

# Scheduling of A Worldwide Transit Passengers Airlines Networks

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**Abstract**—This paper focuses on hub-and-spoke prototyping for universal transit passenger transportation. This paper examines the schedules that optimally allocate an airline's fleet across universal transit routes to achieve the least Generated Energy Consumption and thus the lowest possible Energy costs, while also taking into account the transit passengers satisfaction. To achieve a reasonable solution, the network is simplified into an operational research transportation problem. The air transit traffic network is solved with the minimal total Generated Energy Consumption, has been solved using Microsoft Excel Solver while verifying the results using Tora optimization software for windows software version 2.0, 2006.

The prototyping of the Hub and Spoke model was tested on a national airliner in Saudi Arabia using discrete event simulation software named Arena Rockwell for a run length of 4 days (24 Hours/day).

**Keywords**—Spoke-hub modelling; Linear models; Airlines Fuel consumption; Passengers scheduling; airline routing.

## I. INTRODUCTION

Stated by IATA that more than 29.5% of the total operational cost of international passenger transportation services is estimated to be attributable to the cost of fuel for airlines; by 2023, this figure is projected to reach nearly \$229.3 billion [1]. In addition, oil continues to represent a substantial proportion of traded commodities due to rising volumes and price, which will reach \$92.3 per Brent barrel in 2023.

Hub-and-Spoke paradigm known for using hub airports to route airlines transit passengers flows connecting interacting airports have become realized with successful applications in the aviation industry.

This paper considers the hub-and-spoke network paradigm for a national airliner in Saudi Arabia. The network has ( $n$ ) hubs, and a number of spoke airports ( $m_n$ ) that are considered to be connected via the case airliner. The minimization of the total energy consumption by airliners to transport transit passengers has a significant effect on the airliners success.

## II. THEORITICAL BACKGROUND

Hub airports may face airliners passenger capacity constraints and it may need to expand the airport capacity. Two network models are usually employed in the airliners industry. Hub and spoke (HS) model and point-to-point (P2P) model. There is competition not only from already established hubs, but also from newly established hubs, for

example, Airport closures in the Middle East may have led to the eviction of other airports due to hub airports bankruptcies [2];[3];[4]. The potential mitigation of airports has led to concerns about the economic loss in areas surrounding airports [5]. Budd and others reported a 77% popularity rate for low-cost airlines, citing route selection and (airline) fleet size as important success reasons [6].

According to a study conducted by Dobrusquez and others, smaller airlines and airliners serving smaller destinations have higher rates of failure. To avoid that, Air Berlin provided drinks and food and ran frequent flyer programs to motivate customers. Ryanair now offers services such as VIP boarding and a ticket flexibility program and is adding major hub airports to its network. It is found that lower-cost airliners are increasing their destinations to major airports, thereby increasing the demand for hub airports [7].

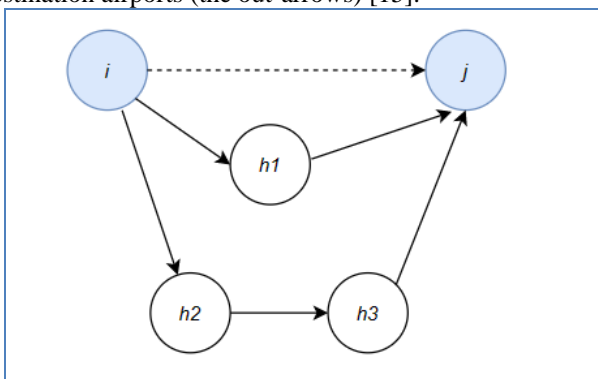
Concluded by O'Kelly and others that the studies regarding Hub and Spoke have in common the aircraft types, flight routes and carbon emissions in the studies analysis. Their study also concludes that a fuel efficient paradigm may require a large number of small regional aircraft [8], whereas a study performed by Baumeister suggests that regional aircraft are, wide body, or turboprop and are significantly less fuel efficient [9].

In a case study of network effects on Southwest Airlines, it was found that Point-to-Point airliner network offers similar advantages to the HS network. Academic debate is still going on about the feasibility of long-distant low-cost strategies that can replace hub carriers in intercontinental markets in the event of hub carrier failure, but the model is already being applied in practice. If a low-cost airline could thrive without offering chauffeur service, the above "spoil effect" wouldn't matter. Newer aircraft may have lower economic and/or environmental costs per seat or passenger, and hybridization reduces fixed costs, reducing the need to use indirect passengers to reduce average costs. While this poses little problem for the local economy, airlines may lose profits from indirect travelers who are more willing to pay [10].

A study notified that the prices of airliners tickets are affected by the fuel prices and fluctuations [11]. Frick has simulated and modeled the hub and spoke networks using a simulation program [12].

Although there are similarities between passenger and freight networks, Chestler and O'Kelly point out that with airliner passenger service, customers are not willing to take long detours throughout their journey, which leads airliners to set up many hubs [13];[14].

Origin-destination (OD) flights from point *i* to point *j* are completed with OD pairs, which can be routed through direct trips (not through hub airports) or through routes that visit hubs (see Figure 1). This figure shows three possible routes from origin point *i* to destination point *j*: path 1 represents a direct flight (the direct node-to-node arrow from origin *i* to destination *j*); path 2 is a one stop flight through hub airport *h1*; and route 3 is a two stop flight through hubs *h2* and *h3*. It can be noted that the traffic (e.g., passengers) transiting via *h1*, or through *h2* and *h3*, has switching alternatives at the hubs where it can be joined by other traffic (e.g., other airline’s passengers) arriving from other airports (the in-arrows) and can depart to other destination airports (the out-arrows) [15].



**Figure 1: Alternative Paths Between Origin *i* and Destination *j***

A study highlighted out that the term “hub” is usually used in different ways: it can be a facility which connect large amounts of traffic (but does not include any originating or terminating traffic) or a facility which holds large number of traffic pass, including all originating and terminating traffic. They also claimed that the Hub and spoke networks are very common in large-scale transportation and telecommunication systems, and that by consolidating/concentrating flows, hub facilities helps to decrease costs, increase service frequency, and hedge against uncertainties [8].

A study reported that, despite rising aviation costs and competition, airlines are seeking partnership partners to strategically cut expenses while raising revenue. By restructuring and centralizing hub networks and operations, a merged airline can improve fuel efficiency by taking advantage of economies of scale [16].

Al-Tahat have built an equation to estimate the fuel consumption per passenger with the regard to the distance and the time flown, the non-linear model built estimates the fuel consumption in a hub and spoke airliner network [17]. The suggested mathematical equation for the computation of the fuel consumption per passenger seat occupied for a travelling passenger between airport (*i*) and airport (*j*) is shown below:

$$C_{ij} = \frac{d_{ij} a c_{ij} f_{ij}}{os_{ij}^2 s_{ij}} \dots \dots \dots \text{Equation (1)}$$

Where;

$os_{ij}$ : Occupied Seats (OS) of an airplane travelling from origin city (*i*) to destination city (*j*).  $\frac{\text{Seats}}{\text{Airplane}}$

$ac_{ij}$ : Airplane Capacity travelling from origin city (*i*) to destination city (*j*).  $\frac{\text{Seats}}{\text{Airplane}}$

$d_{ij}$ : Travelling distance in kilometers from origin city (*i*) to destination city (*j*).

$f_{ij}$ : Rate of Fuel Consumption per hour of the airplane travelling from origin city (*i*) to destination city (*j*).  $\frac{\text{Litre}}{\text{hr.AP}}$

$s_{ij}$ : Speed of the airplane travelling from origin city (*i*) to destination city (*j*).  $\frac{\text{Km}}{\text{hr}}$ .

$k_{ij}$ : Number of Aircrafts intended to be utilized between origin city (*i*) to destination city (*j*).  $k_{ij} = \frac{x_{ij}}{ac_{ij}}$

Al-Tahat have minimized the Hub and Spoke routing costs using the Transshipment model. Where, the objective is to minimize the total fuel consumption, while considering the supply and demand constraints for the Royal Jordanian Airlines [17].

A mathematical model for the environmental impact is built by [18], and the emissions of pollutants produced by aircraft are proposed in equation (2):

$$E = \int \text{Time Flown (hr)} * \text{Number of Engines} * \text{Fuel Flow Rate per Engine per hour} \left(\frac{\text{kg}}{\text{hr}}\right) * \text{Emission for Burning} \left(\frac{\text{Carbon}}{\text{kg}}\right) \dots \dots \dots \text{Equation (2)}$$

A study used aircraft inventory databases to standardize aircraft fuel consumption in kilograms per nautical mile per seat [kg/seat NM] [19].

A study presented a multi-objective optimization model for airline stable flight schedules, considering gradual changes in flight schedules and aircraft maintenance schedules. The model is solved based on simulation. This approach has been tested on real-world data from KLM Royal Dutch Airlines and shows significant improvement against the target under consideration [20].

**III. MODEL FORMULATION AND REAL-LIFE CASE**

An attempt is made to formulate a general model of a global hub-and-spoke aviation network consisting of (*n*) hubs, each of which consists of ( $m_n$ ) spokes. The proposed model should yield a route matrix that yields a sufficient number of aircraft to meet the travel needs of long-haul passengers with the best Energy Consumption among the entire airliner networks. In this model, a passenger is proposed to travel from any airport (spoke or hub) to his final destination airport via one or more airports (spokes or hubs).

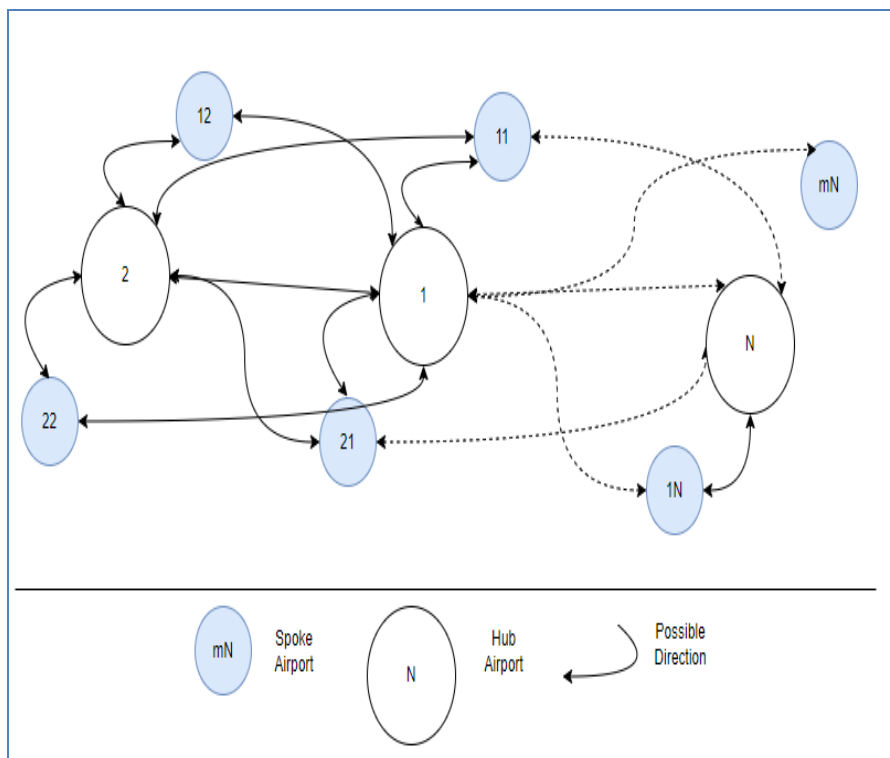


Figure 2: (n) Hub (mn) Spoke Network.

The following notations are used when formulating the required model:

Given:

- N: Total Number of Hubs in the Airliner Network
- n: Total nth Hub in the Airliner Network,  $n = \{1, 2, 3, \dots, N\}$ .
- $M_n$ : The Number of Spokes of the nth hub in the Network.
- $m_n$ : The mth Spoke of the nth hub in the Network,  $m_n = \{1_n, 2_n, \dots, M_n\}$ , where  $n = \{1, 2, 3, \dots, N\}$
- i: Origin city airport index (Supply node),  $i = \{1, 2, 3, \dots, L\}$ ,  $L = N + \sum_{n=1}^N m_n$
- j: Destination city airport index (Demand node),  $j = \{1, 2, 3, \dots, L\}$ ,  $L = N + \sum_{n=1}^N m_n$
- $c_{ij}$ : Energy required to transport passenger from origin city airport (i) to destination city airport (j) and represents the energy consumed to transport each passenger.  $\frac{Kj}{Passenger}$
- $x_{ij}$ : Number of Passengers flying from city airport (i) to city airport (j)
- $a_i$ : Potential Number of weekly Passengers flying from city airport (i),  $a_i = \sum_{j=1}^L x_{ij}$ , where  $j = \{1, 2, \dots, L\}$ .
- $b_j$ : Potential Number of weekly Passengers flying to city airport (j),  $b_j = \sum_{i=1}^L x_{ij}$ , where  $i = \{1, 2, \dots, L\}$ .
- $os_{ij}$ : Occupied Seats (OS) of an airplane travelling from origin city (i) to destination city (j).  $\frac{Seats}{Airplane}$

$ac_{ij}$ : Airplane Capacity travelling from origin city (i) to destination city (j).  $\frac{Seats}{Airplane}$

$LF_{ij}$ : The Ratio of the Occupied Seats to Airplane Capacity travelling from origin city (i) to destination city (j).  $\frac{os_{ij}}{ac_{ij}}$

$d_{ij}$ : Travelling distance in kilometers from origin city (i) to destination city (j).

$f_{ij}$ : Rate of Fuel Consumption per hour of the airplane travelling from origin city (i) to destination city (j).  $\frac{Litre}{hr.AP}$

E: Energy generated by burning one Litre of airplane fuel.  $\frac{Kj}{Litre}$

$s_{ij}$ : Speed of the airplane travelling from origin city (i) to destination city (j).  $\frac{Km}{hr.}$

$k_{ij}$ : Number of Aircrafts intended to be utilized between origin city (i) to destination city (j).  $k_{ij} = \frac{x_{ij}}{ac_{ij}}$

$FH_{ij}$ : Flying Hours to travel between origin city (i) to destination city (j).  $FH_{ij} = \frac{d_{ij}}{s_{ij}}$

Based on the equation derived by Al-Tahat et. al. (2019), the cost in liter for transporting a single passenger from origin (i) to destination (j) is based on:

$$c'_{ij} = \frac{(d_{ij}) \cdot (ac_{ij}) \cdot (f_{ij})}{(os_{ij}^2)(s_{ij})} \dots \dots (3)$$

When the distance over speed yields the time needed in hours to reach to the destination:

$$c'_{ij} = \frac{(FH_{ij}) \cdot (f_{ij})}{(LF_{ij}) \cdot (os_{ij})} \dots\dots (4)$$

The equation yields the cost of fuel in litres per passenger. This thesis aims to get the cost in Mega joules. While, this is multiplied by the energy generated by burning each liters.

A suggested mathematical equation for the required Energy for a travelling passenger between airport (i) and airport (j) is shown below:

$$c_{ij} = \frac{(FH_{ij}) \cdot (f_{ij}) \cdot (E)}{(LF_{ij}) \cdot (os_{ij})} \dots\dots (5)$$

Accordingly, the objective function (z) and the model constraints are represented as follow:

$$\text{Min } Z = \sum_{i=1}^L \sum_{j=1}^L c_{ij} x_{ij} \dots\dots (6)$$

Subject to:

$$L = N + \sum_{n=1}^N m_n \dots\dots (7)$$

$$a_i = \sum_{j=1}^L x_{ij} \dots\dots (8)$$

$$b_j = \sum_{i=1}^L x_{ij} \dots\dots (9)$$

$$\sum_{i=1}^L a_i = \sum_{j=1}^L b_j \dots\dots (10)$$

Where;

$$x_{ij} \geq 0, c_{ij} = \infty \text{ when } i=j, j = \{1,2,\dots,L\}, i = \{1,2,\dots,L\}.$$

The objective function presented by equation (6), with the constraints presented by equations (7), (8), (8), (9) and (10) above constitute an operations research model that estimates the Energy consumption. Solution of the above model is obtained using Microsoft Excel Solver Add-ons and the results are Verified using Tora optimization software for windows software version 2.0, 2006.

**Case Study 1:** Real Life Case: On July 2023, the busiest International flights flight routes in the world as shown below. The data was generated by OAG Aviation Data Analysis Platform. This thesis considers replacing Dubai airport by Jeddah and Riyadh Airports. Thus, Dubai is considered a pure transit airport. See table 1 below.

**Table 1: Market Potential of Seven Routes [21].**

#	Traffic	Route	Route Name	Seats
1	5	CAI-DXB	Cairo – Dubai	146,684
2	22	DXB-LHR	Dubai - London Heathrow	244,276
4	46	BOM-DXB	Mumbai – Dubai	238,792
5	47	DXB-KWI	Dubai – Kuwait	235,072
6	48	DEL-DXB	Delhi – Dubai	208,062
7	49	BAH-DXB	Bahrain – Dubai	185,596
8	50	DXB-IST	Dubai – Istanbul	176,160

Accordingly; the solution steps are shown below:

1. Demand-Vector: Demand at each airport is observed as shown in table (2), this is expressed in passengers and is gathered from OAG Database for the world’s busiest routes that is connected via Dubai. The Demand from an airport doesn’t necessarily represent the summation of the number of passengers who arrives to that city; thus it’s possible for the airliner to have an excess of the supply from another origin city resulted from a shortage of demand on the corresponding destination city.

**Table 2: The Demand from Each City Expressed in Numbers of Passengers**

Node	Cairo	London Heathrow	Mumbai	Kuwait City	New Delhi	Manama	Istanbul
j	1	2	3	4	5	6	7
a <sub>j</sub>	146,684	244,276	238,792	235,072	208,062	185,596	176,160

2. Supply- Vector: Supply at each airport is observed and presented in table (3), this is expressed in passengers. The supply from an airport doesn’t necessarily represent the summation of the number of passengers who departs from that city; thus, it’s possible to have an excess of demand from another destination city while there are a shortage of supply from the airliner who can’t afford for the seats needed.

**Table 3: The Supply from Each City Expressed in Numbers of Passengers**

Node	Cairo	London Heathrow	Mumbai	Kuwait City	New Delhi	Manama	Istanbul
i	1	2	3	4	5	6	7
b <sub>i</sub>	146,684	244,276	238,792	235,072	208,062	185,596	176,160

3. Network Balance Check: In this step we need to check whether the total supply equals to the total demand, if it happened that they aren’t equal we should add a dummy source/destination, while in this case:

$$\text{Total supply} = \sum_{i=1}^L b_i = 146,684+244,276+238,792+235,072+208,062+185,596+176,160=1,678,918$$

$$\text{Total Demand} = \sum_{j=1}^L a_j = 146,684 + 244,276 + 238,792 + 235,072 + 208,062 + 185,596 + 176,160 = 1,678,918$$

4. Buffer computation: In this step we need to allocate a sufficient space at each transit (hub) airport for passengers coming from another airport cities and vice versa. These buffers can be thought of as low-cost trip to a stopping destination, this destination is thought to have low-cost trips to the world. Depending on the efficiency and attractiveness of the airliner’s hubs, the

passengers from spoke airports will be forced to go through these hub airports, which will yield to force the airliner to provide additional capacity at the hub levels.

$$\text{Buffer (B)} = \text{Total supply} = \text{Total Demand} =$$

$$\sum_{j=1}^L a_j = \sum_{i=1}^L b_i = 1,678,918$$

5. Adjusted Demand Vector: To adjust the demand vector, buffer amount (B = 1,678,918) is added to the original demand of each transshipment demand airport. This step extends the number of Airports to include Hub airports (L+2), thus, airports are classified into pure demand airport and transshipment demand airport as follows:

$$\left\{ \begin{array}{ll} \text{Pure demand airport} & \{1,2,3,4,5,6,7\} \\ \text{Transshipment demand airport} & \{\text{KAIA (8), OERK (9)}\} \end{array} \right\}$$

Accordingly, results are shown in table (4), the computations has been carried out using equation (8). This step will enable to pass the demand of other airports through the hubs airports, which will handle the capacities of those transit passengers.

**Table 4: The Adjusted Demand Vector**

Node	Cairo	London Heathrow	Mumbai	Kuwait City	New Delhi	Manama	Istanbul	Jeddah	Riyadh
j	1	2	3	4	5	6	7	8	9
aa <sub>j</sub>	146,684	244,276	238,792	235,072	208,062	185,596	176,160	1,678,918	1,678,918

6. BB’ Vector: To adjust the supply vector, buffer amount (B = 1,678,918) is added to the original supply of each transshipment supply airport. This step extends the number of Airports to include Hub airports (L+2), thus, Airports are classified into pure supply airport and transshipment supply airport as follows:

$$\left\{ \begin{array}{ll} \text{Pure demand airport} & \{1,2,3,4,5,6,7\} \\ \text{Transshipment demand airport} & \{\text{KAIA (8), OERK (9)}\} \end{array} \right\}$$

Accordingly, results are shown in table (5), the computations has been carried out using equation (9). This step will enable to pass the supply of other airports through the hubs airports, which will handle the capacities of those transit passengers.

**Table 5: The Adjusted Supply Vector**

Node	Cairo	London Heathrow	Mumbai	Kuwait City	New Delhi	Manama	Istanbul	Jeddah	Riyadh
I	1	2	3	4	5	6	7	8	9
bb <sub>j</sub>	146,684	244,276	238,792	235,072	208,062	185,596	176,160	1,678,918	1,678,918

7. FH<sub>ij</sub>- Matrix: flight duration (FH<sub>ij</sub>) between each pair of origin-destination airports are observed, the duration are given in hours. When i = j, FH<sub>ij</sub> does not exist so a very large objective coefficient (∞) is assigned when i = j. Flight duration and Aircraft type information is downloaded for each flight involved in the considered paradigm in one minute increments from the publicly available website:

www.flightaware.com. Flights were selected randomly with respect to day of the week, time of day, airline. However, an effort was made to select flights representing a range of aircraft types. Accordingly FH<sub>ij</sub> for i = 1, 2, 3, 4, 5, 6, 7, 8, 9 and for j = 1, 2, 3, 4, 5, 6, 7, 8, 9 are listed in table (6).

**Table 6: Flight Durations (FH<sub>ij</sub>) in hours Between Cities Airports, Based on data from Flightaware website.**

		Destination airport								
		j								
i		1	2	3	4	5	6	7	8	9
Origin airport	1	-	-	-	-	-	-	-	2:20	2:55
	2	-	-	-	-	-	-	-	6:30	7:05
	3	-	-	-	-	-	-	-	5:15	4:20
	4	-	-	-	-	-	-	-	2:10	1:16
	5	-	-	-	-	-	-	-	5:03	4:59
	6	-	-	-	-	-	-	-	2:10	1:06
	7	-	-	-	-	-	-	-	3:50	4:20
	8	2:20	6:30	5:15	2:10	5:03	2:10	3:50	-	1:50
	9	2:55	7:05	4:20	1:16	4:59	1:06	4:20	1:50	-

8.  $f_{ij}$ - Matrix: Rate of fuel consumption ( $f_{ij}$ ) per hour for the aircraft travelling between airports is observed, thus it is expressed in consumed Liters per hour for the operating of the aircraft on a specific destination. See table 7 below.

**Table 7: Rate of Fuel Consumption per hour ( $f_{ij}$ ) Travelling between Airports.**

		Destination airport								
		j								
i		1	2	3	4	5	6	7	8	9
Origin airport	1	-	-	-	-	-	-	-	5,400	6,000
	2	-	-	-	-	-	-	-	6,000	6,800
	3	-	-	-	-	-	-	-	6,000	6,000
	4	-	-	-	-	-	-	-	6,000	2,500
	5	-	-	-	-	-	-	-	6,000	6,800
	6	-	-	-	-	-	-	-	2,500	2,500
	7	-	-	-	-	-	-	-	6,000	6,000
	8	5,400	6,000	6,000	6,000	6,000	2,500	6,000	-	6,000
	9	6,000	6,800	6,000	2,500	6,800	2,500	6,000	6,000	-

9.  $LF_{ij}$ - Matrix: Load Factor ( $LF_{ij}$ ) for aircraft travelling between airports is observed, expressed in seats occupied per destination. The data shown in table (8) are based on hypothesized historical results. Thus accurate data is confidential for each airliner. When  $i = j$ ,  $LF_{ij}$  does not exist.

**Table 8: Load Factor ( $LF_{ij}$ ) of a Airplane Traveling from i to j.**

i	j	Destination airport								
		1	2	3	4	5	6	7	8	9
Origin airport	1	-	-	-	-	-	-	-	51%	88%
	2	-	-	-	-	-	-	-	92%	58%
	3	-	-	-	-	-	-	-	83%	76%
	4	-	-	-	-	-	-	-	54%	40%
	5	-	-	-	-	-	-	-	96%	90%
	6	-	-	-	-	-	-	-	29%	37%
	7	-	-	-	-	-	-	-	85%	83%
	8	88%	65%	55%	82%	56%	40%	76%	-	68%
	9	57%	95%	34%	76%	76%	36%	68%	66%	-

10.  $os_{ij}$ - Matrix: Number of passengers ( $os_{ij}$ ) travelling between airports is observed, these are expressed in seats per an allocated airplane. Note that the Airplane capacity differs for the same airplane type, thus it is based on the airliner preference for the utilizing of the seating space. When  $i = j$ ,  $os_{ij}$  does not exist.

**Table 9: Number of Passengers ( $os_{ij}$ ) for an Airplane Travelling between Airports**

i	j	Destination airport								
		1	2	3	4	5	6	7	8	9
Origin airport	1	-	-	-	-	-	-	-	261	262
	2	-	-	-	-	-	-	-	193	238
	3	-	-	-	-	-	-	-	248	226
	4	-	-	-	-	-	-	-	161	58
	5	-	-	-	-	-	-	-	285	370
	6	-	-	-	-	-	-	-	58	53
	7	-	-	-	-	-	-	-	252	248
	8	153	274	164	244	168	41	228	-	204
	9	168	413	100	109	147	55	204	197	-

11. – C-Matrix: Using equation (5), the Energy generated per passenger for a specific route ( $c_{ij}$ ) is calculated and recorded as shown in table (10).  $E$  is the energy multiplier in Mega Joule (35.3) MJ for Each 1 litre of fuel (A-1 Source: Air BP Handbook). The direct routes between non-hub airports shall be restricted due to the flight regulations and is assigned a high very high cost ( $M=1,000,000$ ). An example for the passengers going from Cairo ( $i = 1$ ) to Jeddah ( $j = 8$ ), the fuel consumption per seat occupied is calculated as follow:

$$C_{18} = \frac{(FH_{18}) \cdot (f_{18}) \cdot (E)}{(LF_{18}) \cdot (os_{18})} = \frac{2.33 \times 6,000 \times 35.3}{(88\%) \times 261} = 2154.9 \text{ MJ}$$

**Table 10: The Calculated Energy Generated for a Passenger (cij) Derived from Equation (5)**

i	j	Destination airport								
		1	2	3	4	5	6	7	8	9
Origin airport	1	M	M	M	M	M	M	M	2154.9	2674.94
	2	M	M	M	M	M	M	M	9915.75	12356.95
	3	M	M	M	M	M	M	M	5397.65	5362.23
	4	M	M	M	M	M	M	M	5267.19	4741.93
	5	M	M	M	M	M	M	M	2292.73	3613.53
	6	M	M	M	M	M	M	M	16141.05	4904.69
	7	M	M	M	M	M	M	M	4670.27	4429.57
	8	6304.44	4919.51	12314.4	2292.73	11326.59	8245.38	3803.41	0	2986.32
	9	6488.12	4520.63	27146.29	1352.02	22971.03	4701.41	6593.1	2777.93	0

There are seven spokes that is attempted to be connected through the two hub airports. See figure 3 below.



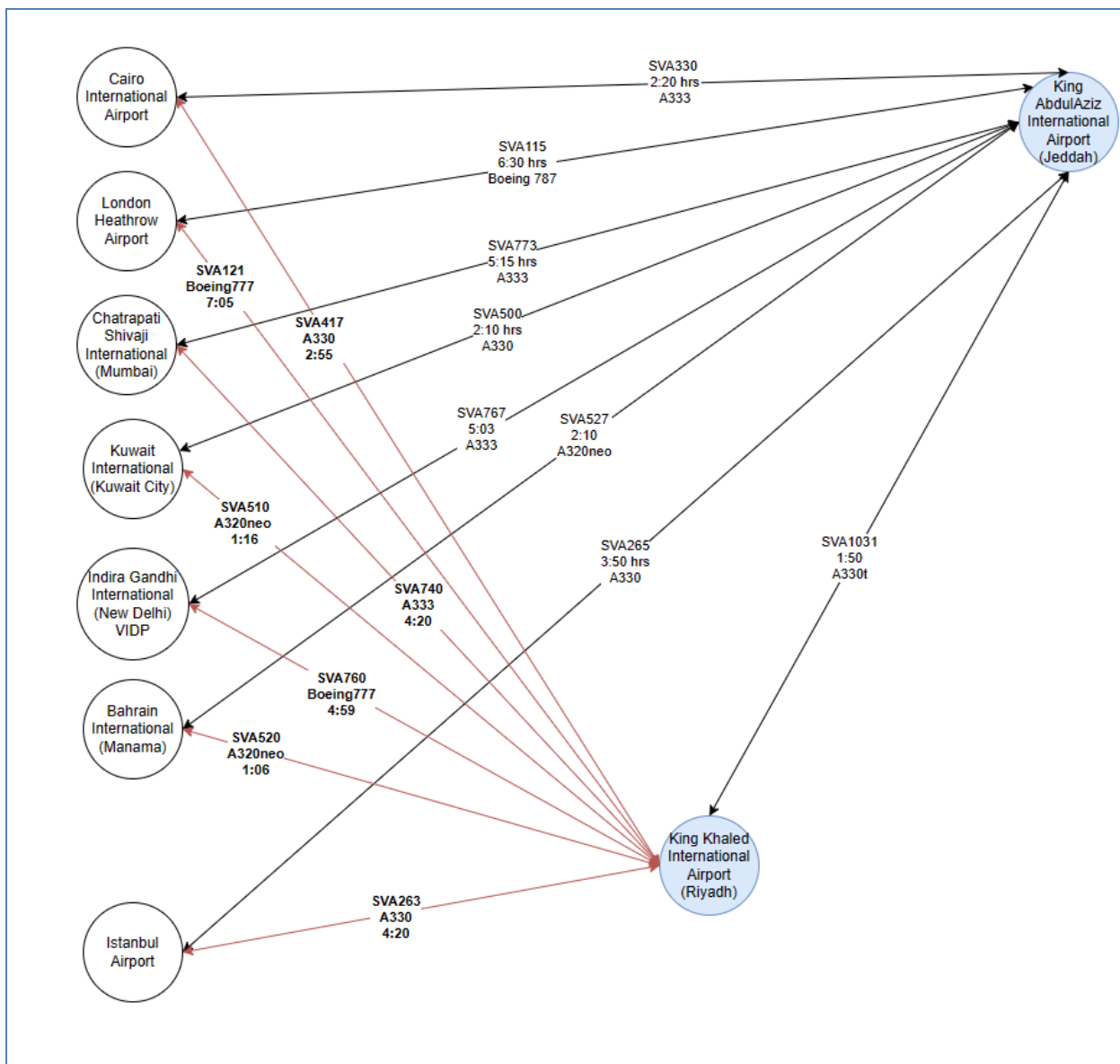


Figure 3: Network modeling of real-life hub and spoke network

The numerical objective function (z) and the model constraints are represented as follow:

$$\begin{aligned}
 \text{Min } Z = & 2154.9x_{18} + 9915.75x_{28} + 5397.65x_{38} \\
 & + 5267.19x_{48} + 2292.73x_{58} + 16141.05x_{68} + \\
 & 4670.27x_{78} + 2777.93x_{98} + 6304.44x_{81} \\
 & + 4919.51x_{82} + 12314.4x_{83} + 2292.73x_{84} \\
 & + 11326.59x_{85} + 8245.38x_{86} + 3803.41x_{87} \\
 & + 2986.32x_{89} + 2674.94x_{19} + 12356.95x_{29} \\
 & + 5362.23x_{39} + 4741.93x_{49} + 3613.53x_{59} \\
 & + 4904.69x_{69} + 4429.57x_{79} + 6488.12x_{91} \\
 & + 4520.63x_{92} + 27146.29x_{93} + 1352.02x_{94} \\
 & + 22971.03x_{95} + 4701.41x_{96} + 6593.1x_{97}
 \end{aligned}
 \tag{9}$$

$$\begin{aligned}
 &X_{12} + X_{13} + X_{14} + X_{15} + X_{16} \\
 &+ X_{17} + X_{18} + X_{19} = 146,684 \\
 &X_{21} + X_{23} + X_{24} + X_{25} + X_{26} \\
 &+ X_{27} + X_{28} + X_{29} = 244,276 \\
 &X_{31} + X_{32} + X_{34} + X_{35} + X_{36} \\
 &+ X_{37} + X_{38} + X_{39} = 238,792 \\
 &X_{41} + X_{42} + X_{43} + X_{45} + X_{46} \\
 &+ X_{47} + X_{48} + X_{49} = 235,072 \\
 &X_{51} + X_{52} + X_{53} + X_{54} + X_{56} \\
 &+ X_{57} + X_{58} + X_{59} = 208,062 \\
 &X_{61} + X_{62} + X_{63} + X_{64} + X_{65} \\
 &+ X_{67} + X_{68} + X_{69} = 185,596 \\
 &X_{71} + X_{72} + X_{73} + X_{74} + X_{75} \\
 &+ X_{76} + X_{78} + X_{79} = 176,160 \\
 &X_{81} + X_{82} + X_{83} + X_{84} + X_{85} \\
 &+ X_{86} + X_{87} + X_{89} = 1,678,918 \\
 &X_{91} + X_{92} + X_{93} + X_{94} + X_{95} \\
 &+ X_{96} + X_{97} + X_{98} = 1,678,918
 \end{aligned}$$

(10)

$$\begin{aligned}
 &X_{21} + X_{31} + X_{41} + X_{51} + X_{61} \\
 &+ X_{71} + X_{81} + X_{91} = 146,684 \\
 &X_{12} + X_{32} + X_{42} + X_{52} + X_{62} \\
 &+ X_{72} + X_{82} + X_{92} = 244,276 \\
 &X_{13} + X_{23} + X_{43} + X_{53} + X_{63} \\
 &+ X_{73} + X_{83} + X_{93} = 238,792 \\
 &X_{14} + X_{24} + X_{34} + X_{54} + X_{64} \\
 &+ X_{74} + X_{84} + X_{94} = 235,072 \\
 &X_{15} + X_{25} + X_{35} + X_{45} + X_{65} \\
 &+ X_{75} + X_{85} + X_{95} = 208,062 \\
 &X_{16} + X_{26} + X_{36} + X_{46} + X_{56} \\
 &+ X_{76} + X_{86} + X_{96} = 185,596 \\
 &X_{17} + X_{27} + X_{37} + X_{47} + X_{57} \\
 &+ X_{67} + X_{87} + X_{97} = 176,160 \\
 &X_{18} + X_{28} + X_{38} + X_{48} + X_{58} \\
 &+ X_{68} + X_{78} + X_{98} = 1,678,918 \\
 &X_{19} + X_{29} + X_{39} + X_{49} + X_{59} \\
 &+ X_{69} + X_{79} + X_{89} = 1,678,918
 \end{aligned}$$

(11)

$$x_{ij} \geq 0 \quad (12)$$

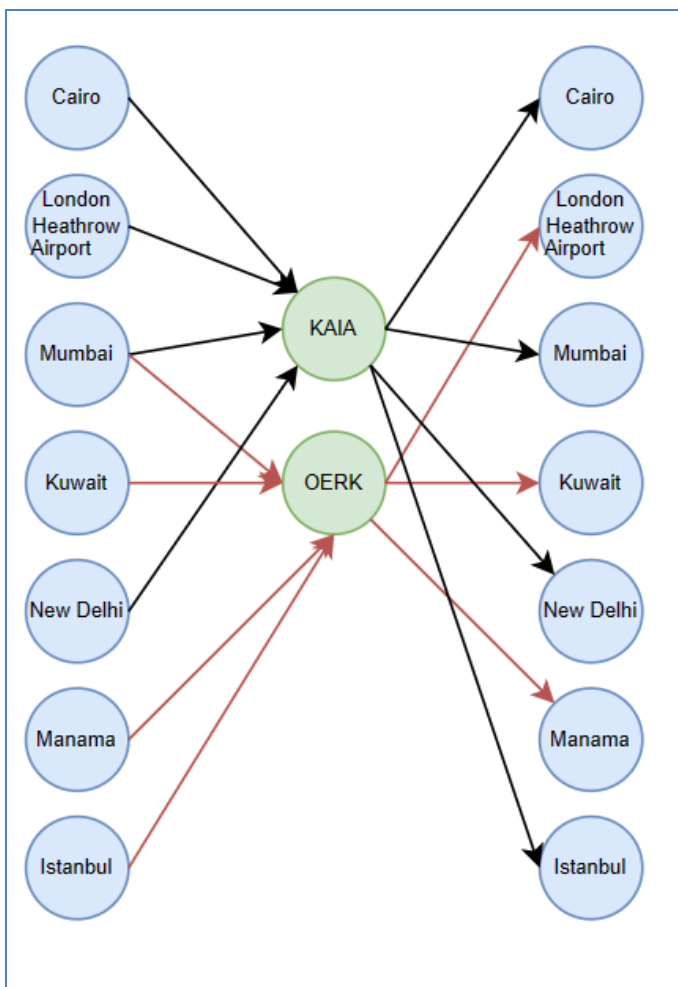
Consequently, the obtained optimal solution is presented in table 11 below, solution has been generated through three starting feasible solution methods, namely North-west corner method, least cost method and Vogel approximation method.

Table 11: Optimal solution based on least cost method.

Objective Value (minimum cost) =16493749628.96 MJ	
TRANSPORTATION MODEL OUTPUT SUMMARY	
Title:	Airliner
Final Iteration	No.:23
Objective Value (minimum cost) =16493749628.96	

From	To	Amt Shipped	Obj Coeff
S1: x1	D8:x8	146684	2154.90
S2: x2	D8:x8	244276	9915.75
S3: x3	D8:x8	170676	5397.65
S3: x3	D9:x9	68116	5362.23
S4: x4	D9:x9	235072	4741.93
S5: x5	D8:x8	208062	2292.73
S6: x6	D9:x9	185596	4904.69
S7: x7	D9:x9	176160	4429.57
S8: x8	D1:x1	146684	6304.44
S8: x8	D3:x3	238792	12314.40
S8: x8	D5:x5	208062	11326.59
S8: x8	D7:x7	176160	3803.41
S8: x8	D8:x8	909220	0.00
S9: x9	D2:x2	244276	4520.63
S9: x9	D4:x4	235072	1352.02
S9: x9	D6:x6	185596	4701.41
S9: x9	D9:x9	1013974	0.00

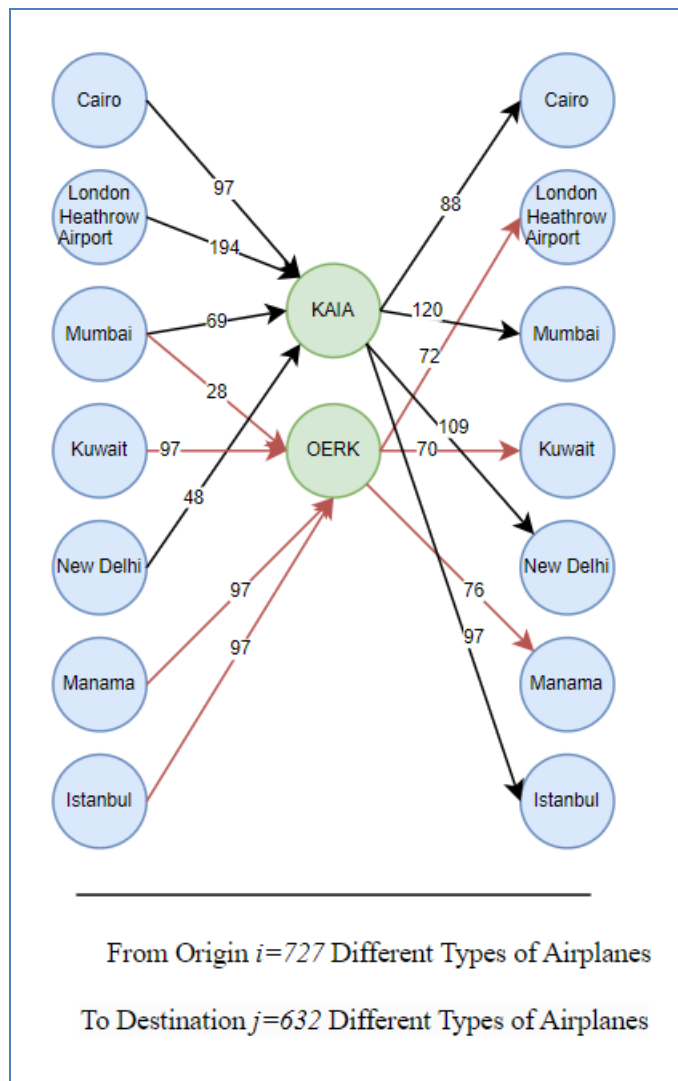
The optimal solution showed in chapter three that the Transit Passengers coming from Cairo, London Heathrow, New Delhi and 71.5% of Mumbai passengers have to stop in Jeddah (King Abd-Aziz International Airport). The transit passengers originated from Kuwait, Manama, Istanbul and 28.5% of Mumbai Passengers are routed via Riyadh (King Khaled International Airport). Jeddah Airport is the stop to passengers intending to reach Cairo, Mumbai, New Delhi and Istanbul as their final destination. However, King Khaled International Airport in Riyadh will be the stop for passengers intending London Heathrow, Kuwait and Manama. See figure 4 below.



**Figure 4: Case Airliner Service Optimal Routes**

The output results shown in table 11 using TORA Windows © version 2.00, 2006. Illustrates the number of passengers who must be transferred between each route, the Energy cost coefficients and the contribution of the route.

The simulation results shows that the number of airplanes intended to be utilized in a time interval of 4 days, 24 hours per day. 97,194,97,97,48,97,97 airplanes are connected from Cairo, London Heathrow, Mumbai, Kuwait, New Delhi, Manama, and Istanbul via their intended hubs, respectively. While 88,120,109,97 airplanes are connected via KAIA to Cairo, Mumbai, New Delhi and Istanbul, respectively. 72,70,76 airplanes are connected via Riyadh to London Heathrow, Kuwait and Manama, respectively. See figure 3 below. The Differences are due changes in the Types of Airplanes and the uncompleted trips (Buffer). See figure 5.



From Origin  $i=727$  Different Types of Airplanes  
To Destination  $j=632$  Different Types of Airplanes

**Figure 5: Optimal Number of airplanes through the transit network**

Table 12 below shows the queue length of the optimal routes based on the simulation results. In the hub airports London Heathrow Passengers has the highest queue length with an average length of 40.636 passengers, Mumbai passengers (17.13), New Delhi passengers (15.71), Istanbul passengers (14.82), Cairo passengers (13.1), Kuwait passengers (9.15), and Manama passengers (8.49). This shows that the Airliner needs to purchase/lease new airplanes in order to raise the capacity and lower the number of transit passengers waiting in the queue.

**Table 12: WIP (Queue Length) for the Airlines optimal solution Using Arena Rockwell simulation**

WIP (Queue Length)	Average	Maximum
Cairo Passengers	13.1048	16
Istanbul Passengers	14.8229	21

<b>Kuwait Passengers</b>	9.1579	15
<b>LHR Passengers</b>	40.636	48
<b>Manama Passengers</b>	8.4977	15
<b>Mumbai Passengers</b>	17.1307	24
<b>New Delhi Passengers</b>	15.7111	21

#### IV. RESULTS CONCLUSIONS, AND RECOMMENDATIONS

The modeling process of applying the transit model to the Hub and Spoke model was tested on a specific Saudi Arabian airliner. The modeled transit network has been simplified into a transportation network in which airports are interconnected by direct point-to-point connections. The model's objective function is to minimize total fuel burned (MJ) for transporting model passengers through a simplified route network, the solution of which determines the number of passengers transported per route  $x_{ij}$  as a decision variables, the binding parameters are; fuel burned to displace per passenger per route  $c_{ij}$ ; This is calculated from a proposed mathematical model that takes into account several factors: Flight Duration in hours, fuel consumption for the assigned aircraft per hour, aircraft capacity, the occupied seats and the Energy Burned per a liter of jet fuel (MJ); total supply and total demand are balanced. Then the problem is solved using the Microsoft Excel Solver and Tora optimization application. The result of the solution for the number of passengers carried on each route is simplified into proportional numbers using the allocated airplane capacities for the same route, these proportional numbers give the planned distribution of the aircraft fleet in the route matrix, while simulating the final results using Arena Rockwell Simulation Software for Windows.

Based on the conclusions and limitations, it is recommended by the end of this paper to Consider cost of Available seat per kilometer-CASK in future optimization equations, Consider the Design of experiment of the fuel type used by the case saudi airline in the Energy consumption with factors such as the airplane payload.

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**Conflicts of Interest:** The authors declare no conflict of interest.

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