Sensitivity Analysis of MEMS Capacitive Pressure Sensor with Different Diaphragm Geometries for High Pressure Applications

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***Abstract*— In this paper three MEMS capacitive pressure sensors with different diaphragm geometries are designed and simulated. The sensors modelled have square, circular and rectangular diaphragms, with some fixed area. The diaphragm thickness of the three sensors is 63µm. The sensors are designed for high pressure sensing, over a range of pressure varying from 1Mpa to 100Mpa. The paper presents a rectangular diaphragm designed using a golden ratio of rectangle design widely used in image processing application with the ratio (b/a) equal to 1.618. Silicon<100> is used as a diaphragm material, because of its excellent properties. The paper provides a thorough analysis and discussion on different performance parameters for capacitive pressure sensing, such as the total displacement, capacitance, PRCC (Percentage Relative Change in Capacitance), electrical sensitivity. The design and simulation of the pressure sensors have been done based on Finite Element Method using Multiphysics simulation platform. Such kind of pressure sensors can be used in harsh environments involving high pressure applications.**

Keywords*— *MEMS, Capacitive Pressure Sensors, Si<100>, Golden rectangle, Golden ratio.

I. INTRODUCTION

The combination of micromaching techniques of silicon and the advent of high expertise in silicon integrated circuits have paved way for MEMS and Microsystems concept. MEMS pressure sensors have gained lot of interest as they have a wide horizon of applications [1, 2]. Pressure sensors are now being used in harsh environments, involving pressure ranging from few Pascal (Pa) to several Mega Pascal (MPa). Amongst various transduction principles of MEMS pressure sensors, piezoresistive and capacitive transduction mechanism have been used widely [3]. Capacitive pressure sensors provide high sensitivity as compared to piezoresistive pressure sensors, as their performance is invariant with the temperature. MEMS pressure sensors typically use a flexible diaphragm that deforms in the presence of a pressure and this deformation is converted to an electrical signal. Capacitive sensors come with square, circular and rectangular diaphragms [4,5,6].

This paper presents three different diaphragm structures, viz. square, circular and rectangular keeping the area same. Here rectangular diaphragm model with, a concept of golden ratio is utilized, which is widely used in image processing applications. A ratio (b/a) of 1.618 called golden ratio has been used to design the rectangular diaphragm. The performance study of the designed pressure sensors is done using COMSOL Finite Element Method based Multiphysics simulation tool. Displacement, capacitance and sensitivity of the sensor are the performance parameters considered in this paper. A mathematical validation has been done using the mathematical models given in section II. Section III describes the details of the sensor design. Section IV shows the simulation setup of the pressure sensors using COMSOL. In section V results are presented and section VI provides the conclusion.

II. SENSOR MODEL

The capacitive pressure sensor with square, circular and rectangular diaphragm has been modelled using COMSOL/Multiphysics.

A. Sensor description

The designed pressure sensor consists of five layers viz. diaphragm, electrode, followed by dielectric material (air gap), bottom electrode and substrate. The top electrode is free, where as the bottom electrode is fixed. Figure 1 shows the model of the designed capacitive pressure sensor. Figure 2 shows square & circular diaphragm were, the square diaphragm has dimensions of 783µm*783µm with thickness of 63µm & circular diaphragm has radius of 442µm with thickness of 63µm. Figure 3 describes the model of rectangular diaphragm of thickness of 63µm, length and width of 1000µm*620µm with the golden ratio (b/a) of 1.618 & the model of normal rectangular diaphragm of 950µm*645µm with same thickness.

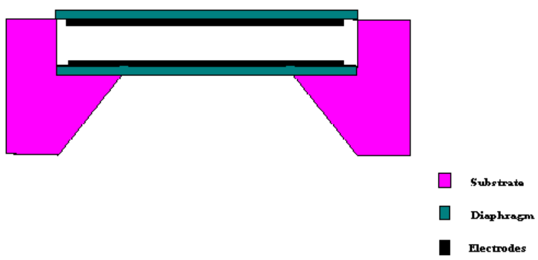

Figure 1 Model of the capacitive pressure sensor

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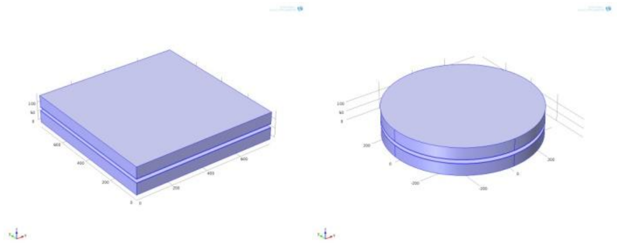

Figure 2 square & circular diaphragm model

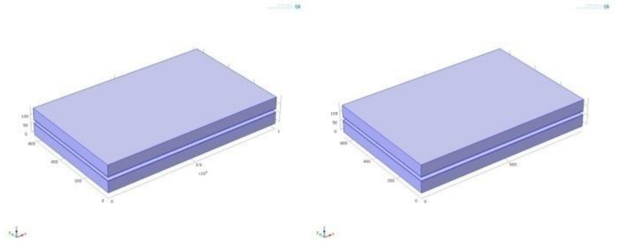

Figure 3 normal & golden rectangular diaphragm model

The two electrodes are of gold material with dimensions of the length, radius and width as mentioned in Table 1. The thickness for the electrodes has been taken as 1µm. The details of the design are given in Table 1. Were, b=length of diaphragm, a=width of diaphragm, h=thickness of diaphragm and A=area of the diaphragm (all values are in microns).

TABLE I
SENSOR DESIGN DETAILS

Si. No	Diaphragms		
	Square	Circular	Rectangular
1.	a=783µm*783µm h=63µm A=0.61 μm^2	r=442µm h=63µm A=0.61 μm^2	Golden rectangle b*a= 1000µm*620µm h=63µm, A≈0.61 μm^2 Normal rectangle b*a=950µm*645µm h-63 µm, A=0.6127 μm^2
2.	Electrodes thickness 2µm, lateral dimensions are same as the diaphragm length and width		

B. *Material Properties of silicon <100>*

Silicon <100> material is used to design the diaphragms of the pressure sensors. The material properties of silicon have been presented in Table. 2. Silicon <100> has been selected because of its Young's modulus.

TABLE III
SILICON MATERIAL PROPERTIES

Si. No	Silicon<100> Diaphragm	
	Material Properties	Value
1.	Young's Modulus	129.5GPa
2.	Poisson's ratio	0.27
3.	Density	2329[Kg/m^3]
4.	Thermal Expansion Coefficient	2.6e-6[K^{-1}]
5.	Thermal Conductivity	130[W/(m*K)]
6.	Relative permittivity	11.7

III. MATHEMATICAL BACKGROUND OF CAPACITIVE PRESSURE SENSOR

Here the theory of 'thin plates' and 'Small deflection' is used for the design consideration of the diaphragms [6]. Thin plates refers to the condition $h \cong \frac{a}{10}$ (a-side of a diaphragm) were as small deflections refers to $W_{max} \cong \frac{h}{4}$ and also it's a maximum central deflection. Hence suitable structure for a sensor would be a case $W_{max} \cong \frac{h}{4}$ [7]. The general equation for displacement w(x,y) for the diaphragm with clamped edges with applied pressure 'p' is given by the following fourth order differential equation (1).

$$\frac{\partial^4 w(x,y)}{\partial x^4} + 2\alpha \frac{\partial^4 w(x,y)}{\partial x^2 \partial y^2} + \frac{\partial^4 w(x,y)}{\partial y^4} = \frac{P}{Dh^3} \quad \textbf{(1)}$$

w(x,y) is the displacement of the diaphragm with realistic boundary conditions, whereas w_{max} is the maximum centre displacement of the diaphragm. Were the square diaphragm is of side 'a' and thickness 'h. Rectangular diaphragm is of length 'b' and width 'a' and thickness of 'h' and for circular diaphragm, has a radius of 'r' and thickness of 'h'.

A. *Square diaphragm*

The design consists of side of 783µm*783µm and 'h' of 63 µm. The maximum centre displacement w_{max} for the square diaphragm is given by (2).

$$W_{max} = 0.01512(1 - v^2)\frac{P_i a^4}{Eh^3} \quad (2)$$

Were the above equation has been formulated for a flexural density, which is given by (3).

$$D = \frac{Eh^3}{12(1-v^2)} \quad (3)$$

Capacitance is calculated using the equation (4).

$$C = \iint \frac{\epsilon dxdy}{d - w(x,y)} \quad (4)$$

Where'd' is the initial gap between the electrodes, w(x,y) is the total centre deflection. The equation (4) provides non-linear capacitance with deflection, binomial expansion method can be used to solve the above non-linearity. Equation (4) can be elaborated using binomial expansion mentioned [8].

$$C = C_0\left(1 + \frac{12.5Pa^4}{2015dD}\right) \quad (5)$$

Here, C- new capacitance, C_0-initial capacitance, P-Pressure applied, a-half the length of diaphragm, d-gap between the electrodes, D-flexural density.

B. *Circular diaphragm*

The circular diaphragm has been modeled with radius of the diaphragm of 442 µm and thickness of 63 µm. The maximum centre displacement of the circular diaphragm can be calculated using (5) [7].

$$w(0, P_i) = \frac{P_i r^4}{64D} \quad (6)$$

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C. Rectangular diaphragm

The paper provides design of two rectangular diaphragms, 1) based on the golden ratio used in image processing techniques and 2) normal selection method. Golden ratio rectangular diaphragm has dimensions of 1000 µm*620 µm, whereas the normal rectangular diaphragm has dimensions of 950 µm*645 µm. The equation used to find the deflection of the rectangular diaphragms is mentioned in equation (7). The calculation using equation (7) is formulated for b/a ratio of 1.61 for golden rectangle and 1.5 for the normal [7].

$$W_{max} = \alpha \frac{Pa^4}{D} \qquad (7)$$

Where, α is 0.0023 for golden rectangle and 0.0022 for normal rectangle. The selection of $\alpha$is based on b/a ratio [9, 10]. D is flexural density same for all models with value of $2.91X10^{-3}$. For all the above modelled diaphragms, Percentage Relative Change in Capacitance PRCC is calculated using the formula in equation (8).

$$PRCC = \frac{C - C_0}{C_0} \times 100 \qquad (8)$$

Sensitivity: -Sensitivity of the diaphragm is defined as the change in the capacitance to the change in the applied pressure. The equation used to find the sensitivity of the designed models is given in equation (9) [8].

$$S = \frac{49\varepsilon a^6}{2025 d^2 D} \qquad (9)$$

Where, a=half the length of diaphragm, D= Flexural density, d= gap between the electrodes

IV. SIMULATION SETUP OF PRESSURE SENSOR

The sensor has been modeled and simulated using COMSOL/Multiphysics. Three models viz., square, circular and rectangular diaphragms have been modeled using the tool with silicon <100> as diaphragm material and air as the dielectric material. The gap between the two electrodes i.e., the diaphragm and substrate is 19µm. The square diaphragm is modeled with dimensions of 783µm*783µm and thickness of 63µm. Circular diaphragm has been modelled using silicon and radius of 442µm and 63µm thickness. The rectangular diaphragm model consists of a golden rectangular diaphragm and normal rectangular diaphragm. The golden rectangular diaphragm has dimensions of 1000µm*620µm and normal rectangular diaphragm has dimensions of 950µm and 645µm. Both rectangular diaphragms are models with the thickness of 63µm. The models were subjected to electromechanical analysis with the application of load varying from 1MPa to 100MPa.Three models were subjected to Finite Element Analysis with maximum element size of 117µm, minimum element size of 21.9µm.

V. PERFORMANCE PARAMETERS

The following parameters are used for the analysis of the designed models.

1) **Total Displacement**: Deflection of the diaphragm against the pressure applied.

2) **Total Capacitance**: Capacitance of the model against the pressure applied (Capacitance changes with change in the applied pressure).

3) **PRCC**: Percentage Relative Change in Capacitance defined as the ratio of difference between the new capacitance and initial capacitance to that of initial capacitance equation (8).

4) **Sensitivity**: Sensitivity, defined as the ratio of change in capacitance with respect to per unit change in the pressure applied.

VI. RESULTS AND DISCUSSION

The results are taken for the diaphragm thickness of 63µm. Further the results were taken by varying the diaphragm thickness. The pressure range applied is between 1MPa to 100MPa. This section shows the simulated and analytical results for the diaphragm displacement, capacitance, change in capacitance and sensitivity of the models. Equations in section III are used to manually calculate and verify the simulated results. The simulated results and the analytical/theoretical results show similarities.

Figures 4, 5, 6 and 7 show the plot of the displacement versus applied pressure in MPa for the square, circular, golden rectangular and normal rectangular diaphragms respectively. The four graphs show the simulated and analytical results for displacement at the centre of diaphragm, which are almost near.

Figure 6 gives the displacement plot of the circular diaphragm, the results are shown till the pressure of 80MPa, after that pressure the diaphragm touches the bottom electrode due to large displacement. Whereas the other two models operates normally till 100MPa of applied pressure. All the four graphs show linear increase in displacement of the diaphragm with the applied pressure. The gap between the electrodes is 19µm.

Figure 8 shows the plot of capacitance of a square diaphragm with 63µm thickness for a gap of 19µm. The plot provides both simulated and analytical results for the capacitance using the equations given in section III. Figure 9 show the graph of capacitance of square diaphragm with varying thickness against applied pressure. The gap between the electrodes is kept as 19µm. All plots in the graph show near linear behaviour, but the plot for capacitance of the diaphragm with thickness 60µm shows a sudden increase in capacitance form 5.382e-13F at 90MPa pressure to 8.366e-13F at 100MPa. This is because at the diaphragm at that thickness for a pressure of 100MPa has a large displacement.

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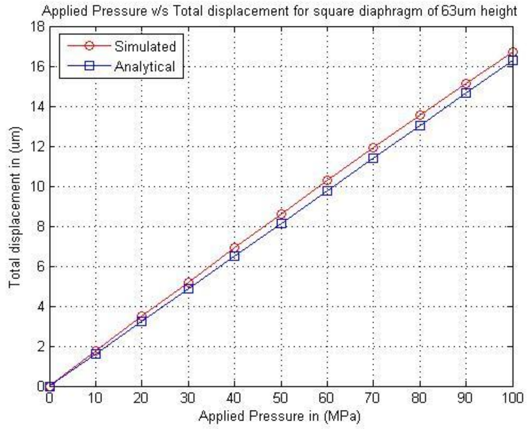

Figure 4 graph of Applied Pressure v/s Total displacement for a square diaphragm of thickness 63μm

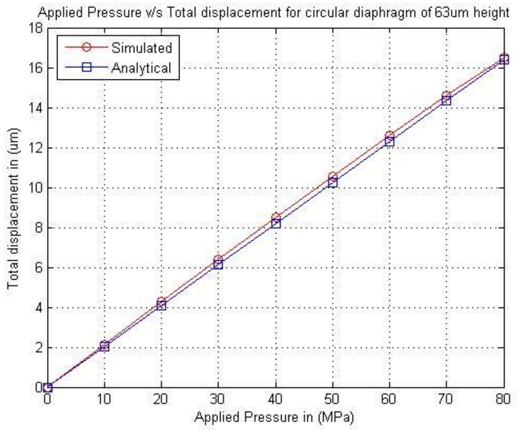

Fig.ure 5 graph of Applied Pressure v/s Total displacement for a circular diaphragm of thickness 63μm

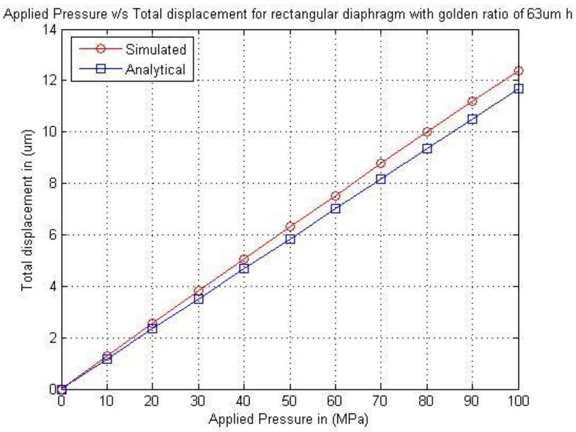

Figure 6 graph of Applied Pressure v/s Total displacement for a golden ratio rectangular diaphragm of thickness 63μm

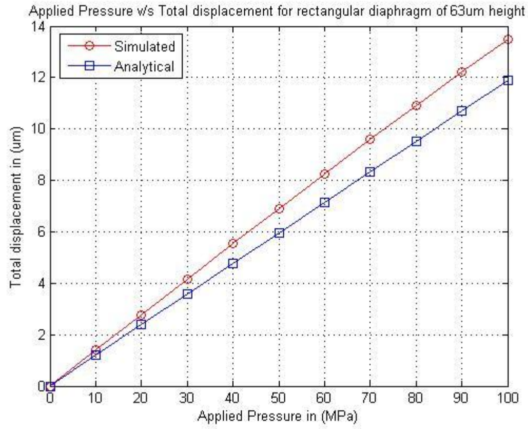

Figure 7 graph of Applied Pressure v/s Total displacement for a normal rectangular diaphragm of thickness 63μm

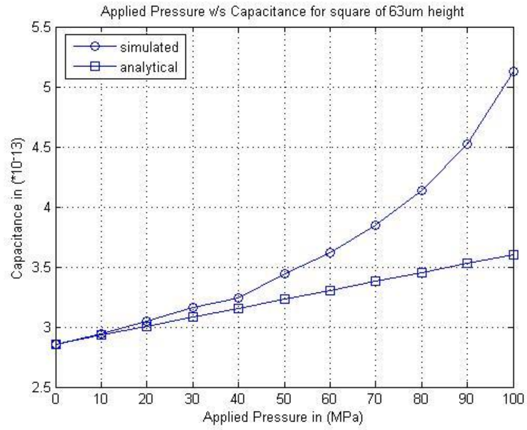

Figure 8 graph of Capacitance v/s Applied Pressure square diaphragm of thickness 63μm, gap 19μm.

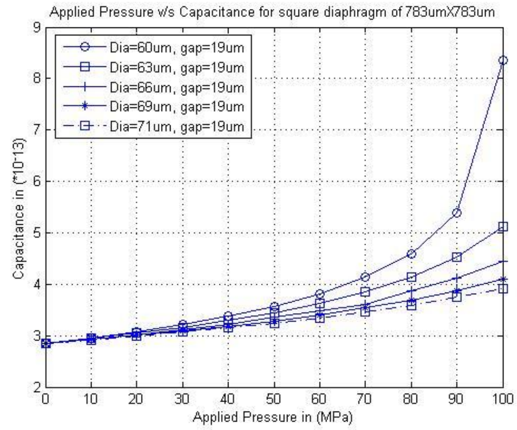

Figure 9 graph of Applied Pressure v/s Capacitance with different thickness of square diaphragm, with the gap between electrodes equal to 19μm

Figure 10 and 11 show the capacitance plot of circular diaphragm with thickness of 60μm and 63μm respectively. The two plots are shown separately because circular diaphragm at 60μm thickness diaphragm touches the bottom electrode 70MPa of applied pressure whereas the circular diaphragm at 63μm thickness touches at 80MPa of applied pressure. Figure 12 provides the plot of circular diaphragm with thickness of diaphragm for 69 & 71μm.

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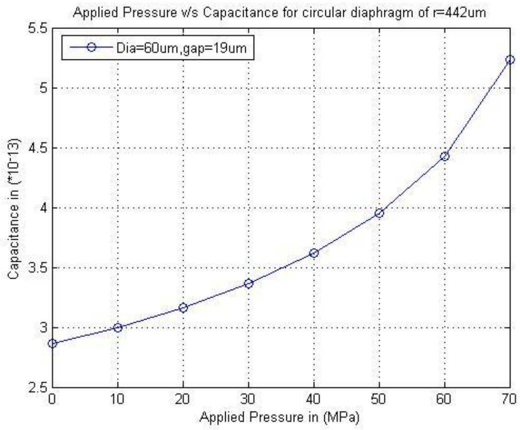

Figure 10 graph of Applied Pressure v/s Capacitance for a circular diaphragm of 60µm.

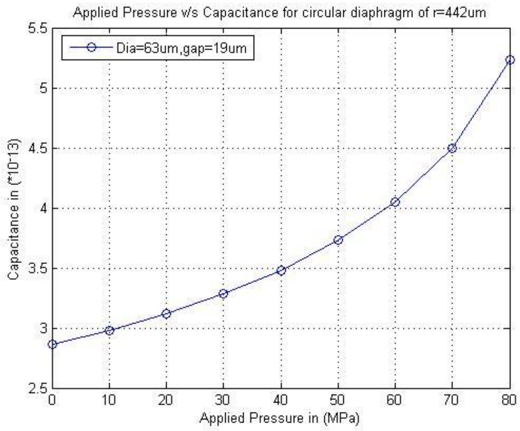

Figure 11 graph of Applied Pressure v/s Capacitance for a circular diaphragm of 63µm.

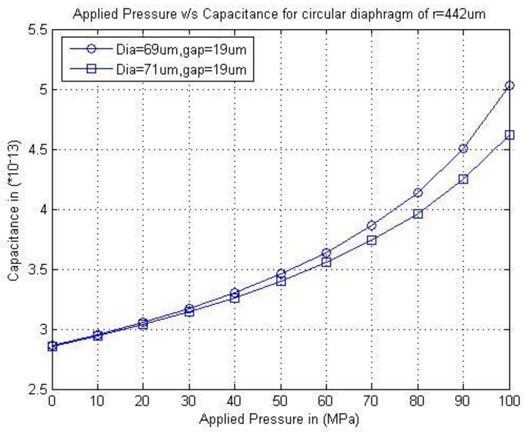

Figure 12 graph of Applied Pressure v/s Capacitance with different thickness of circular diaphragm, with the gap between electrodes equal to 19µm.

Figure 13 shows the capacitance plot against the pressure applied for a golden rectangular diaphragm. The plot presents the capacitance for varying diaphragm thickness from 60µm to 71µm. The gap between the electrodes is set at 19µm. The plot shows near linear behaviour for all the diaphragm thickness variations.

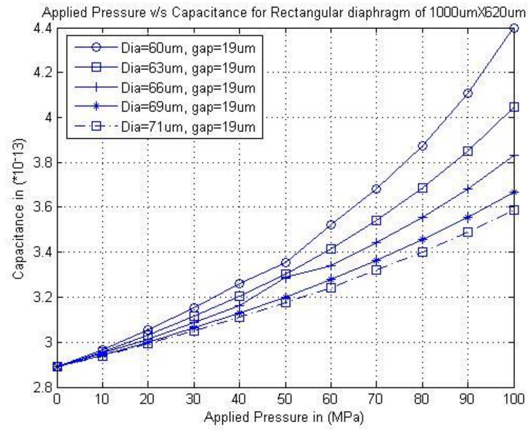

Figure 13 graph of Applied Pressure v/s Capacitance with different thickness of golden ratio rectangular diaphragm, with the gap between electrodes equal to 19µm.

Figure 14 shows the plot of capacitance against applied pressure for the normal rectangular diaphragm. The graph provides the plot for capacitance of the normal rectangular diaphragm with varying diaphragm thickness from 60µm to 71µm. It can be observed that the plot show the near linear behaviour for the diaphragm with varying thickness. The gap between the electrodes is set at 19µm.

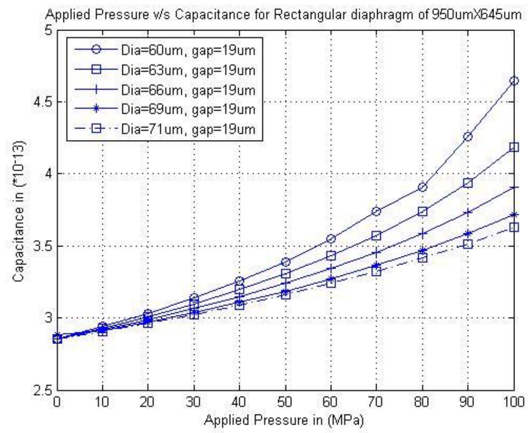

Figure 14 graph of Applied Pressure v/s Capacitance with different thickness of normal rectangular diaphragm, with the gap between electrodes equal to 19µm.

Figure 15 provides the plot of capacitance of the four models against the pressure applied. The diaphragm thickness for all the four models is kept at 63µm and the gap of 19µm. From the figure it can be observed that the circular diaphragm provides the better capacitance readout compared to the other models. Square diaphragm is next to circular diaphragm in capacitance readout. Rectangular diaphragm doest show the amount of capacitance as the other two, but provides better linearity compared over the range 1MPa to 100MPa, to the other two diaphragm models. From the plot it can be observed that, the golden rectangular diaphragm provides better capacitance in the range 0 to 30 MPa. The normal rectangular diaphragm provides better capacitance in the range 60 to 100 MPa. Between 30 to 60 MPa both diaphragms behave same.

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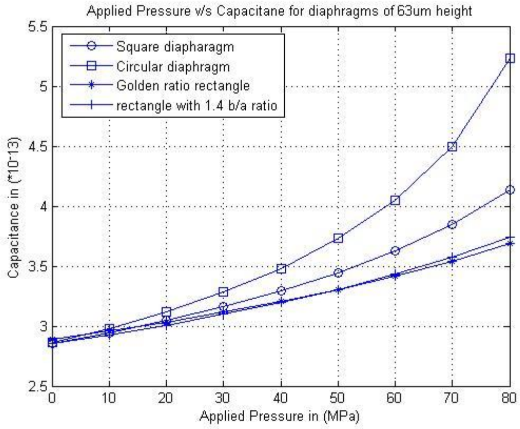

Figure 15 graph of Applied Pressure v/s Capacitance of square, golden ratio rectangular, normal rectangular and circular diaphragm, with the gap between electrodes equal to 19μm.

Figure 16 shows the PRCC against the applied pressure for the two diaphragm models viz., square and rectangular (golden & normal rectangular diaphragms). Square diaphragm shows better PRCC compared to the rectangular models. Figure 17 gives the plot of PRCC of circular diaphragm. Circular diaphragm shows better PRCC as compared to other two diaphragm models.

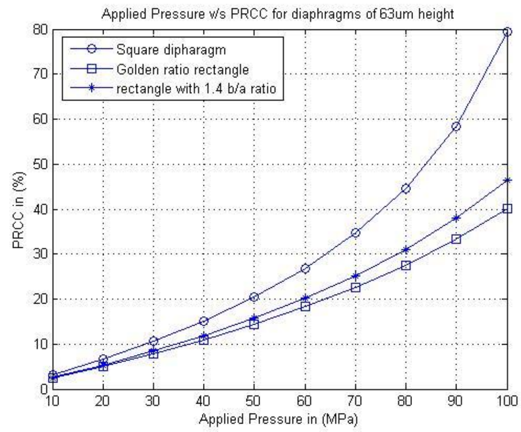

Figure 16 graph of Applied Pressure v/s Percentage relative change in capacitance for square, golden and normal rectangular diaphragm of thickness 63μm

Sensitivity Analysis: Sensitivity is an important parameter of capacitive pressure sensor. It is defined as the change in the capacitance with respect to change in the pressure applied. In the model the pressure is varied in terms of MPa. Sensitivity was calculated for the square diaphragm and it is found to be **7.34e-22 F/MPa**. To elaborate if suppose the initial capacitance is 2.856e-13 F and if we aim at finding the capacitance for 10MPa change in the applied pressure, 7.34e-22X10MPa gives 7.34e-15. When 7.34e-15 is added to initial capacitance of 2.856e-13 the next capacitance for 10MPa change in applied pressure will be 2.929e-13, this value is same as the analytical values abtained.

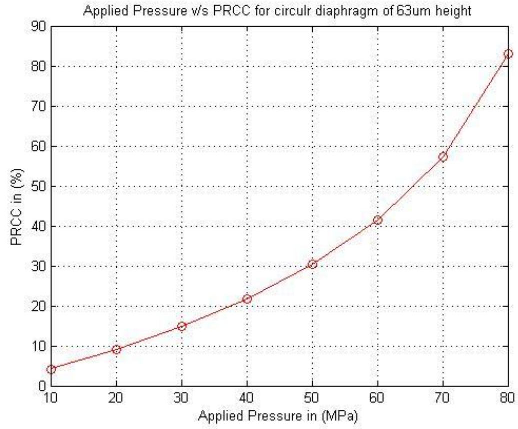

Figure 17 graph of Applied Pressure v/s Percentage relative change in capacitance for a circular diaphragm of thickness 63μm

VII. CONCLUSION

The paper presents modeling, simulation and analysis of the capacitive pressure sensor consisting of three different diaphragms. Both simulation results & theoretical results are provided in the paper which shows near resemblance. COMSOL/Multiphysics has been used to model and simulate the models. Silicon <100> material has been used in the design of the diaphragms. The circular diaphragm shows better capacitance readout compared to the other models, whereas the square and rectangular diaphragm models provide more linear outputs.

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