotor Multi Input Multi Output System

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***Abstract*—This paper deals with a deadbeat PID control of Non-linear higher order system. The method used in this paper uses feedback arrangement which helps to improve system response. TheDeadbeat control method is applied to Non-linear model of Twin Rotor MIMO Laboratory setup (Helicopter model).This helicopter model is first simulated in matlab with simple PID controller and then with deadbeat controller. Comparative study shows that Deadbeat PID controller is capable of providing robust performance.**

Keywords— MIMOsystem, TRMS, PID, Deadbeat control.

1. INTRODUCTION

The TRMS system is nonlinear system similar to helicopter as shown in Fig. 1.The conventional controller like PID is used to control such nonlinear system[1-4], as it has simple structure and easy to use. In recent years new methods like feedback linearization [5] and gain scheduling [6] have been applied with good outcomes. Intelligent control theories like Fuzzy control , Neural network and GA applied, but designing logic for such intelligent controller is not easy[7,8,9].

Many methods are tested in order to get an acceptable performance of the controller. This includes minimum settling time, rise time, zero steady state error, less oscillation, robustness against disturbances. TRMS system is very complex and it includes uncertainties' thus finding gains of PID become difficult task. Also modeling of TRMs system is difficult [10, 11, and 12]. Even if we get system model, for entire input range it will not represent system exactly. In order to control TRMS system here, we use the method proposed in [13, 14] which include PID and Deadbeat controller. In [13] writer claims that "response will remain unchanged when all parameters change by as much as 50%".

The main problem with TRMS is that of coupling effect between two rotors, to encounter this problem we are going to decouple [15] into two SISO system and effect is considered as disturbances to each of the SISO system. Then the Deadbeat control scheme is applied to the SISO system.

This paper is organized as follows. Section 1 introduction, Section 2 system model and its mathematical representation, section 3 Deadbeat control method and controller design, section 4 then system is simulated in MATLAB. Finally, comparative analysis between PID and Deadbeat PID controller and this study is summarized in conclusion.

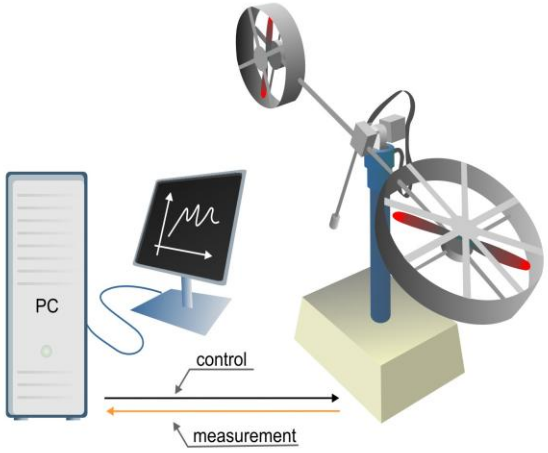

Fig.1 Schematic diagram of TRMS control system

2. TWIN ROTOR SYSTEM

The TRMs is a laboratory platform designed for control experiment by Feedback Instruments Ltd [16]. It consists of a beam pivoted on its base in such way that it can rotate in both vertical and horizontal planes. Two DC motors are used for two rotors (main and tail rotor), at each end of the beam. In actual Helicopter, aerodynamic force is controlled by changing the angle of attack of the blades but TRMS model designed such that force is controlled by controlling speed of motors. The main rotor produces force, which allows the beam to raise vertically making rotation along pitch axis. While tail rotor produces horizontal force to make the beam turn left or right around the yaw axis.

Usually, phenomenological models tending to be nonlinear, this means that at least one of the states is an argument of a nonlinear function. So as to present such a model as a transfer function (a form of linear plant dynamics representing a control system), it has to be linearized. The following non-linear model equations can be derived.

Mathematical equation in vertical plane is given as
$I_1.\ddot{\psi} = M_1 - M_{FG} - M_{B\psi} - M_G$ (1)
Where
$M_1 = a_1.\tau_1^2 + b_1.\tau_1$ Nonlinear static characteristic(2)
$M_{FG} = M_g \sin\psi$ Gravity momentum (3)
Friction forces momentum
$M_B = B_{1\psi}.\dot{\psi} + B_{2\psi}.\sin(\dot{\psi})$ (4)

$M_G = K_{gy}.M_1.\dot{\varphi}.\cos\psi$ Gyroscopic momentum (5)

The motor and the electrical control circuit is approximated as a first order transfer function, thus the rotor momentum in Laplace domain is described as

$$\tau_1 = \left(\frac{K_1}{T_{11}s+T_{10}}\right)u_1 (6)$$

Mathematical equation in horizontal plane is given as

$$I_2.\ddot{\varphi} = M_2 - M_{B\phi} - M_R \quad (7)$$

Where

$M_2 = a_2.\tau_2^2 + \boldsymbol{b_2}.\tau_2$ Nonlinear static characteristic (8)

Friction forces momentum

$$M_{B\varphi} = B_{1\varphi}.\dot{\psi} + B_{2\varphi}\sin(\dot{\varphi}) \quad (9)$$

$$M_R = \frac{K_c(T_0s+1)}{(T_ps+1)}\tau_1 \quad \text{Cross reaction momentum} \quad (10)$$

Rotor momentum in Laplace domain is given as

$$\boldsymbol{\tau_2 = \left(\frac{K_2}{T_{21}s+T_{20}}\right)u_2 \ (11)}$$

The model parameters used in above (1)-(11) equation are chosen experimentally, which makes the TRMS non-linear model a semi-phenomenological model.

The boundary for the control signal is set to [-2.5 to +2.5].

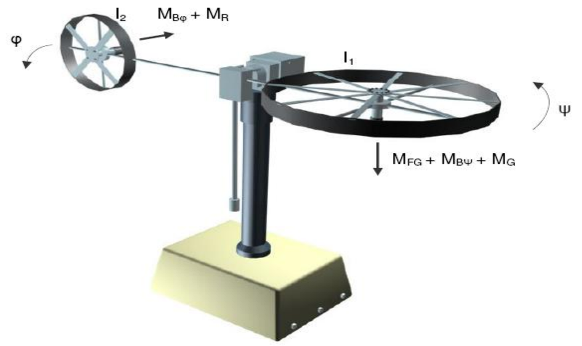

Fig.2

Table 1 gives approximated values of parameters.

Parameters	Value
I_1- moment of inertia of vertical rotor	$6.8 * 10^{-2} kgm^2$
I_2- moment of inertia of horizontal rotor	$2 * 10^{-2} kgm^2$
a_1-static characteristic parameter	0.0135
b_1-static characteristic parameter	0.0924
a_2-static characteristic parameter	0.02
b_2-static characteristic parameter	0.09
M_g-gravity momentum	0.32 Nm
$B_{1\psi}$-friction momentum function parameter	$6 * 10^{-3} Nms/rad$
$B_{2\psi}$- friction momentum function parameter	$1 * 10^{-3} Nms^2/rad$
$B_{1\varphi}$- friction momentum function parameter	$1 * 10^{-1} Nms/rad$
$B_{2\varphi}$- friction momentum function parameter	$1 * 10^{-2} Nms^2/rad$
K_{gy} - gyroscopic momentum parameter	0.05 s/rad
k_1-motor 1 gain	1.1
k_2-motor 2 gain	0.8
T_{11}-motor 1 denominator parameter	1.1
T_{10}- motor 1 denominator parameter	1
T_{21}- motor 2 denominator parameter	1
T_{20}- motor 2 denominator parameter	1
T_{11}- cross reaction momentum parameter	2
T_{11}- cross reaction momentum parameter	3.5
k_1-cross reaction momentum gain	-0.2

Table 1

As we know there is cross coupling between main and tail rotor. With the help of model equation nonlinear Simulink model of TRMS is obtained as shown in Fig.3

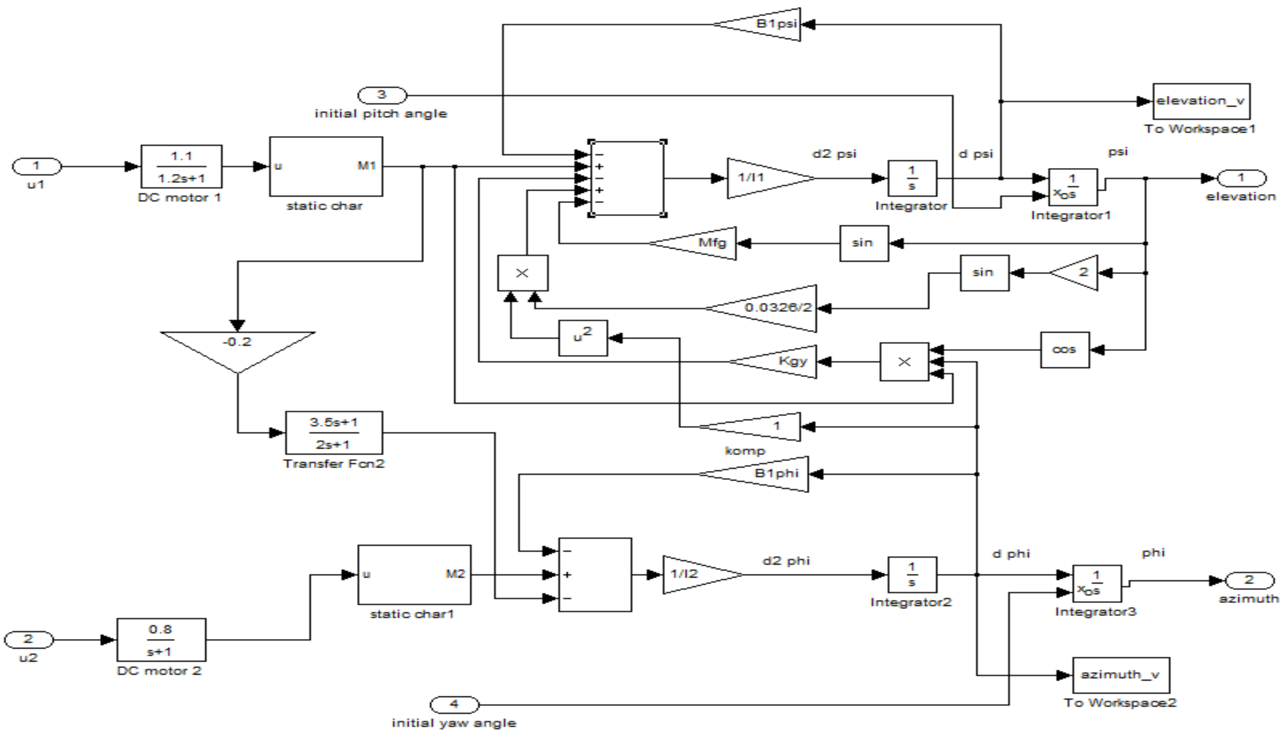

Fig. 3 Nonlinear simulink model of TRMS

3. DEADBEAT CONTROLLER DESIGN METHODOLOGY

The main goal of control system is to reach the desired value with zero steady state error within specified settling time. A deadbeat response should have zero steady state error, minimum rise time, controllable settling time, % overshoot from 0 to 2%, % undershoot < 2% . Also have robustness against external disturbance and parameter uncertainty.Refer following fig.4 for the basic structure of the controlled system design.

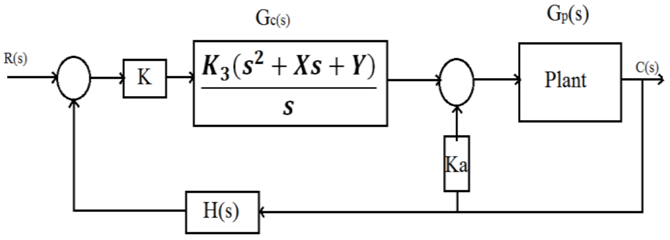

Fig.4 Basic structure

1. Use PID controller as $G_c(s)$.
2. Add cascade gain K before the PID controller.
3. Add state variable feedback gain Ka. This will make the system over specified by at least one variable
4. Determine n_p for $G_cG_p(s)$, where n_p equals the no of poles in $G_cG_p(s)$.
5. Add the feedback

H1(s) =1 for $n_p = 2$
H1(s) =1+$K_b s$ for $n_p = 3\ or\ 4$
H1(s) = $1 + K_b s + K_c s^2$ for $n_p = 5$

6. select gain, using the coefficient from table 2 to achieve response with the following requirement:

(a)set k=1

(b)set $\omega_n = \frac{T_s'}{80\%\ of\ the\ desired\ settling\ time}$

(c)set C.E. of closed loop equal to:

$$s^{n_p} + \alpha\omega_n s^{n_p-1} + \beta\omega_n^2 s^{n_p-2} + \ldots + \omega_n^{n_p}$$

(d)The roots of H(s) must be real & negative

(e)Smallest root of H(s) will set the desired

$$t_s \cong \frac{4}{80\%\ of\ the\ desmallest\ rootsired\ settling\ time}$$

7. Increase K until the response becomes deadbeat and the settling time is approximately equal to the desired value.

Order n_p	α	β	γ	δ	ε	Ts'
2nd	1.82	-	-	-	-	4.82
3rd	1.90	2.20	-	-	-	4.04
4th	2.20	3.50	2.80	-	-	4.81
5th	2.70	4.90	5.40	3.40	-	5.43
6th	3.15	6.50	8.70	7.55	4.05	6.04

Table 2 Deadbeat coefficients and response time

4. CONTROLLER DESIGN

First using the decouple techniques we separate the system into two SISO ones as shown below

Vertical part (Tail Rotor)

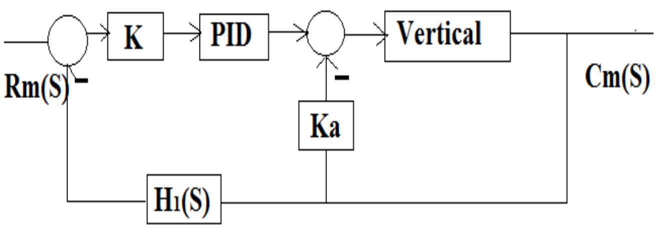

Fig.5

Horizontal part (Main Rotor)

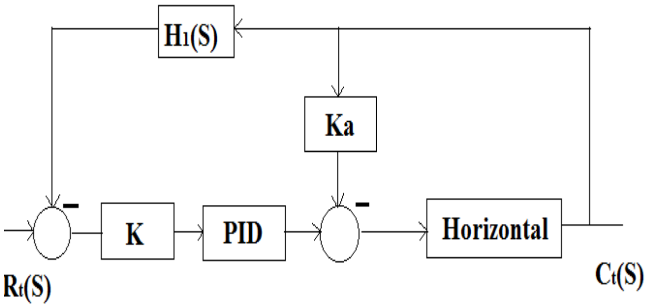

Fig.6

TF of theTRMS in vertical and horizontal movement are given as

$$G_t(s) = \frac{15.02}{s^3+3.458s^2+2.225s}.\ \text{Tail rotor} \qquad (12)$$

$$G_m(s) = \frac{1.519}{s^3+0.748s^2+1.533s+1.046}.\ \text{Mainrotor} \quad (13)$$

This TF will be utilized throughout this work.

4.1 Tail rotor controller design

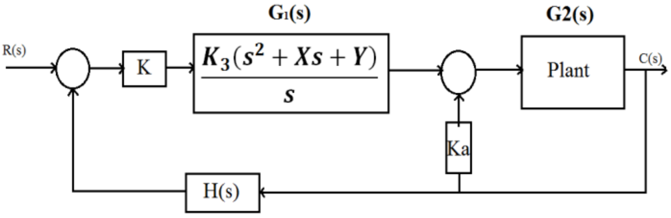

Fig.7 Tail rotor control

The close loop transfer function

$$\frac{C(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1+G_2(s)H_2(s)+G_1(s)G_2(s)H_1(s)} \qquad (14)$$

Where $G_1(s) = G_c(s)$ and $G_2(s) = G_t(s)$

n_pEquals the no of poles in $G_cG_p(s)$ = 4

Thus

$H_1(s) = 1 + K_b s \text{And} H_2(s) = K_a$

Finally

$$\frac{C(s)}{R(s)} = \frac{15.02K[K_3(S^2+XS+Y)]}{S^4+\{3.458+15.02K_bKK_3\}S^3+\{2.225+15.02KK_3+15.02KK_bK_3X\}S^2+\{15.02K_a+15.02KK_3X+15.02KK_bK_3Y\}S+\{15.02KK_3Y\}} \qquad (15)$$

Then CE of above transfer function compared with Standard CE

$$S^4+\alpha\omega_n S^3+\beta\omega_n^2 S^2+\gamma\omega_n^3 S+\omega_n^4 \qquad (16)$$

From design procedure we get coefficient values from Table 2, also we get value of

$$\omega_n = \frac{T_s'}{T_s\ *\ 80\%} = \frac{4.81}{1.6} = 3.00625$$

Therefore CE of deadbeat TF is:

$$S^4+6.613S^3+31.6314S^2+76.07355S+81.6771 \quad (17)$$

Comparing characteristic equation (16) and (17) we have

$$3.485 + 15.02KK_bK_3 = 6.6138$$

$$2.225 + 15.02KK_3 + 15.02KK_bK_3X = 31.6314$$

$$15.02K_a + 15.02KK_3X + 15.02KK_bK_3Y = 76.0735$$

$$15.02KK_3Y = 81.6771$$

Then cascade gain K is set equal to 1. After some trial and error we get values for $K_3 = 17$

$$K_b = 0.243K_a = -45.848$$
$$X = 14.21Y = 38.65$$

Select k until deadbeat response not reached.

4.2 Main rotor control

Similarly,

$$\frac{C(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1 + G_2(s)H_2(s) + G_1(s)G_2(s)H_1(s)}$$

Where $G_1(s) = G_c(s)$ and $G_2(s) = G_m(s)$

n_pEquals the no of poles in $G_cG_p(s)$ = 4
Thus
$H_1(s) = 1 + K_b s$And$H_2(s) = K_a$

$$\frac{C(s)}{R(s)} = \frac{K[K_3(S^2 + XS + Y)]}{S^4 + \{0.748 + 1.519K_bKK_3\}S^3 + \{1.533 + 0.1549KK_3\}S^2 + \{1.046 + 1.519K_a + 0.1549KK_bK_3Y\}S + \{1.519KK_3Y\}}$$

Then CE of above transfer function compared with Standard CE

$$S^4 + \alpha\omega_n S^3 + \beta\omega_n^2 S^2 + \gamma\omega_n^3 S + \omega_n^4 \qquad (18)$$

From design procedure we get coefficient values from Table 2, also we get value of

$$\omega_n = \frac{T_s'}{T_s * 80\%} = \frac{4.81}{1.6} = 3.00625$$

Therefore CE of deadbeat TF is:

$$S^4 + 6.613S^3 + 31.6314S^2 + 76.07355S + 81.6771 (19)$$

Comparing characteristic equation (18) and (19) we have

$$0.748 + 1.519K_bKK_3 = 6.6138$$

$$1.533 + 0.1549KK_3 = 31.6314$$

$$1.046 + 1.519K_a + 0.1549KK_bK_3Y = 76.0735$$

$$1.519KK_3Y = 81.6771$$

Then cascade gain K is set equal to 1. After some trial and error we get values for $K_3 = 7.723$

$$K_b = 0.5K_a = -2.5453$$
$$X = 3.131Y = 6.963$$

5. SIMULATION AND RESULTS

5.1 For Step input

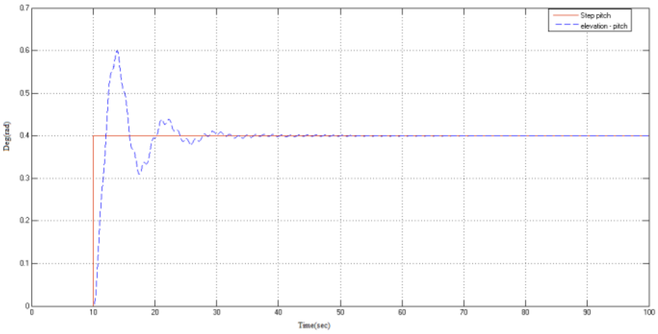

Fig. 8 PID response to main rotor(pitch)

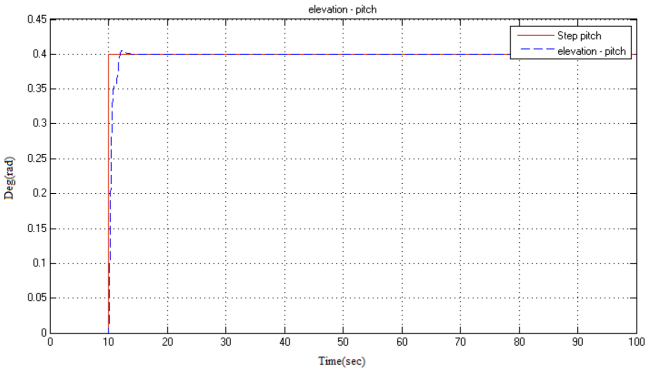

Fig. 9 Deadbeat PID response to main rotor(pitch)

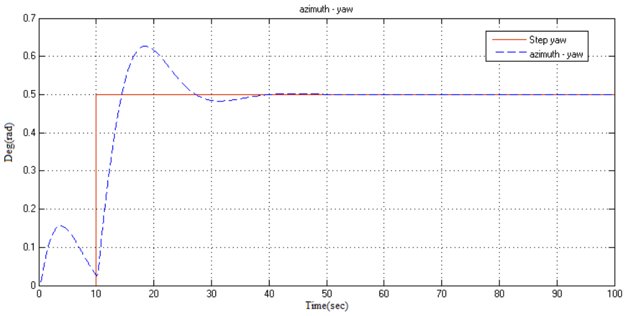

Fig. 10 PID response to tail rotor

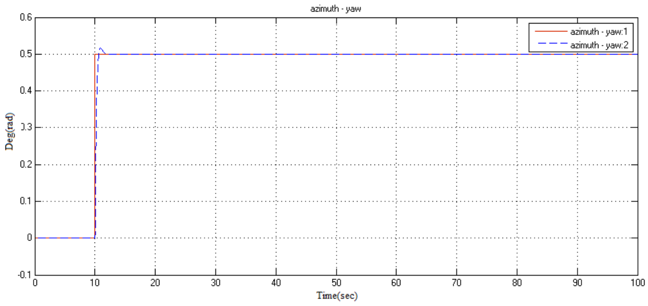

Fig.11 Deadbeat PID response to tail rotor

5.2 For sin wave input

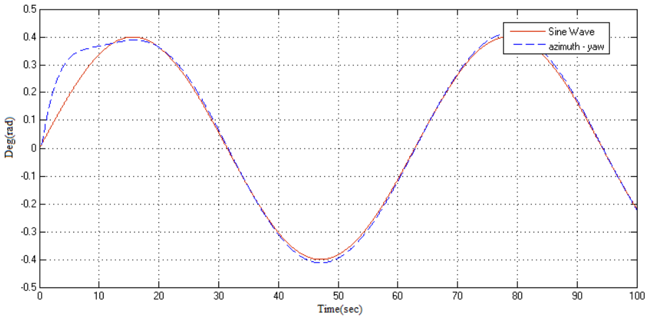

Fig. 12 PID response to tail rotor (yaw)

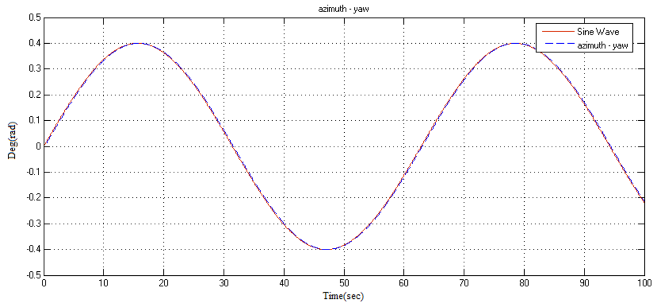

Fig.13 Deadbeat PID response to tail rotor (yaw)

5.3 For square input

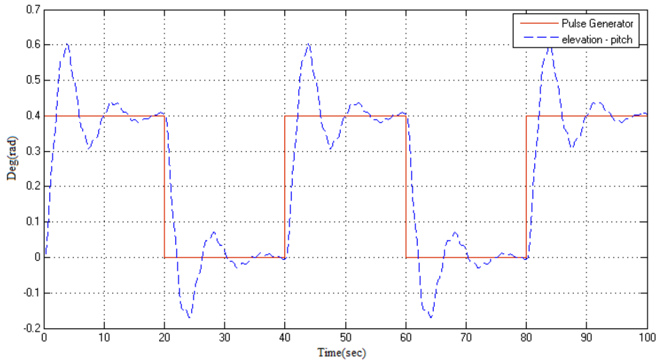

Fig. 14 PID response to main rotor(pitch)

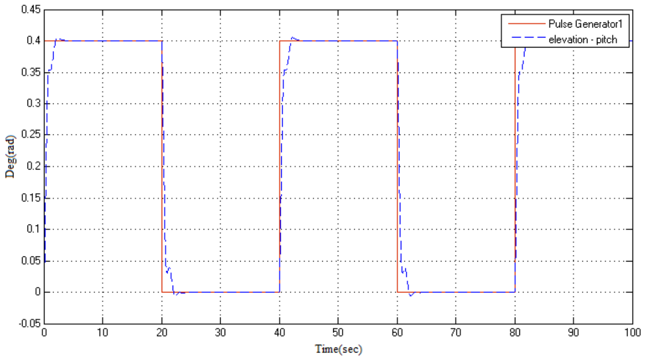

Fig. 15 Deadbeat PID response to main rotor(pitch)

Above Figures shows the controller effect for different type of inputs. From results it is clear that traditional PID controller responses have overshoots and its settling time is more than designed Deadbeat controller. Here with Deadbeat control approach result shows remarkable improvement in system behavior.

6.CONCLUSION

In this study, we have successfully modeled TRMS system and successfully applied robust Deadbeat controller scheme. In system performance, the settling time has been reduced also overshoot has been decreased.Implemented scheme does not deal with much harder mathematics. it is easily understood method .

But as this method is model based, we require accurate system transfer function.

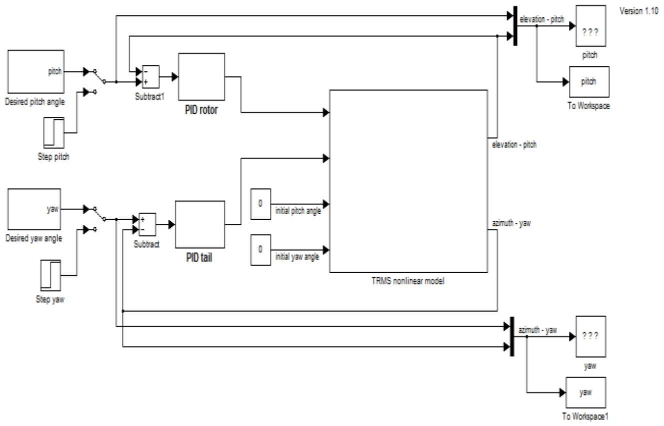

Fig. 16 simulation diagram of PID controller scheme

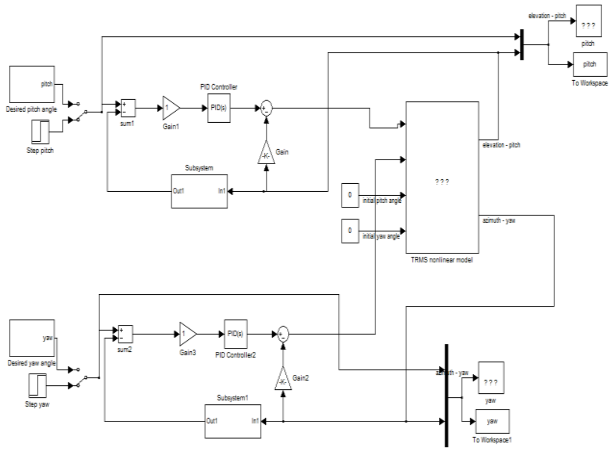

Fig. 17 simulation diagram of deadbeat controller scheme

7. REFERENCES

1. K. Turkoglu, U. Ozdemir, M. NIkbay, E. Jafarov, "PID parameter otimization of UAV longitudinal Flight control system", World Academy of Science, Engineering and Technology 45, 2008.
2. S. Yang, K. Li and J. Shi, "Design and Simulation of the Longitudnal Autopilot of UAV Based on Self-Adaptive Fuzzy PID Control", International Conference on Computational Intelligence and Security, 2009.
3. H. Takaaki, Y. Kou, A. Yoshinori, M. Wanori, A. Santoshi and M. Shun, "A Design Method for Modified PID Control System for Multiple Input Multiple Output Plants to Attenuate Unknown Disturbances", World Automation Congress 2010 TSI Press.
4. F, Sajjad, G. Dawei, K. Naeem and P. Ian, "Fault Detection, Isolation and Accommodation in a UAV Longitudinal Control System", Conference on Control and Fault Tolerent System, Nice, France, October 2010.

5. T. J. Koo, S. Sastry, “Output Tracking Control Design of a Helicopter Model Based on Approximate Linearization, Decision and Control”, 1998. Proceedings of the 37th IEEE
6. N. Wada, M. Minami, Y. Matsuo, M. Saeki, “Tracking Control of a Twin Rotor Helicopter Model Under Thrust Constrains Using State Dependent Gain Schedulling and Reference Management ”, Mechatronics and Automation, 2008. ICMA 2008.
7. B. U. Islam, N. Ahmed, D. L. Bhatti and S. Khan, “Controller Design Using Fuzzy Logic for a Twin Rotor MIMO System”, pp.264-268, 2003.
8. F. M. Aldebrez, M. S. Alam and M. O. TOkhi, “Input-shaping with GA tuned PID for target tracking and vibration reduction”, Limassol, Cyprus, pp.485-490,2005.
9. Juang, J. G., Liu, T. K. “Real Time Neural Network Control of Twin Rotor System”,Adv. Mater. Res., 2011, 271-273, pp. 616-621
10. I. Z. Mat Darus, F. M. Aldebrez and M. O. Tokhi, “Parametric modelling of a twin rotor system using genetic algorithms”, Hammamet, Tunisia, pp.115-118, 2004.
11. S. M. Ahmad, M. H. Shaheed, A. J. Chipperfield and M. O. Tokhi, “Nonlinear modelling of a twin rotor MIMO system using radial basis function networks”, pp.313-320, 2000.
12. F. M. Aldebrez, I. Z. M. Darus and M. O. Tokhi, “Dynamic modelling of a twin rotor system in hovering position”, pp. 823-826, 2004.
13. J Dawes, L. Ng, R. Dorf and C. Tam, “Design of Deadbeat Robust System”, Glasgow, UK, pp1597-1598, 1994
14. L. Chien, “IMC-PID Controller Design-An Extension”, IFAC Proceeding Series,6, pp.147-152, 1988.
15. Stephanopoulos, Chemical Process Control.
16. Twin Rotor MIMO System Control Experiment 33-949S User Manual, Feedback Instrument Ltd, UK

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