Second Minimum Weight Spanning Tree of A Network with Triangular Intuitionistic Fuzzy Number As Edge Weight

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***Abstract*— Determination of minimum weight spanning tree of a network is very significant in the field of operations research. The next option for minimum weight spanning tree is second minimum weight spanning tree. In this paper a new algorithm to find the second minimum weight spanning tree of a network has been suggested where edge weights are considered as triangular intuitionistic fuzzy number.**

***Keywords*— Network, Spanning tree, Minimum Weight Spanning Tree, Triangular intuitionistic fuzzy number.**

I. INTRODUCTION

Computing minimum weight spanning tree of a network is one of the most fundamental algorithmic problems in graph theory. In 1957, Prim [10] proposed the method for determining the minimal spanning tree of a network. The determination of the optimal path tree was efficiently determined by Bellman [2], Ford [5] and Moore [9]. Hassan [6] presents a new algorithm based on the distance matrix to solve the least-cost minimum spanning tree problem in 2012. In real world applications, it is not always possible to use the Minimum weight spanning tree. In such situations, second MWST is of equal importance to MWST. In crisp environment, it is assumed that the decision maker is certain about the parameters like distance, cost, time etc. But in real situations there always exists uncertainty about the parameters. In such cases, parameters can be represented by triangular intuitionistic fuzzy numbers. Second Minimum weight spanning tree of a network with crisp values as edge weight was discussed by Ismail Mohideen and Rajesh [7]. Applications of Minimum Weight Spanning Tree (MWST) were found in Foulds [4]. The concept of intuitionistic fuzzy sets was proposed by Atanassov [1] in 1986. In 2004, Mitchell [8], proposed a method to solve intuitionistic fuzzy numbers. In 2010, Deng Feng Li et al. [3] proposed a ranking method for triangular intuitionistic fuzzy number.

II. PRELIMINARIES

2.1 Fuzzy numbers

A fuzzy subset of the real line R with membership function is called a fuzzy number if

i. $\tilde{A}$ is normal, (i.e.) there exists an element x_0 such that $\mu_{\tilde{A}}(x_0) = 1$
ii. $\tilde{A}$ is fuzzy convex, (i.e.) $\mu_{\tilde{A}}[\lambda x_1 + (1-\lambda)x_2] \geq \mu_{\tilde{A}}(x_1) \wedge \mu_{\tilde{A}}(x_2), x_1, x_2 \epsilon\, R, \lambda \epsilon\, [0,1]$
iii. $\mu_{\tilde{A}}$ is upper continuous and
iv. Supp$\tilde{A}$ is bounded, where Supp $\tilde{A} = \{x\epsilon\, R: \mu_{\tilde{A}}(x) > 0\}$.

2.2 *Fuzzy tree*

Fuzzy tree is a tree in which the weight of the edges constituting the tree is considered as fuzzy number.

2.3 *Weight of a fuzzy tree*

Weight of a fuzzy tree is a fuzzy number with each of its component representing the sum of the corresponding components of the fuzzy number of the edges constituting the tree.

2.4 *Intuitionistic fuzzy number*

Let $\tilde{A} = \{(x, \mu_{\tilde{A}}(x), \gamma_{\tilde{A}}(x)), x\epsilon X\}$ be an intuitionistic fuzzy set, then the pair $(\mu_{\tilde{A}}(x), \gamma_{\tilde{A}}(x)$ is refered here as an intuitionistic fuzzy number.

2.3. Triangular intuitionistic fuzzy number

A triangular intuitionistic fuzzy number $\tilde{A}$ in R, written as $(a_1, b_1, c_1; a_1', b_1, c_1')$ where $a_1' \leq a_1 \leq b_1 \leq c_1 \leq c_1'$ has the membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{b_1-a_1} & a_1 \leq x \leq b_1 \\ \dfrac{x-c_1}{b_1-c_1} & b_1 \leq x \leq c_1 \\ 0 & \text{othererwise} \end{cases}$$

and non-membership function of $\tilde{A}$ is given by

$$\gamma_{\tilde{A}}(x) = \begin{cases} \dfrac{b_1 - x}{b_1 - a_1'} & a_1' \le x \le b_1 \\ \dfrac{x - b_1}{c_1' - b_1} & b_1 \le x \le c_1' \\ 1 & \text{othererwise.} \end{cases}$$

III. ALGORITHM

Following is the algorithm to find second minimum weight spanning tree of a network, where triangular intuitionistic fuzzy number is considered as weight of the edges.

Step 1: Initialization
Let k = 0. Let E_k be the set of all edges and n be the number of nodes of a network N_k where $n \geq 3$.
Let E_{ij} be the weight of the edge (i,j). Let T_k be the MWST of a network N_k.
For all $(i,j) \in E$, where $i \neq j$,
let $E_{ij} = (d_{ij}{}^1,\ d_{ij}{}^2,\ d_{ij}{}^3;\ d'_{ij}{}^1,\ d_{ij}{}^2,\ d'_{ij}{}^3)$

Step 2: Value of the membership function of the triangular intuitionistic fuzzy number
If $E_{ij} = (d_{ij}{}^1,\ d_{ij}{}^2,\ d_{ij}{}^3;\ d'_{ij}{}^1,\ d_{ij}{}^2,\ d'_{ij}{}^3)$ and $E_{pq} = (d_{pq}{}^1,\ d_{pq}{}^2,\ d_{pq}{}^3;\ d'_{pq}{}^1,\ d_{pq}{}^2,\ d'_{pq}{}^3)$ be two edge weight of fuzzy network with triangular intuitionistic fuzzy number, then the value of the membership function of the triangular intuitionistic fuzzy number can be calculated as follows
If $v_\mu(E_{ij}) > v_\mu(E_{pq})$ then $E_{ij} > E_{pq}$
If $v_\mu(E_{ij}) < v_\mu(E_{pq})$ then $E_{ij} < E_{pq}$
If $v_\mu(E_{ij}) = v_\mu(E_{pq})$ then $E_{ij} = E_{pq}$
Where $v_\mu(E_{ij}) = v_\mu(d_{ij}{}^1,\ d_{ij}{}^2,\ d_{ij}{}^3;\ d'_{ij}{}^1,\ d_{ij}{}^2,\ d'_{ij}{}^3)$
$= 1/6\,(d_{ij}{}^1 + 4\, d_{ij}{}^2 + d_{ij}{}^3)$

Step 3:
Using step 2, select an edge (i,j) from E_k such that $v_\mu(E_{ij})$ is minimum. Tie can be broken arbitrarily. Remove this edge (i,j) from E_k and include it as a part of T_k unless it creates a cycle with the edges already in T_k.

Step 4:
If T_k has $n-1$ edges, then store its weight in $W(T_k)$ and go to step 5.
Else go to step 3.

Step 5:
If $k = n-1$ then go to step 7. Else go to step 6

Step 6:
Let $k = k + 1$
Arbitrarily select an edge (p,q) from T_o.
$T_o = T_o - (p,q)$ and $N_k = N_0 - (p,q)$. Go to step 3.

Step 7:
Minimum of $v_\mu(W(T_k))$, $k = 1\ to\ n-1$, is found and its corresponding T_k is the second MWST.

Step 8:
Path = $< N^{k-j} > \oplus$ Path
Let $x = N^{k-j}$
Go to step 7.

Step 9:
Let $j = j + 1$
If $k - j = 0$, then
{Path = < s >$\oplus$ Path
If $TLN \neq \phi$, go to step 6
Else Terminate}
Else go to step 7.

IV. COMPUTATIONAL COMPLEXITY

In 2012, Zhan Ning and Wu Longshu [11] discussed the computational complexity of finding a spanning tree in a network with minimum total expenses. In the proposed algorithm of section III, from step1 to step3, the computational complexity for determining the MWST of a given network is $O(n^2 logn)$. According to step 4 and step 5, MWST is found for (n-1) networks. Therefore the computational complexity involved from step1 to step 6 is $O(n^3 logn)$. Therefore the computational complexity of the proposed algorithm in Section III is $O(n^3 logn)$.

V. NUMERICAL ILLUSTRATION

Consider a simple undirected network given in figure 5.1 with six vertices and nine edges. Here triangular intuitionistic fuzzy numbers are considered as weight of the edges and is given in table 5.1.

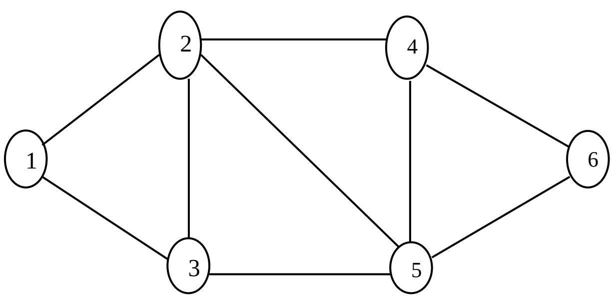

Fig 5.1 Network N_0

Edge	Weight of the edge
$(1,2)$	$(40,45,50;\ 35,45,55)$
$(1,3)$	$(50,52,56;\ 48,52,58)$
$(2,3)$	$(5,10,15;\ 4,10,17)$
$(2,4)$	$(15,20,24;\ 13,20,30)$
$(2,5)$	$(60,64,70;\ 58,64,75)$
$(3,5)$	$(12,18,20;\ 10,18,25)$
$(4,5)$	$(10,14,16;\ 8,14,20)$
$(4,6)$	$(30,34,40;\ 25,34,45)$
$(5,6)$	$(22,28,30;\ 20,28,34)$

Table 5.1 Edge weights of the Network N_0 corresponding to fig 5.1

Weight of MWST of the given network is $(89,115,131;\ 77,115,151)$ and its edges are $(2,3),(4,5),(3,5),(5,6)\ and\ (1,2)$
For constructing network N_1, the edge $(1,2)$ is removed from the network N_0.
Edges of MWST T_1 corresponding to Network N_1 are $(2,3),(4,5),(3,5),(5,6) and (1,3)$. Weight of T_1 is $W(T_1) = (99,122,137;\ 90,122,154)$
For constructing network N_2, the edge $(2,3)$ is removed from the network N_0.
Edges of MWST T_2 corresponding to Network N_2 are $(4,5),(3,5),(2,4),(5,6) and (1,2)$. Weight of T_2 is $W(T_2) = (99,125,140;\ 86,125,164)$
For constructing network N_3, the edge $(3,5)$ is removed from the network N_0.
Edges of MWST T_3 corresponding to network N_3 are are $(2,3),(4,5),(2,4),(5,6) and (1,2)$. Weight of T_3 is $W(T_3) = (92,117,135;\ 80,117,156)$
For constructing network N_4, the edge $(4,5)$ is removed from the network N_0.
Edges of MWST T_4 corresponding to Network N_4 are $(2,3),(3,5),(2,4),(5,6) and (1,2)$. Weight of T_4 is $W(T_4) = (94,121,139;\ 82,121,161)$
For constructing network N_5, the edge $(5,6)$ is removed from the network N_0.
Edges of MWST T_5 corresponding to Network N_5 are $(2,3),(4,5),(3,5),(4,6) and (1,2)$.
Weight of T_5 is $W(T_5) = (97,121,141;\ 80,121,162)$
Hence the second MWST is determined from $T_1, T_2, T_3, T_4\ and\ T_5$.
Min of
$\{v_\mu(\ w(T_1)), v_\mu(w(T_2)), v_\mu(w(T_3), v_\mu(w(T_4)), v_\mu(w(T_5))\ \}$
$= v_\mu(w(T_3))$.
Therefore T_3 is the second MWST of the given network.
Weight of second MWST of the given network is $(92,117,135;\ 80,117,156)$ and its edges are $(2,3),(4,5),(2,4),(5,6)\ and\ (1,2)$.

VI. CONCLUSION

In this paper a new algorithm is proposed to find the second minimum weight spanning tree of a given network with triangular intuitionistic fuzzy numbers as edge weights. Computational complexity of the proposed algorithm in section III is $O(n^3 logn)$. An example to illustrate the method is provided.

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