

Robustness Study for Longitudinal and Lateral Dynamics of RLV with Adaptive Backstepping Controller

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Abstract-This paper presents a robustness study of Adaptive backstepping method applied to a Reusable Launch vehicle longitudinal dynamics. RLV's are subjected to large parameter uncertainties and disturbances in the atmosphere. Adaptive controller provides a consistently updating algorithm to cope with the parameter uncertainties and small disturbances. Longitudinal dynamics of the RLV is controlled using adaptive backstepping method and robustness study was performed by giving some inputs. The simulation is done through MATLAB and results indicate the necessity of a Robust controller in the presence of disturbances.

Index Terms-Backstepping control, Adaptive backstepping control, Reusable Launch Vehicles (RLV), Longitudinal dynamics, pitch rate, yaw rate, sideslip angle, Lateral dynamics, etc..

I. INTRODUCTION

Reusable launch vehicles (RLV's) are used to deliver satellites and other celestial objects into space. After the deliver it returns back. The dynamics of the RLV in the decent phase is just like controlling an unmanned air vehicle in the atmosphere. The flight Reusable Launch Vehicle during its descent phase is subjected to a huge variation in Mach numbers and adverse flight envelopes and the system must be stabilized in the midst of these uncertainties. The control surfaces used for the orientation in space are ailerons and Rudders. The control surfaces are represented in Fig.1. The longitudinal dynamics are controlled by pitch rate and the latitude control is provided by yaw rate. Latitude control and longitudinal control can be seen from Fig.2 and Fig.3.

The RLV considered here is X-38 vehicle which was developed by NASA . The equations of dynamics are obtained from from Diagroro Ito et al, [1] which involves the latitudinal and longitudinal dynamics. The nonlinear sets of equations are converted into a strict feedback form and pure feedback form by some assumptions [2]. This modified sets of equations are considered for the control purpose . Adaptive backstepping controller design for strict feedback systems are proposed by Kritic et al, [3]. Another application to the flight control was proposed by Ola Håkegard[4] which was very helpful in formulating this work. Other applications to this adaptive backstepping

controller was found to be applied on Inverted Pendulum [5][6] and Electrohydraulic actuators [7]. The theory of robust adaptive control was proposed by Ionnu et al, [8] which deals with the robustness of adaptive control systems was used to verify the robustness stability of the systems under disturbances

The Paper has been organised as follows. Section II deals with the mathematical modelling of RLV, X-38. Section III deals with the Theory of Adaptive backstepping controller. Section IV deals with the Design of Adaptive controllers for the prototype model. In Section V the Simulation results are shown with some discussions on it. Section VI is the Conclusion part and Section VII deals with Future Works.

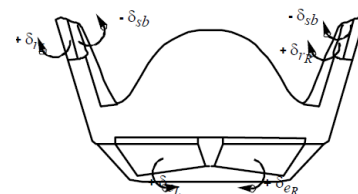


Fig.1: Control surfaces of RLV

II. MATHEMATICAL MODELLING OF RLV

The basic assumptions considered for the formulation are, the atmosphere is considered to be fixed w.r.to the earth and the disturbances are considered to be act from control surface or atmospheric turbulence. The basic Newton's Second law of motion is considered.

$$\sum F = \frac{d}{dt} (m V_T) \quad \text{and} \quad (1)$$

$$\sum M = \frac{dH}{dt}$$

Where m is the mass of the aircraft and V_T is the terminal velocity of the aircraft. M is the moment of inertia.

The derivative $\frac{dV_T}{dt}$ can be resolved as

$$\frac{dV_T}{dt} = 1_{V_T} \frac{dV_T}{dt} + wV_T \quad (2)$$

The final equations of motions are developed as

$$\sum \Delta F = i \sum \Delta F_x + j \sum \Delta F_y + k \sum \Delta F_z \quad (3)$$

The angular moment equation is also resolved as

$$\frac{dV_T}{dt} = 1_H \frac{dH}{dt} + wH \quad (4)$$

The final moment equation is

$$\sum \Delta M = i \sum \Delta L + j \sum \Delta M + k \sum \Delta N \quad (5)$$

Where L, M and N are the moment along X, Y, Z axis respectively.

The control surfaces are modelled to obtain the control input vectors. The elevon deflections are averaged to give the total elevon angle or elevator angle (δ_e) for pitch control and the average of the difference gives aileron angles for roll control

$$\delta_e = \frac{(\delta_{eL} + \delta_{eR})}{2} \text{ And } \delta_a = \frac{(\delta_{eL} - \delta_{eR})}{2} \quad (6)$$

By commanding the deflections either symmetrically or asymmetrically, these two pairs of surfaces provide the same control effects as that of conventional control surfaces. The non-linear set of equations of longitudinal motion for the X-38 vehicle is

$$\dot{q} = \frac{1}{I_y} \{M_\alpha \sin \alpha + M_q q + M_{\delta_e} \sin \delta_e + M_{\delta_r} \sin \delta_r\} \quad (7)$$

$$\dot{\alpha} = \frac{Z_\alpha}{V_T} \sin \alpha - \frac{g \sin \gamma \theta}{V_T} + \left[1 + \frac{Z_q}{V_T}\right] q + \frac{Z_{\delta_e}}{V_T} \sin \delta_e + \frac{Z_\alpha}{V_T} \sin \alpha$$

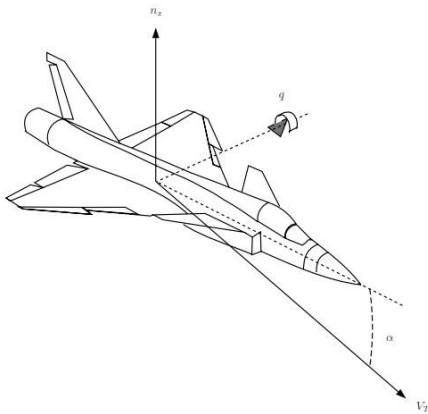


Fig.2: longitudinal dynamics for aircraft

These nonlinear set of equations are converted into strict feedback form by some assumptions [2] and the new set of equations are in the form of strict feedback form and is given by

$$\dot{\alpha} = \frac{Z_\alpha}{V_T} \sin \alpha + q$$

$$\dot{q} = \frac{1}{I_y} \{M_\alpha \sin \alpha + M_q q + M_{\delta_e} \sin \delta_e\} \quad (8)$$

Where α is the angle of attack, q , the roll rate of the aircraft and Z_α is the uplift force provided by the control surface. The equations are further modified as

$$\dot{\alpha} = \Phi_1 \sin \alpha + q$$

$$\dot{q} = \Phi_2 \sin \alpha + \Phi_3 q + \Phi_4 \sin \delta_e \quad (9)$$

The lateral dynamics equations are provided as

$$\dot{\beta} = \frac{Y_B}{V_T} \sin \beta - r$$

$$\dot{r} = \frac{1}{I_z} [N_\beta \sin \beta + N_{\delta_r} \sin \delta_r] \quad (10)$$

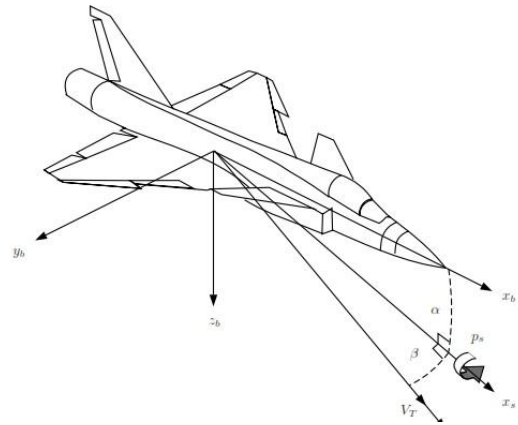


Fig.3: Lateral dynamics of RLV

where r is the yaw rate, V_T is the terminal velocity, β is the side slip angle. The set of equations are further modified as

$$\dot{x}_1 = \Phi_5 \sin x_1 - x_2$$

$$\dot{x}_2 = \Phi_6 \sin x_1 + \Phi_7 \sin u \quad (11)$$

These final sets of equations are used for the controller designing purpose

III. THEORY OF ADAPTIVE BACKSTEPPING CONTROLLER

Adaptive backstepping controller is a nonlinear control technique which allows the designer to construct controllers for a wide range of nonlinear systems in a structured, recursive way. The dynamic feedback part constantly updates the static feedback control part to deal with parametric uncertainties.

The tuning functions are introduced in adaptive backstepping technique [3] to reduce the over parameterization so that only one update law for each parameter is required.

Consider a second order system

$$\dot{x}_1 = \varphi(x_1)^T \theta + x_2$$

$$\dot{x}_2 = u \quad (12)$$

Where $(x_1, x_2) \in \mathbb{R}^2$ are the states and $u \in \mathbb{R}$ is the control input, $\varphi(x_1)$ is a smooth non linear function vector and θ is the vector of unknown constant parameters. The control objective is to regulate the states with any initial conditions. The adaptive backstepping starts by introducing regulating errors $e_1 = x_1 - x_{ref}$ and $e_2 = x_2 - \alpha$. The virtual control α is defined in terms of parameter estimate $\hat{\theta}$ as

$$\alpha(x_1, \hat{\theta}, x_{ref}, \dot{x}_{ref}) = -C_1 e_1 - \varphi(x_1)^T \hat{\theta} + \dot{x}_{ref}, \quad (13)$$

$$C_1 > 0;$$

The virtual control reduces the (e_1, e_2) dynamics to

$$\dot{e}_1 = \varphi(x_1)^T \check{\theta} + e_2 - C_1 e_1$$

$$\dot{e}_2 = u - \frac{d\alpha}{dx_1} \dot{x}_1 - \frac{d\alpha}{dx_{ref}} \dot{x}_{ref} - \frac{d\alpha}{dx_{ref}} \ddot{x}_{ref} - \frac{d\alpha}{d\hat{\theta}} \dot{\hat{\theta}} \quad (14)$$

Where $\check{\theta} = \theta - \hat{\theta}$ is the parameter estimation error. A closed loop function is defined that not only penalizes the tracking errors, but also the estimation errors as

$$V(e_1, e_2, \check{\theta}) = \frac{1}{2} (e_1^2 + e_2^2 + \frac{1}{\gamma} \check{\theta}^2) \quad (15)$$

Which is the lyapunov function that should be pdf and its negative should be ndf. For that the control law should be

$$u = -C_2 e_2 - e_1 - \frac{d\alpha}{dx_{ref}} \dot{x}_{ref} - \frac{d\alpha}{dx_{ref}} \ddot{x}_{ref} - \frac{d\alpha}{d\hat{\theta}} \dot{\hat{\theta}} - \frac{d\alpha}{dx_1} x_2 - \frac{d\alpha}{dx_1} \varphi(x_1)^T \hat{\theta} \quad \text{Where } C_2 > 0 \quad (16)$$

And the update law for $\hat{\theta}$ is given by

$$\dot{\hat{\theta}} = \gamma \varphi(e_1 - \frac{d\alpha}{dx_1} x_2) \quad (17)$$

For the practical applications the plant will be subjected to low frequency unmodelled dynamics, measurement noises, computational round-off errors and sampling delays etc.. The uncertainties will hardly affect the robustness of the adaptive backstepping design. The lack of robustness is primarily due to the control laws which are nonlinear in general and therefore subjected to modelling error effects.

IV. ADAPTIVE CONTROLLER DESIGN FOR THE LONGITUDINAL DYNAMICS OF RLV WITH DISTURBANCES

IV.A. LONGITUDINAL DYNAMICS

Reusable launch vehicles are very much subjected to the disturbances and the responses may drift from the global boundness. This paper proposes a controller which can bring the system back inside the variation limits. An adaptive backstepping controller is designed for equation

(9) and the system is made robust by incorporating leakage terms in it.

Let $e_1 = \alpha - \alpha_{ref}$ (18) for regulation reference must be zero. We define the first lyapunov function as

$V_1 = \frac{1}{2} e_1^2 + \frac{1}{2\gamma} \hat{\theta}_1^2$, so as to make e_1 zero as time tends to infinity. The control vector is selected as q and q should possess the value

$$q_{des} = \alpha_{ref} - \hat{\theta}_1 \sin \alpha - C_1 e_1, \quad \text{and} \quad (19)$$

$$\dot{\hat{\theta}}_1 = \gamma_1 \sin \alpha e_1 \quad (20)$$

The second error variable is defined as $e_2 = q - q_{des}$ and the aim is to make e_2 zero. So that the second lyapunov function is described as

$$V_2 = \frac{1}{2} e_1^2 + \frac{1}{2} e_2^2 + \frac{1}{2\gamma} \hat{\theta}_2^2 + \frac{1}{2\gamma} \hat{\theta}_3^2 + V_1 \quad (21)$$

Select the control vector δe to make V_2 negative definite.

For the adaptive controller the parameters are replaced by parameter updates so that the equation will be

$$\delta e = \sin^{-1} \left\{ \frac{1}{\theta_4} [q_{des} - \hat{\theta}_3 q - \hat{\theta}_2 \sin \alpha - C_2 e_2 - C_1 e_1] \right\} \quad (22)$$

The other parameter update laws or tuning functions are given by

$$\dot{\hat{\theta}}_2 = \gamma_2 \sin \alpha e_2 \quad (23)$$

$$\dot{\hat{\theta}}_3 = \gamma_3 q e_2$$

IV.B. LATERAL DYNAMICS

Let the error between x_1 and x_{1des} is e_1 . For the regulation e_1 should be zero. For that we introduce the lyapunov function

$$V_1 = \frac{1}{2} e_1^2 + \frac{1}{2\gamma} \hat{\theta}_5^2 \quad (24)$$

We get $x_{2des} = \theta_1 \sin x_5 - \dot{x}_{5des} + C_1 e_1$ to make \dot{V}_1 negative definite so the second error term arises which is

$$e_2 = x_2 - x_{2des} \quad (25)$$

Then

$$V_2 = \frac{1}{2} e_1^2 + \frac{1}{2} e_2^2 + \frac{1}{2\gamma} \hat{\theta}_6^2 + V_1 \quad (27)$$

From (27) the control law is given as

$$u = \sin^{-1} \left\{ \frac{1}{\theta_7} [x_{2des} - \hat{\theta}_6 \sin x_1 - C_2 e_2] \right\} \quad (28)$$

The other update laws are given by

$$\dot{\hat{\theta}}_5 = \gamma_5 \sin x_1 e_2 \quad (29)$$

$$\dot{\hat{\theta}}_6 = \gamma_6 \sin x_1 e_2$$

The controllers with adaptive gains and tuning functions are designed and the system along with the controller is subjected to test for the regulation and robustness.

V. SIMULATION RESULTS AND DISCUSSIONS

The RLV X-38 is modelled in this paper was used with the adaptive backstepping control. The variations of the angle of attack with different values of gains are shown in Fig.4. The variations of roll rate with different values of gains are plotted in Fig.5

curves which was given as Fig.8. The roll rate is approaching about 250 and angle of attack is found to be approaching about 60 in 10 seconds. Here the need for a robust controller arises to bring back the system to robust stability.

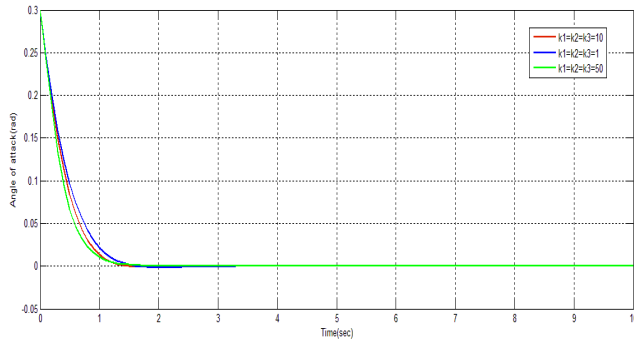


Fig.4: Regulation curve for angle of attack

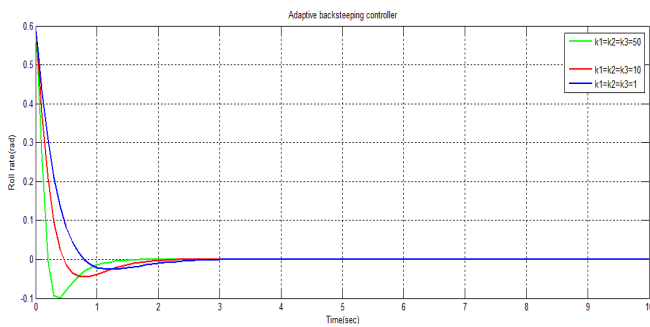


Fig.5: Regulation curve for roll rate

From the responses it is found out that as gain value increases adaption will be very fast and regulation can be achieved early. Gain values used are 1, 10, 50 and corresponding results are shown above.

A bounded disturbance which is the combination of Gaussian noise and Band limited white noise (Fig.6) is added with the input of the system and the response is found to be exponentially growing after some time. This could be due to the variations of tuning functions $\hat{\theta}_1, \hat{\theta}_2$ & $\hat{\theta}_3$. So the corresponding variations are plotted in Fig.7 and the variations are found to be approaching higher values, due to this the robust stability gets reduced. The problem with robust stability can be seen from the response

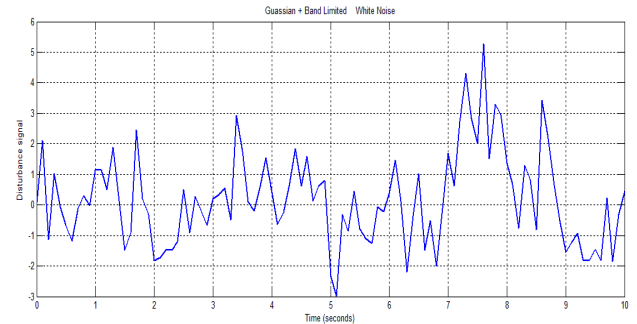


Fig.6: Disturbance signal applied to the system

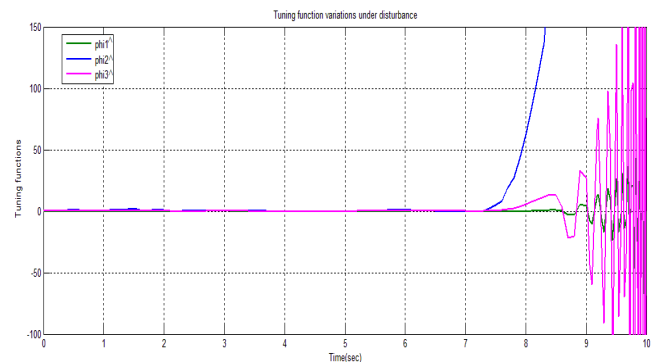


Fig.7: Tuning Function variations

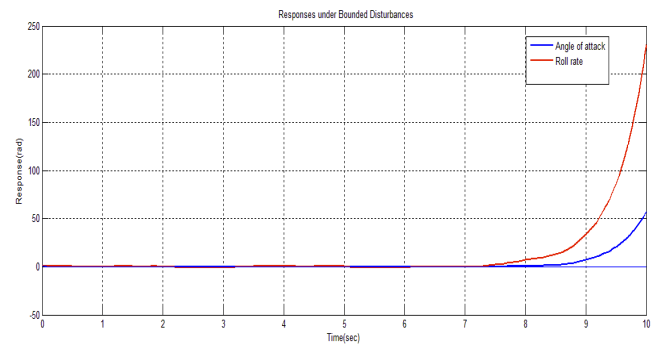


Fig.8: Responses under Disturbance conditions

Similarly tests are conducted for the lateral dynamics of the system. The yaw rate and side slip angle variations with time are plotted in Fig.9 and Fig.10 respectively

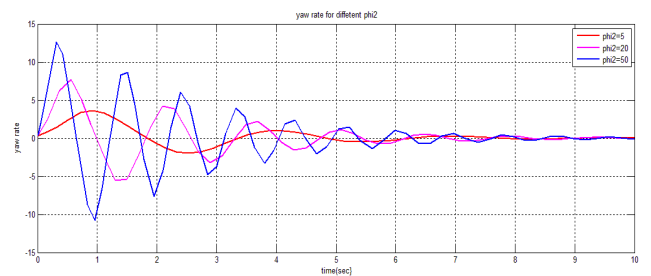


Fig.9: yaw rate regulation for different θ_2

VI. CONCLUSION

In this paper, a nonlinear adaptive backstepping controller is designed for lateral and longitudinal dynamics of RLV based on the adaptive state feedback and parameter dependant Lyapunov function are proposed for parameter uncertainties with unknown input disturbances. We listed out the longitudinal dynamics equation for the RLV, X-38. The error between angle of attack with its desired value is regulated subjected to adaptive feedback controller and final control laws are derived. After giving a disturbance in the input of both dynamics, the longitudinal dynamics are found to be much worse than lateral dynamics. The tuning functions in the longitudinal dynamics are found to be responsible for the instability. The robustness against unknown disturbance has to be achieved. The simulation results show the performance of proposed control system.

VII. FUTURE SCOPE

The Lateral dynamics are much affected to the disturbances. So a controller which also gives robust stability must be added to the system. One of such controller was proposed by Petros Iannou[8]. The different methods suggested can be used to ensure parametric boundness to the tuning functions and thereby keeping the responses within limits.

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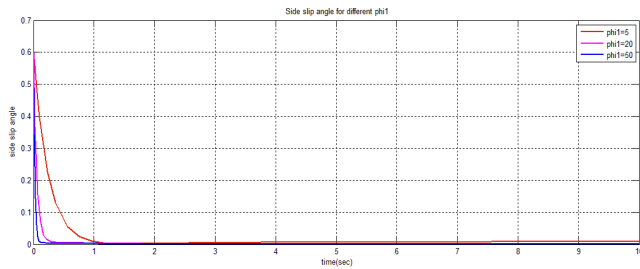


Fig.10: Side slipregulation for different ϕ_1

The variation of the tuning functions when disturbance doesn't acts is given in Fig.11.

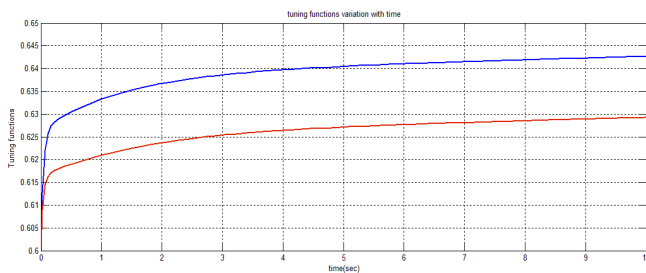


Fig.11: Tuning functions without disturbance

The combination of bound noises Fig(6) are also applied to the system and the variation of tuning functions and corresponding responses are plotted in Fig.12 and Fig.13 respectively

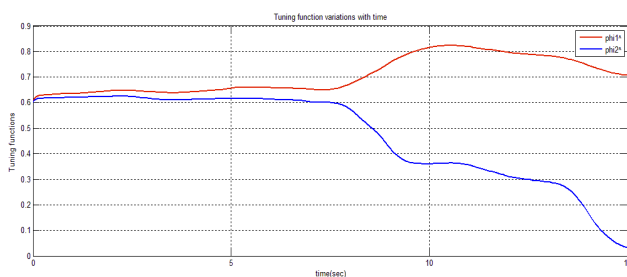


Fig.12: Tuning function variation with disturbance

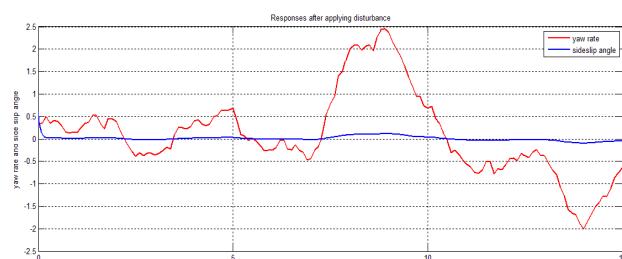


Fig .13: Responses under disturbance