Simplex Tool for Engineering Optimization

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Abstract— **Engineering design and analysis significant fields that are requiring a lot of support from computational methods. Decision support systems and tools those are readily amenable for the engineering community is need of the hour. Optimization algorithms that are specific to engineering applications need focus for enabling high performance computation. Algorithms that are simple enough to implement in desk based excel application has many advantages. Current research makes an attempt to develop an excel based algorithm that can readily be coupled with commercially available tools like ansys, abaqus, nastran leading to high performance in terms of decrease of no of iterations. Engineering optimization helps in meeting requirements of design and analysis of structures that are expected to operate under stringent operating conditions along with requirements on low weight and low operating cost. Genetic algorithm has been a powerful technique which is popularly used for optimization of engineering components. In current research an optimization tool based on genetic algorithm is developed with a random selection of initial population with cross over and mutation probability. A fitness function is build using function to be optimized and constraints like stress and deflection along with limits on variables based on available sizes of components. Typical cross sections for a beam like C-Section, I Section, T-Section are optimization for a given stress and strain. The results are compared with published results and are in agreement with published results. The developed tool enables the engineer with options for probability of mutation and cross over along with a termination criteria based on user choice. Standard benchmark problems like pressure vessel design, tension spring etc are solved using this simplex tool. This attempt could successfully develop a tool for performance improvement in engineering computation.**

Keywords— **Optimization, Simplex, GA, Computational performance**

I. Introduction

In recent times, engineering design is one of the significant fields that are requiring a lot of support from computational methods. Decision support systems and tools those are readily amenable for the engineering community is need of the hour. Optimization algorithms that are specific to engineering applications need focus for enabling high performance computation. Algorithms that are simple enough to implement in desk based excel application has many advantages. Current research makes an attempt to develop an excel based algorithm that can readily be coupled with commercially available tools like ansys, abaqus, nastran leading to high performance in terms of decrease of no of iterations. Standard benchmarks problems like pressure vessel design, tension spring etc can be solved using this simplex tool. This attempt could successfully develop a tool for performance improvement in engineering computation. one of the chief procedures involved in diverse fields of engineering or commerce, where objective is to minimize or maximize desirable set of properties pertaining to the problem, subjected to pre-defined constraints. The intricacy of the process depends on the objective and the type of problem. Majority of real life optimization problems are solved with the help of computers. Therefore people are more intrigued in devising new methods or algorithms for solving the optimization problems and develop the existing ones in terms of efficiency, accuracy or precision.

Genetic algorithm which is basically a search heuristic can be used for a broad spectrum of optimization problems. It has several applications in computational science, engineering, economics, chemistry and other fields of research and development. John Holland wrote the first book on Genetic Algorithms ‘Adaptation in Natural and Artificial Systems’ in 1975. After that many researchers have been making their debut in this field with their new approaches and perspectives. Continuous efforts are still carried out to improve the strengths and reduce the shortcomings of these algorithms.

This paper demonstrated basic working principles of genetic algorithm using some simple and standard examples. The GA approach developed under this project was applied to all the problems which enabled us to come up with a common simple tool in VBA (integrated with Excel). Essential conclusions were derived related to optimal GA parameters and strategies.

II. Need for a Simple Tool

There are some in built tools available using dedicated software. As a decision support a simple excel based tool would help a lot during initial phases of preliminary sizing of components. Many engineering calculation while coming out with a design alternative need a handy tool which can be amenable by user.

A. General Optimization Problem:

Several kind of optimization problems are encountered in different fields of research and development. Basic objective of such problems is to find a solution which is feasible for given set of conditions and suits best to our purpose in all respects.

In general key goal of such problems is to optimize a given function called as objective function. The various parameters involved in determination of concerned function are referred to as design variables and various conditions that are desired to be fulfilled in the process are called as constraints. There may be sometimes various set of design variables satisfying an optimum value of the function. One of the basic objective functions in structural optimization problems is weight.

General optimization problem can be formulated in mathematical form as follows:

- Objective: Minimize $f(x)$
- Design Variables: $x = (x_1, x_2, x_3, x_4, \dots\dots\dots, x_n) \in R^n$
- Constraints : Total number is 'p'
- Inequality Constraints - $g_i(x) \leq 0 \quad i = 1, 2, 3 \dots\dots\dots m$
- Equality constraints - $h_i(x) \leq 0 \quad i = m+1, m+2, \dots\dots\dots, p$

B. Objective Function:

It is the function which needs to be minimized or maximized for given optimization problem. In general the functions dealt by genetic algorithms are highly non-linear and complex in nature along with having several optimal solutions. In structural optimization problems weight minimization is one of the most important objectives. The problem can be single or multi-objective.

C. Design Variables:

These are parameters on which objective function depends. We need to determine the set of parameter values corresponding to optimal value of objective function. In optimization process we repeatedly change value of these parameters and compare the respective outcomes to reach the optimal solution. These variables can exhibit continuous or discrete values.

In structural optimization, there are three types of design variable. These are:

- Size design variables
- Shape design variables
- Topology design variables

D. Constraints:

These are conditions which check feasibility of the problem in all respects

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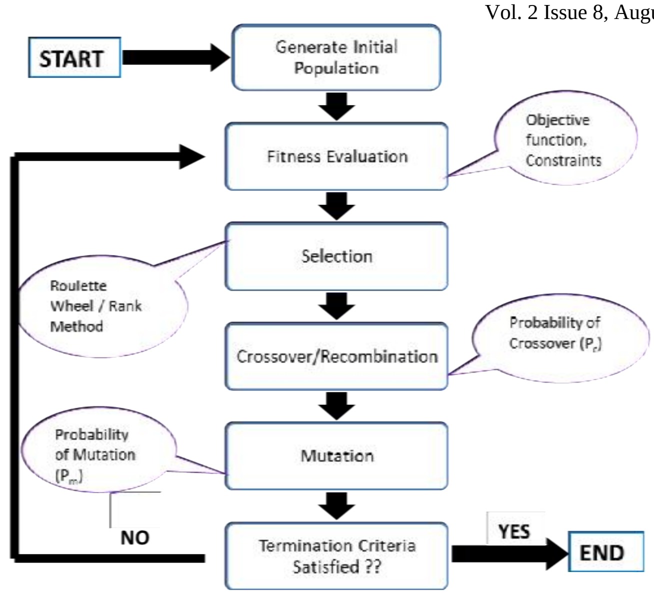

Fig 1. Structure of the tool

The constraints are basically of two type equality constraints and inequality constraints. The intricacy of the problem exceeds with the increase in number of constraints in the problem. These constraints may imply to the extremes of the objective function value or some other function(s) that depends on a subset of design variables and is meaningful for given problem. The values of design variables are also generally constrained. If the problem is having no constraints it is called unconstrained optimization problem while, the one with constraints is called constrained optimization problem.

In structural optimization problems the constraints are generally related to stress and deflection of their elements.

III. METHODOLOGY

A typical optimization problems starts with a choosing a right cross section for a given load scenario. Usually cross section dimensions are variables.

A. Optimization of T- and I-section Beams:

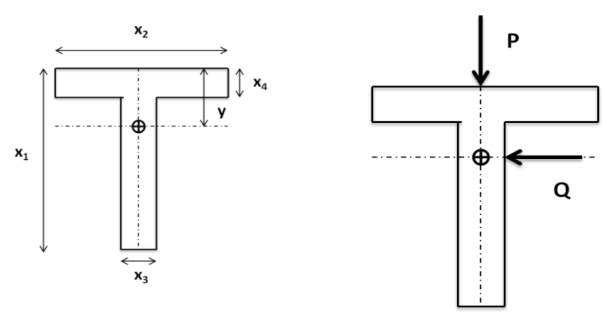

Fig 2: T, I Section Properties

The problem [1] consists of a simply supported beam with given loading conditions and our objective is to minimize the cross-sectional area of the beam (both T- and I-section) subjected to given stress and deflection constraints. Two loads P and Q are applied at center of the beam passing through centroid of the cross-section. Stresses and deflections due to bending are taken into consideration.

The following specifications are stipulated:

- Maximum allowable stress = 16 kN/cm^2
- Maximum allowable deflection = 0.10 cm
- Length (L) of the beam = 200 cm
- Loads: P = 75 kN vertical load and
- Q = 7.5 kN transverse force
- Young's Modulus of Elasticity (E) = 20,000 kN/cm^2
- Simply supported beam

Design elements, objective function and constraints are provided for individual sections given below.

T-section:

❖ **Design Elements:** There are 4 design variables in this problem (x_1 to x_4) which are different dimensions of the cross-section as shown in Figure below.

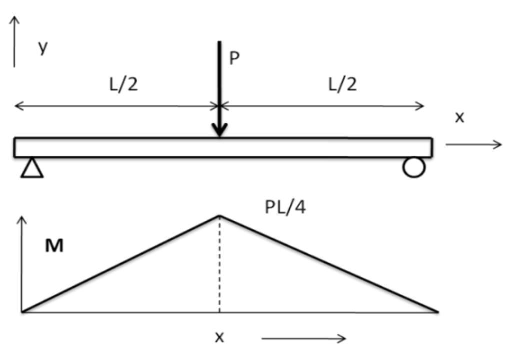

Fig 2. Simply supported beam with T-section under given loading conditions.

IV. Methodology

The design elements required for the problem are calculated as follows:

- Cross sectional area:

$$A = x_2 x_4 + x_3 (x_1 - x_4)$$

- Neutral axis from top of section:

$$\bar{y} = \frac{\frac{1}{2}\left(x_2 x_4^2 + x_3(x_1^2 - x_4^2)\right)}{A}$$

- Moment of inertia about the horizontal neutral axis (y axis) of section:

$$I_{yy} = \left[\frac{1}{12} x_2 x_4^3 + x_2 x_4 \left(\bar{y} - \frac{1}{2} x_4\right)^2\right] + \left[\frac{1}{12} x_3 (x_1 - x_4)^3 + x_3 (x_1 - x_4)\left(\frac{1}{2}(x_1 + x_4) - \bar{y}\right)^2\right]$$

- Moment of inertia about the vertical neutral axis (x axis) of section:

$$I_{xx} = \frac{1}{12}\left[(x_1 - x_4) x_3^3 + x_4 x_2^3\right]$$

- Maximum bending moment at center span due to P:

$$M_y = \frac{PL}{4} = 3750 \; kN \; cm$$

- Maximum bending moment at center span due to Q:

$$M_x = \frac{QL}{4} = 375 \; kN \; cm$$

- Maximum combined stress due to P and Q:

$$\sigma_{max} = \frac{M_y \; y}{I_{yy}} + \frac{M_x \; x}{I_{xx}} = \frac{3750 \; \bar{y}}{I_{yy}} + \frac{187.5 \; x_2}{I_{xx}}$$

❖ **Objective:** Minimize cross-sectional area of the beam i.e. the function given below.

$$A(x) = x_2 x_4 + x_3 (x_1 - x_4)$$

❖ **Constraints :**

- Limits of the dimensions (in cm):

$$5.28 \le x1 \le 47.50$$
$$10.31 \le x2 \le 42.49$$
$$0.71 \le x3 \le 2.84$$
$$0.88 \le x4 \le 5.11$$

- Stress constraint:

$$g_1(x) = \frac{3750 \; \bar{y}}{I_{yy}} + \frac{187.5 \; x_2}{I_{xx}} \le 16$$

- Deflection constraint:

$$g_2(x) = \frac{PL^3}{48EI_{yy}} = \frac{625}{I_{yy}} \le 0.1$$

Similar formulation was done for I-section beam optimization. The results are discussed on next section. Same problem is solved using ansys giving rise to same stress and deflection.

Figure 4 shows the user options for the developed tool which contains both GA parameters and engineering parameters.

Benchmark Problems:

Various problems are solved in optimization literature by different researchers to verify the effectiveness of their approaches (may be GA based or some mathematical programming technique) and show how they work. There are some problems which have been always solved by researchers to verify their algorithms. Two such optimization problems were solved using the GA technique formulated for T- and I-section problems. These are single objective problems with multiple constraints (both linear and non-linear).

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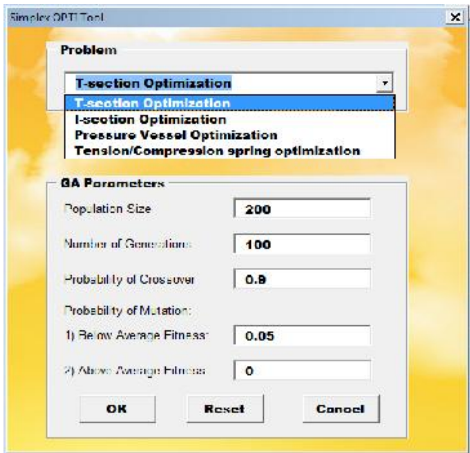


Fig 3. Simple structural optimization tool

This whole GA technique was coded in Visual Basic and integrated with M S Excel to output the results. One simple tool was created which enables user to simulate the optimization problem of his choice and takes important GA parameters as input. Snapshot of tool is given below.

Suggested GA parameters:

- Population size: 100 - 300
- Number of Generations: 100
- Probability of crossover: 0.6 – 0.9
- Probability of Mutation:
  1) Below Average fitness: 0.05
  2) Above average fitness: 0 or very low value (eg. 0.001)

Optimization of Pressure Vessel Design:

In this problem, the objective is to minimize the total cost, including the cost of material, forming and welding. A cylindrical vessel is capped at both ends by hemispherical heads as shown in Fig. 4.2. There are four design variables:

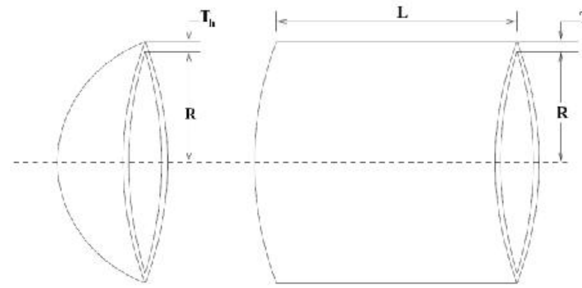
- T_s (x_1, thickness of the shell)
- T_h (x_2, thickness of the head)
- R (x_3, inner radius)
- L (x_4, length of the cylindrical section of the vessel, not including the head).

Among the four variables, T_s and T_h integer multiples of 0.0625 inch, which are the available thickness of rolled steel plates, and R and L are continuous variables lying in ranges [40,80] and [20,60] respectively. The problem can be formulated as follows:

Minimize: $f(x) = 0.6224\, x_1 x_3 x_4 + 1.7781\, x_2 x_3^2 + 3.1661\, x_1^2 x_4 + 19.84\, x_1^2 x_3$

Constraints:

- $g_1(x) = -x_1 + 0.0193\, x_3 \le 0$
- $g_2(x) = -x_2 + 0.0954\, x_3 \le 0$



- **Fig 4** Design Variables of pressure vessel

- $g_3(x) = -\pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 + 1296000 \le 0$
- $g_4(x) = x_4 - 240 \le 0$
- $g_5(x) = 1.1 - x_1 \le 0$
- $g_6(x) = 0.6 - x_2 \le 0$

Ranges of Design Variables' values:

$$1 \times (0.0625) \le x_1, x_2 \le 99 \times (0.0625)$$
$$40 \le x_3 \le 80$$
$$20 \le x_4 \le 60$$

Minimization of Weight of Tension/Compression Spring:

This problem is described in, and the aim is to

minimize the weight f(x) of a tension/compression spring (as shown in Fig 4.3) subject to constraints on minimum deflection, shear stress, surge frequency, limits on outside diameter and on design variables. The design variables are:

- The wire diameter d(x_1)
- The mean coil diameter D(x_2)
- The number of active coils P(x_3).

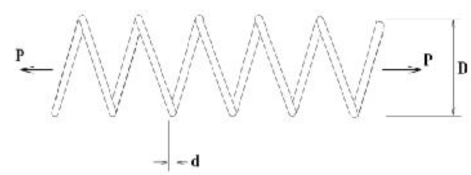


Fig 5 Design of spring

The mathematical formulation of this problem can be described as follows:

Minimize: $f(x) = (x_3 + 2)\, x_2 x_1^2$

Constraints:

- $g_1(x) = 1 - \frac{x_2^3 x_3}{71785\; x_1^4} \le 0$
- $g_2(x) = \frac{4x_2^2 - x_1 x_2}{12566\,(x_2 x_1^3 - x_1^4)} + \frac{1}{5108\, x_1^2} - 1 \le 0$

- $g_3(x) = 1 - \frac{140.45\, x_1}{x_2^2\, x_3} \leq 0$
- $g_4(x) = \frac{x_1 + x_2}{1.5} - 1 \leq 0$

Ranges of Design Variables' values:

$$0.05 \leq x_1 \leq 2.00$$
$$0.25 \leq x_2 \leq 1.30$$
$$2.00 \leq x_3 \leq 15.00$$

Table .1 Optimization comparison of T-section design

Results and Discussion

T-section Optimization:

Item	Nadela et. Al [1]	Present Method
x_1 (cm)	29.81	29.6
x_2 (cm)	13.4	12.8
x_3 (cm)	0.71	0.71
x_4 (cm)	0.88	0.96
Area (cm^2)	43.61	44.2288

All of the four optimization problems were solved using same GA technique. The algorithm was run 10

Times for each of them and the best and mean values are reported in this section.

Our GA algorithm checks the feasibility of output intrinsically. It only outputs feasible solution. For T-section problem the algorithm was run for population of 200 solutions and 100 generations for 10 times (using the Tool).

Every new run produces a new optimum value slightly different from others because of randomness involved in procedures of selection, crossover and mutation. While each time new initial population was generated, this played key role in bringing this variation in the optimized results per run. The best value obtained out of 10 runs iterated by tool is compared with that of Nadela et. al [1]approach.

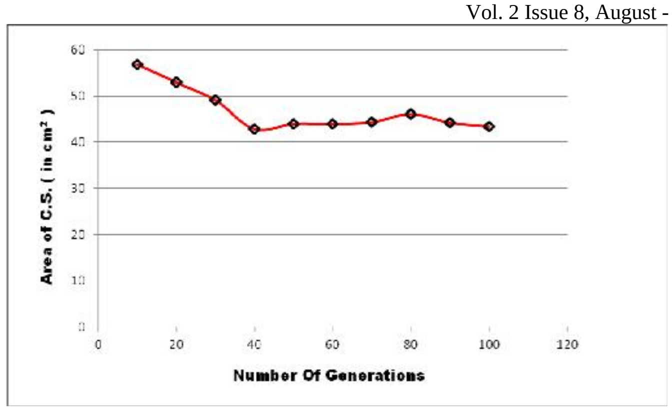


Fig 6 Solutions with no of runs.

The maximum stress constraint and deflection constraint (for mid-point) are properly satisfied. The average value of optimum area obtained in 10 runs is 45.628 cm^2.

The variation of value of optimized T-section area with number of generations is plotted as shown in Fig 6 .From this plot it is observed that for very small number of generations the value of optimum area is higher than that in case of larger number of generations. With increase in number of generations in the run it decreases rapidly up to some extent then, it varies within a particular band of area values. It implies that there exists a lower limit for number of generations below which we can't obtain best optimal solution. While at the same time there lies an upper limit above which no benefit can be acquired. Therefore it is necessary for user to input appropriate number of generation for a run.

Table .2 Optimization comparison of I-section design

Item	Nadela et. Al[1]	Present Method
x_1 (cm)	37.45	36.9
x_2 (cm)	17.15	13.9
x_3 (cm)	0.71	0.72
x_4 (cm)	0.88	1.28
Area (cm^2)	41.12	43.4384

I-section Optimization:

The algorithm was run for this problem in similar manner as for T-section problem using the Tool. The results obtained are compared with that of Nadela et. Al [1]. The variation of optimum area obtained with number of generations is plotted in the figure below.

The trend here is similar to that in case of T-section optimization. Similar interpretations can be derived in this case also. The best value obtained out of 10 runs iterated by tool is compared with that of nadela et al.

The average value of optimum area obtained in 10 runs is 46.675 cm^2.

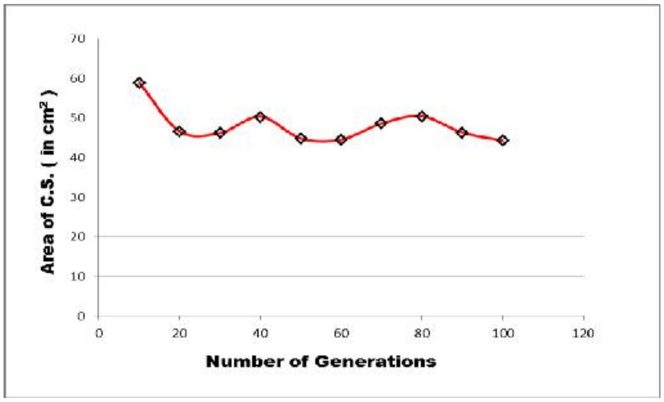


Fig 7. Variation of optimum area of I-section with no. of generations

Pressure Vessel Optimization [2]:

Same GA technique was used for pressure vessel optimization and obtained results were compared with those obtained by others with their approaches. The precision of values of design variables is up to 4th place after decimal.

Optimization of Spring[3]:

Results are obtained for 10 runs of 100 generations and 300 solutions in a population of solutions (shown in Table 5.3). Precision of individual values of design variables is up to 5 digits after decimal.

Table . 2 Optimization comparison of best solutions Pressure Vessel design

Item	Sandgren	Fu et. Al	Wu and Chow	HSIA	Present Method
x_1 (cm)	1.125	1.125	1.125	1.125	1.1875
x_2 (cm)	0.625	0.625	0.625	0.625	0.6875
x_3 (cm)	48.67	48.38	58.19	58.29	61.2522
x_4 (cm)	106.72	111.75	44.29	43.7	31.5713
$g_1(x)$	-0.179	-0.191	-0.001782	-.000003	-0.00533
$g_2(x)$	-0.1578	-0.163	-0.06979	-0.0689	-0.10315
$g_3(x)$	-3	-75.875	-974.58	-69.24	-39276.9
$g_4(x)$	-133.28	-128.255	-195.7	-196.3	-208.429
f(x)	**7982.5**	**8048.62**	**7207.50**	**7197.9**	**7870.323398**

Table.3 Optimization comparison of best solutions spring

Item	Belegundu	Arora	Coello	Present Method
x_1 (cm)	0.05	0.053396	0.05148	0.05488
x_2 (cm)	0.3159	0.39918	0.351661	0.43556
x_3 (cm)	14.25	9.1854	11.632201	8.01384
$g_1(x)$	N/A	N/A	-0.00208	-0.01694
$g_2(x)$	N/A	N/A	-0.00011	-0.00548
$g_3(x)$	N/A	N/A	-4.026318	-4.0699
$g_4(x)$	N/A	N/A	-4.02318	-0.67304
f(x)	**0.0128334**	**0.0127303**	**0.0127048**	**0.0131364**

V. SALIENT FEATURES OF GA TECHNIQUE

In the GA technique conventional static penalty method is adopted which imparts penalty to the unfeasible solutions in proportion to their degree of unfeasibility. This leads to high variation of fitness values among the individuals. That's why we used Rank method in place of Roulette Wheel method for selection procedure.

- The uniform crossover technique enabled sharing of information between mating parent solutions at bit level.

Mutation was not freely applied to whole population. It was controlled properly and applied according to the fitness value of new individuals generated after crossover operation. In this technique different probabilities of mutation were allotted for different ranges of fitness values. Those having fitness value

- Above average are mutated with probability as input by user (see the Tool).
- Though this value is advised to be kept very low (generally 0). The second input in tool under mutation probability is for individuals with fitness value below average but up to some range as a factor of average value. The solutions with fitness values too deviant from average are mutated with probability of 0.5. While those left in between are mutated with probability of 0.2.
- Important point to notice here is that the average fitness value mentioned above doesn't refer to that for new individuals generated from crossover operation but it is for the parent generation.

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References

[1] Federico M. Nadela and Jose Ernie C. Lope, *Comparative Strength of Common Structural* Shapes Using Genetic Algorithms, Member, IAENG

[2] Kalyanmoy Deb and Surendra Gulati, *Design of Truss-Structures for MinimumWeight using Genetic Algorithms*, KanGAL Report No. 99001, IIT Kanpur

[3] Leticia C. Cagnina and Susana C. Esquivel, Carlos A. Coello Coello, *Solving Engineering Optimization Problems with the Simple Constrained Particle Swarm Optimizer*, Informatica 32 (2008) 319–326.