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# Sheath formation in multi-component plasma

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## Abstract

*Using a multi-fluid model, the sheath formation criterion in electronegative plasma is examined. It is shown that in a collisional sheath there are both upper and lower limits for the Bohm entrance velocity ions. The Sagdeev potential in the whole sheath expansion were used. The criterion was specified for several special cases, viz., constant ion mean-free path and constant ion mobility.*

*The sheath-edge potential as well as the floating potential are also calculated.*

**Introduction.**

The plasma sheath is the localized electric field that separates plasma from a material boundary. Sheath formation is one of the old problems in plasma physics **[1]**.That is why, many authors paid attention to the sheath structure formation in electron-ions plasmas **[2]**.Recently, much effort has been invested in studying the sheath of multicomponent plasmas [**3**]. Due to electron attachment, a large number of negative ions are generated in the course of plasma discharge. Plasmas containing negative ions usually referred to as electronegative plasmas are of practical interest in plasma processing as well as their significance in theoretical research [**4**]. A plasma sheath involving negative ions is called an electronegative plasma sheath. Electronegative plasma sheaths have been extensively used in many areas such as material surface treatment, etching and thin-film deposition processing, and plasma chemistry **[5,6]**. Thus, investigation of electronegative plasma sheaths is considerably significant. The electrodynamics properties of plasma-sheath boundary are of great importance in a wide range of applications, such as electrostatic plasma probes, electrode phenomena, and surface of materials processed in dc and RF discharges**[7].** In these cases, the electrical characteristics and plasma parameter profiles of the sheath are defined by the condition of the plasma-sheath boundary. Therefore, a sheath criterion is needed to provide the boundary condition to calculate the plasma parameter profiles. For an electron-ion plasma, the necessary condition for steady-state charged sheath existence was first obtained by Bohm1 and is commonly known as the Bohm criterion **[8].** This criterion determines the ion velocity at the sheath entrance, which is necessary for steady-state sheath existence in two-component plasma. The presence of the negative ions modifies the structure of theplasma–sheath considerably. Indeed, in electronegative plasmas the negative ion density often exceeds the density of electrons, and the presence of negative ions introduces complications into discharge analysis **[9]**. In surface treatment by such plasmas, the knowledge of the sheath structure is essential since processes taking place at the surface will depend on the particle distribution at the vicinity of the wall. Additionally, the effect of negative ion in the formation of a sheath is considered to be important for potential control of an electrode in etching and deposition plasmas **[10].** Several authors derived **[11]**conditions

model of the plasma with Boltzmann-distributed electrons. In all of these works, the conditions for steady-state sheath existence were obtained by expanding solutions for small electric potentials in the sheath. Their validity may be verified by analyzing the solution of Poisson's equation in all ranges of the potential. Besides, Das *et al*.**[13]** derived the plasma sheath equation, and studied the effect of the finite ion temperature on the required condition for the sheath formation in electropositive plasma. Liu *et al*.**[14]** investigated the sheath

of the sheath existence using a fluid criterion in a collisional plasma sheath by using a two fluid model. Ghomi*et al*. **[15]** studied the boundary condition for collisional thermal electropositive plasma. Motivated by previous works, in this paper, we use the multi-fluid model to examine the effects of density of negative ions on the positive ion transition velocity in the plasma sheath boundary and on the sheath formation in collisional plasmas. The objective of the present investigation is to derive the steady-state sheath criterion by analyzing the Sagdeev potential throughout the entire sheath.

## 1.Model equation.

We consider a one dimensional plasma sheath treating electron and negative ions as thermal Boltzmann particles, but positive ions as cold fluid. The sheath edge at x=0 is the plasma-sheath interface (interface between essentially neutral and non-neutral regions).Both electrons and negative ions in the sheath are assumed to obey the Boltzmann distribution

$$n_e = n_{e0} exp\left(\frac{e\phi}{kT_0}\right) \quad (1)$$

$$n_- = n_{-0} exp\left(\frac{e\phi}{kT_-}\right) \quad (2)$$

Where $\phi$ Is the local potential, $T_e$ $(T_-)$is the electrons (negative ions) temperature and $n_{e0}$ $(n_{-0})$ is the electron (negative ion) density at the sheath edge where $x = 0$ $and$ $\phi = 0$.

The cold positive ions satisfy the continuity and momentum equations:

$$\nabla(n_i v_i) = 0. \quad (3)$$

$$m_i v_i \nabla(v_i) = -e\nabla\phi - m_i(n_n \sigma v_i)v_i. \quad (4)$$

Where $m_i$is the ion mass, $n_n$ is the neutral gaz density, and $\sigma = \sigma_s(v_i/c_s)^\gamma$ is the momentum-transferring cross sections for collision between ions and neutrals. $c_s = (kT_e/m_i)^{1/2}$is the ion acoustic velocity, $\sigma_s$ is the cross section measured at ion acoustic velocity,$\gamma$is an index and set as $\gamma = 0$ for constant ion mean-free path and $\gamma = -1$for constant ion mobility. Assuming, in this section, the wall is infinitely long in the y and z direction, the quantities changes only in the $\hat{x}$direction normal to the wall, ie: $\nabla \rightarrow \hat{x}\frac{\partial}{\partial x}$.

The Poisson's equation relates the electrons and ions to the self-consistent potential,

$$\frac{d^2\phi}{dx^2} = -4\pi\rho. \quad (5)$$

Where is the space charge density in the sheath and is given by

$$\rho = -e(n_i - n_e - n_-).$$

Simplifying the equations, we can use the dimensionless variables as follows:

$$\eta = -\frac{e\phi}{kT_e}, \xi = \frac{x}{\lambda_D}, u = \frac{v_i}{c_s}, \quad \alpha = \lambda_D n_n \sigma_s \quad , \quad \theta = \frac{T_e}{T_-},$$

$\delta = \frac{n_{-0}}{n_{e0}}$.As well as $\frac{d}{d\xi} \rightarrow {}'$, where $\lambda_D = \sqrt{\frac{kT_e}{4\pi e^2 n_{e0}}}$ in the Debye electron length.

Substituting them into the governing **Eqs.(4)** and **(5)**one can obtain:

$$u.u' = \eta' - \alpha u^{2+\gamma} \quad (6)$$

$$\eta'' = (\delta - 1)u_0/u - exp(-\eta) - \delta exp(-\theta\eta) \quad (7)$$

$u_0$is the Mach number,$\eta'$ is the dimensionless electric field.

## 3.Sagdeev potential.

Investigation of a collisional unmagnetized plasma sheath consisting of electrons, negative ions as well as positive ions can be carried out by solving **Eqs.[(1)-(7)]**. However, to solve these equations the boundary conditions must be specified. This leads to determine the Bohm's criterion. First we derive the generalized the Bohm's criterion, and then we reduced it to some special cases.First, integrating **Eq.(7)** once, we get

$$\frac{1}{2}(\eta')^2 + V(\eta, u_0) = \frac{1}{2}(\eta_0')^2. \quad \textbf{(8)}$$

where$\eta'$ is dimensionless electric field at the sheath edge.**Eq.(8)** is known as plasma sheath equation for the plasma we have taken into account. The boundary conditions are$\eta \to 0\ at\ |\xi| \to \infty$, which measures the sheath thickness, $\eta \to \eta_w\ at\ |\xi| \to 0$, and determines the wall potential.

$$V(\eta, u_0) = \left(1 + \frac{\delta}{\theta}\right) - exp(-\eta) - \frac{\delta}{\theta} exp(-\theta\eta) - \int_0^u (\delta + 1)\,(u_0/u) d\xi. \quad \textbf{(9)}$$

$V(\eta, u_0)$is the Sagdeev potential satisfying boundary conditions $V(0, u_0) = 0, \frac{\partial V(0,u_0)}{\partial \eta} = 0$.

**4.Bohm sheath criterion.**

The variation of $V(\eta, u_0)$ yields the basic characteristics of the sheath formation in plasma.**Eq.(8)** is analogous the conservation law for a classical particle in a well $V(\eta, u_0)$, with being the analogous total energy. Similar to the analysis of a partial well, it requires $V(\eta, u_0) < 0$ in the sheath, which leads to $\left.\frac{\partial^2 V(\eta,u_0)}{\partial \eta^2}\right|_{\eta=0} < 0$.

Then, from **Eq.(9)**, we get

$$\frac{\partial^2 V(\eta, u_0)}{\partial \eta^2} = exp(-\eta) + \frac{\delta}{\theta^2} exp(-\theta\eta) - (\delta - 1)(u_0 u'/u^2). \quad \textbf{(10)}$$

From **Eq.(10)**, by considering the condition for maximizing the Sagdeev potential at the sheath edge, ie., $\frac{\partial^2 V(0,u_0)}{\partial \eta^2} < 0$, we have

$$u'_0 < \frac{(1 + \delta\theta)}{(1 + \delta)} u_0 \eta'_0, \quad \textbf{(11)}$$

In addition, at the sheath edge ($\xi = 0$), **Eq.(6)** takes the following form:

$$u_0 u'_0 = \eta_0' - \alpha u_0^{2+\gamma}. \quad \textbf{(12)}$$

It is evident that $u'_0 \geq 0$ due to neutral drag to the positive ion in the plasma. Therefore, the necessary condition of entrance of the positive ions into the sheath region is $\eta_0' > 0$ which means that there must exist acceleration force to overcome collision drag.

Considering **Eq.(12)** and the above mentioned condition, we get

$$\eta_0' \geq \alpha u_0^{2+\gamma}. \quad \textbf{(13)}$$

In relation **(11)-(13)**, we find the upper and lower limits of Bohm's sheath criterion:

***a/ For constant cross section:***

$$\left[\frac{\eta_0'(1+\theta)}{\alpha(1+\theta) + (1+\delta\theta)\eta'_0}\right]^{1/2} \leq u_0 \leq \left[\frac{\eta_0'}{\alpha}\right]^{1/2} for\ \gamma = 0. \textbf{(14)}$$

*b/ For collisional mobility:*

$$\left[\sqrt{\left(\frac{\alpha^2}{4\eta_0'^2}\left(\frac{(1+\theta)}{(1+\delta\theta)}\right)^2\right) + \frac{(1+\theta)}{(1+\delta\theta)}} - \frac{1}{2}\frac{\alpha}{\eta_0'}\frac{(1+\theta)}{(1+\delta\theta)}\right] \leq u_0 \leq \left[\frac{\eta_0'}{\alpha}\right] for\ \gamma = -1. \textbf{(15)}$$

Now to investigate the validity of the derived Bohm's criterion, we would like to reduce our general Bohm's criterion to some special cases;

*i/ For collisionless electrostatic plasma*($\alpha = 0, \theta = 0, \delta = 0$) inequalities[**(14)-(15)**] becomes$u_0 \geq 1$, which is the well-known Bohm criterion for a collision-free sheath. It means that the ion flow speed at the plasma boundary must be at least as great as the ion sound speed in order for a sheath to form.

*ii/ For collisional electrostatic plasma*($\alpha \neq 0, \theta = 0, \delta = 0$) inequalities **[(14)-(15)]** reduces to

$\left[\frac{\eta_0'}{\alpha+\eta'_0}\right]^{1/2} \leq u_0 \leq \left[\frac{\eta_0'}{\alpha}\right]^{1/2}$ $(for\ \gamma = 0)$, and

$\left[\left(1 + \frac{\alpha^2}{4\eta_0'^2}\right)^2 - \frac{1}{2}\frac{\alpha}{\eta_0'}\right] \leq u_0 \leq \left[\frac{\eta_0'}{\alpha}\right]$ $(for\ \gamma = -1)$.

The result is in agreement with Jin-Yun Liu *et al.* As mentioned by Liu, in collisional plasma ion Mach number is between upper and lower limits. The lower limit of the collisional sheath criterion shows the reduction of the ion entering speed $u_0$ due the existence of neutral drag $\alpha > 0$, on the other hand, the upper limit shows the balance between the driving (electric field $\eta_0^{'}$) and the drag (the collisionality $\alpha u_0^{2+\gamma}$).

*iii/ For collisional electronegative plasma sheath* ($\alpha \neq 0, \theta \neq 0, \delta \neq 0$), the range of the Mach is determined by inequalities**[(14)-(15)]**. To discuss the result, we consider a quasineutral argon plasma with following parameters:$\alpha = 0.134$ (corresponding to neutral pressure of P=0.1 Torr at temperature 290K), the density of argon plasma is $5.0 \times 10^8 cm^{-3}$ and the electron temperature $T_e = 3\ eV$.For the above-mentioned parameters, **Figure1** shows the upper and lower limits of Bohm's criterion of the collisional electronegative plasma sheath varying with the normalized electric field at the sheath edge, for$(a)\ \gamma = 0$ and$(b)\gamma = -1$.Using inequalities **(14-15)** and assuming$\theta = 10, \delta = 0.01\ and\ \eta_0^{'} = 0.1$, it is found that the range of allowable ion Mach number is $0.86 \leq U_0 \leq 1.29$ (for $\gamma = 0$) and $0.74 \leq U_0 \leq 1.71$ (for $\gamma = -1$).A comparison with ion temperature is made in **figure 2**.

**5. Floating potential.**

In this section we shall be looking for analytically exact solutions of the nonlinear equations systems **Eqs. [3-5].**The normalized governing equations of the plasma evolution are given by

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i v_i)}{\partial x} = 0. \quad \mathbf{(16)}$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = -\frac{\partial \phi}{\partial x} - (n_n \sigma v_i) v_i. \mathbf{(17)}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_i - n_e - n_{-}. \mathbf{(18)}$$

Normalized electrons and negative ions are given by

$$n_e = e^{-\eta}\ , \mathbf{(19)}$$

$$n_{-} = \delta e^{-\theta\eta}. \mathbf{(20)}$$

We confine ourselves to investigate stationary solutions that depend on space and time in thefollowing way: $\zeta = x - ut$.

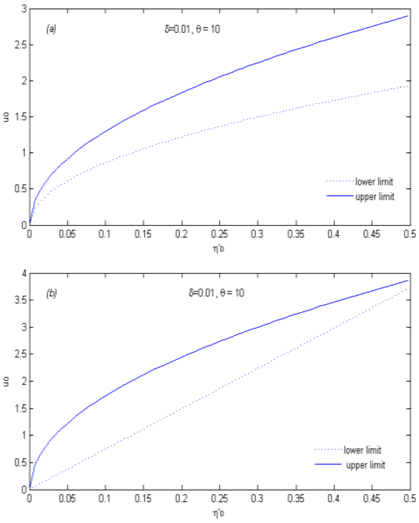


**Figure1.**The lower and upper limits of Bohm's sheath criterion as a function of the normalized electric field$\eta_0^{'}$
For ($\alpha = 0.134, \theta = 10, \delta = 0.01$)*(a)*$\gamma = 0$ and *(b)* $\gamma = -1$

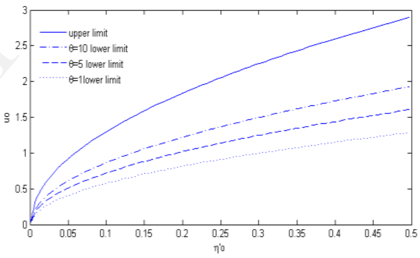


**Figure2.** The lower and upper limits of Bohm's sheath criterion vs.$\eta_0^{'}$
For $\alpha = 0.134, \delta = 0.01$for $\gamma = 0$.

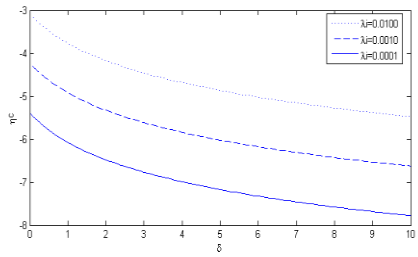


**Figure3.** Floating potential $\eta_c$vs.$\delta$.

In the stationary frame, we obtain from **Eqs.[(3)-(5)]** the density as

$$n_i = \delta_i \left(1 + \frac{2\eta}{u}\right)^{-1/2} \quad \mathbf{(21)}$$

*where* $\delta_i = n_{i0}/n_{e0}$.

Then the Poisson's equation becomes:

$$\frac{\partial^2 \eta}{\partial \zeta^2} = -e^{-\eta} - \delta e^{-\theta\eta} + \delta_i \left(1 + \frac{2\eta}{u}\right)^{-1/2} \quad \mathbf{(22)}$$

The normalized space charge density will be defined as

$$\rho = \delta_i \left(1 + \frac{2\eta}{u}\right)^{-1/2} - e^{-\eta} - \delta e^{-\theta\eta}. \quad (23)$$

So the exact Sagdeev potential reads as,

$$V(\eta, u) = (1 - e^{-\eta}) + \frac{\delta}{\theta}(1 - e^{-\theta\eta}) + u^2 \delta_i \left[1 - \sqrt{\left(1 + \frac{2\eta}{u^2}\right)}\right] \quad (24)$$

From the equation of integral energy, the normalized Maxwell stress reads as

$$\frac{1}{2}E^2 = (e^{-\eta} - 1) + \frac{\delta_i}{\theta}(e^{-\theta\eta} - 1) + u^2 \delta_i \left[1 - \sqrt{\left(1 + \frac{2\eta}{u^2}\right)}\right]. \quad (25)$$

Consequently, at the cathode $(\eta = \eta_c)$ it will be of the form

$$\frac{1}{2}E_c^2 = (1 - e^{-\eta_c}) + \frac{\delta_i}{\theta}(1 - e^{-\theta\eta_c}) + u^2 \delta_i \left[1 - \sqrt{\left(1 + \frac{2\eta_c}{u^2}\right)}\right]. \quad (26)$$

At the floating electrode, the current due to the positive ions $j_i$ is balanced at the electrode by that of the electrons$j_e$ and negative ions$j_-$ . Thus we have

$$j_e + j_- = j_i, \quad (27)$$

Where the current due to the electron is express in the normalized form as

$$j_e = \frac{e^{-\eta_c}}{\sqrt{2\pi}(2\eta_c)^{3/2}}, \quad (28)$$

And the normalized current due to positive ions and negatively charged ions are respectively, as follows:

$$j_i = \frac{\delta_i \sqrt{2\eta_0 \lambda_i}}{\sqrt{2\eta_c}}, \quad (29)$$

$$j_- = \frac{\delta_- \sqrt{2\eta_0 \lambda_-}}{\sqrt{2\eta_c}}, \quad (30)$$

$\lambda_{i(-)} = m_e/m_{i(-)}$.Using **Eqs.(28)-(30)** in **Eq.(27)** we get the floating potential as

$$\eta_c = \frac{1}{2} ln\left(\frac{1}{2\pi\lambda_i}\right) - \left(1 + ln(1 + \delta)\right) \quad (31)$$

The first term of the RHS of **Eq.(31)** gives the floating potential without the negatively charged ions. The second term contains information about the plasma composition in the quasi-neutral region.The effect of the ion densities ratio and masses on the floating potential is given in **Figure 3**. It is seenthat for large concentration of negatively charged ion (larger value of$\delta$) the floating potential will be more negative. In addition, the heavier positive ions have the same effect. Indeed, the same figure reveals that the potential is reduced for heavier ions ($\lambda_i$). Thus our theoretical model may be applicable in RF plasma to decrease the potential difference between an electrode and the plasma.

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