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Single Objective Dynamic Economic Dispatch with Cubic Cost Functions using a Hybrid of Modified Firefly Algorithm with Levy Flights and Derived Mutations

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Abstract: The single objective dynamic economic dispatch (SODED) problem been formulated in quadratic form and solved extensively using pure and hybrid methods. SODED solution can be improved by introducing higher order generator cost functions since the fuel cost functions become more non-linear when the actual generator response is considered. Cubic cost functions models the actual response of thermal generators more accurately, thus it is an industry practice to adopt cubic polynomials for modelling fuel costs of generating units. Previous works have considered cubic static ED (SED).Therefore, there is need to consider the formulation of the dynamic SODED with all the possible constraints. Further the hybrid methods used in the solution of this vital problem need to be revisited and better ones developed. The modern trend of hybrids are the two-method and three-method hybrids. In this paper, cubic SODED is formulated and validated on IEEE 3-unit,5-unit and 26-unit systems using Modified Firefly Algorithm with Levy Flights and Derived Mutation(MFA-LF-DM).The proposed method proved better than Genetic Algorithm (GA), Particle Swarm Optimization (PSO) in determining optimal dispatch in the industry using the fully constrained SODED.

Key words: Cubic cost functions, Modified Firefly Algorithm with Levy Flights and Derived Mutation (MFA-LF-DM), Single objective dynamic economic dispatch (SODED)

I: INTRODUCTION

Economic Dispatch (ED) with cubic cost functions has been extensively studied in the past researches. According Z.X Liang and J.D Glover **,**1991[1],a very crucial issue in SODED studies is to determine the order and approximate the coefficients of the polynomial used to model the cost function. This helps in reducing the error between the approximated polynomial along with its coefficients and the actual operating cost. According to Z.X Liang and J.D Glover,1992 [2] and A.Jiang and S.Ertem,1995 [3] to obtain accurate SODED results, a third order polynomial is realistic in modelling the operating cost for a non-monotonically increasing cost curve. SODED works using cubic cost functions include Bharathkumar.S et al, 2013[4], Hari M.D et al**,** 2014[5], Deepak Mishra et al, 2006[6], and N.A.Amoli et al, 2012[7]. Krishnamurthy**,** 2012 [8] used the static cubic function of the emissions dispatch in the Multi Objective Static ED(MOSED) using the Lagrange method(LM).This provided better results as compared to the quadratic functions. In all these studies, however, the cubic cost function provided more accurate and practical results as compared to lower order cost functions. A summary of ED works using cubic cost functions is provided in Table 1.0, from which, it is clear that static cubic cost functions have been considered in a great extent. Only B.S et al, 2013 [4] has considered the SODED, thus there is need to consider the SODED with all the possible constraints in place**.** The thermal cost functions has been considered with only the work in [4], [5] and [8] incorporating emission cost functions**.** Further, the pure heuristic deterministic methods which are strong and weak at the same time have been applied, only the works in [5] have considered a two method hybrid. Thus there is need to use more advanced hybrid methods for better results in these vital and complex cubic cost functions.

Contribution: In this paper a fully constrained dynamic SODED (with ramp rates, valve points and prohibiting zones) with cubic cost function is formulated. A new method, Modified Firefly Algorithm (MFA) and its hybrids is proposed for its solution. These hybrids include MFA with Levy Flights (MFA-LF) and MFA-LF with Derived Mutation (MFA-LF-DM). The results are compared with those for pure methods, for example, Genetic Algorithm (GA) and Particle Swarm Optimization (PSO). Cubic and Quadratic cost functions results are also compared and presented.

II: PROBLEM FORMULATION

Economic dispatch (ED) may sometimes be classified as a static optimization (SOSED) problem in which costs associated with the act of changing the outputs of generators are not considered.

According to Jizhong Zhu, pp. 87-88, (2009) [13], the single objective function corresponding to the production cost can be approximated to be a quadratic function of the active power outputs from the generating units. This static ED (SED) is formulated as

$$\min F = a_i P_i^2 + b_i P_i + C_i \qquad (1)$$

A general formulation for the n^{th} order SOSED was proposed by Yusuf Sonmez, 2013[9]**.** It can be given by the equation

$$\mathrm{F}(P_{ij}) = a_{0,i} + \sum_{j=1}^{L=n} a_{ji}\, P_{t,i}^{j} + r_i \qquad (2)$$

Table 1.0. ED with Cubic Cost Functions.

Reference	Ob	Nature Ob function	Con	Method
Z.X Liang et al,1991[1]	T	Static	-	Gram-Schmidt(GS), Least Squares(LS)
Z.X Liang et al,1992 [2]	T	Static	3	Dynamic programming(DP)
A.Jiang and S.Ertem,1995 [3]	T	Static	2	Newton Method(NM)
B.S et al ,2013[4]	T,E	DED with ramp rates and valve points	4	Fuzzy Logic (FL), Bacteria Foraging(BF) and Nelder-Mead(NM) (FL-BF-NM)
Hari Mohan D.et al,2014[5]	T,E	Static	5	PSO-General Search Algorithm (PSO-GSA)
Yusuf Somez,2013[9]	T	General static	2	Artificial Bee Colony (ABC)
Deepak Mishra et al,2006 [6]	T	General static	2	OR-Hopfield Neural Network(ORHNN)
N.A Amoli et al,2012 [7]	T	Static	2	Firefly Algorithm(FA)
Krishnamurthy .S et al ,2012[8]	T,E	Static	2	Langrange Method(LM)
T. Adhinarayanan M.Sydulu,2006[10]	T	Static	2	Lambda-logic based(LLB)
T.Adhinarayanan M.Sydulu,2010 [11]	T	Static	2	Lambda-logic based(LLB)
E.B Elanchezhian et al,2014[12]	T	Static	8	Teaching learning based optimization (TLBO)

Key: Ob-Objective function, T-Thermal cost objective function-Emissions cost objective function, Con-Number of constraints

On the other hand, dynamic SODED is one that considers change-related cost and takes the ramp rate limits, valve points and prohibited operating zone of the generating units into consideration. The general form of the SODED is given by

$$F(P_{ij}) = \left\{a_{0,i} + \sum_{j=1}^{L=n} a_{ji}\, P_{t,i}^{j} + r_i\right\} + \left|e_i \sin f_i(P_i^{min} - P_i)\right| \quad (3)$$

Where $a_{0,i}, a_{j,i}, e_i$ and f_i are the cost coefficients of the ith unit, $P_i^{min}$is the lower generation bound for it unit and $r_i$is the error associated with the ith equation.

When L=1 the linear form of the SODED results.

$$F(P_{i,1}) = a_{1,i}P_{t,i} + a_{o,i} + r_i + \left|e_i \sin f_i(P_i^{min} - P_i)\right| \quad (4)$$

This is also called the first order model this is of no practical significance ED studies.

When L=2, the most popular quadratic SODED results. This is given by

$$F(P_{i,2}) = a_{2,i}P_{t,i}^2 + a_{1,i}P_{t,i} + a_{o,i} + r_i + \left|e_i \sin f_i(P_i^{min} - P_i)\right| \quad (5)$$

When L=3, the cubic form of the SODED results. This can be expressed as

$$F(P_{i,3}) = a_{3,i}\, P_{t,i}^3 + a_{2,i}P_{t,i}^2 + a_{1,i}P_{t,i} + a_{o,i} + r_i + \left|e_i \sin f_i(P_i^{min} - P_i)\right| \quad (6)$$

The problem in equation (6) is solved subject to the following constraints:

$$\sum_{i=1}^{N} P_{gi} = P_D + P_L \quad (7)$$

$$P_i^{min} \le P_i \le P_i^{max} \quad (8)$$

$$P_{ij} - P_{ij-1} \le UR_i \quad (9)$$

$$P_{ij-1} - P_{ij} \le DR_i \quad (10)$$

$$-P_l^{max} \le P_{lj} \le P_l^{max} \quad l = 1,2,3 \ldots\ldots L \quad (11)$$

$$P_i \le P^{PZ,LOW} \quad (12)$$

$$P_i \ge P^{PZ,HIGH} \quad (13)$$

III: PROPOSED METHODOLOGY

Introduction to Fireflies

The fireflies are the most charismatic species among the insects and their spectacular display have inspired the poets, writers and scientists. Today more than 2000 species exists and the flashings of the fireflies can be seen in the summer sky in the tropical and temperate regions with warm weather and most active in the nights [15]. These fireflies produce the short rhythmic patterns of flashing lights and these patterns of flashes are unique in species to species, and the flashing light is produced by a *bioluminescence* process. Moreover, flashing is produced to attract their *mating partners*; the first signalers are flying males who tries to attract the females on ground. In response females also emit flashing lights and move towards the brightest firefly. However the flashing lights obey certain physical rules, the light intensity, I , decrease with the increase of distance r according to the term $I\alpha 1/r^2$ [16] .Also the flashing is produced for *communication* purpose among each other and also to *attract prey*, but still the flashing behavior is a topic of discussion among scientists and engineers. Thus the flashing behavior of fireflies plays a key role in reproduction, protection, communication and feeding.

Firefly Algorithm (FA)

Firefly Algorithm (FA) [14] is a new nature inspired algorithm developed by Xin-She Yang in the year 2007, based on the flashing behavior of the fireflies. The flashing signifies the signal to attract other fireflies, where the *objective function* is associated with the flashing light or the light intensity which helps the fireflies to move to brighter and more attractive locations to achieve optimal solution.

The FA has three idealized rules or assumptions which are been developed to define the characteristics of fireflies: i) All fireflies are unisex and they move towards the more attractive and brighter one irrespective of their sex. ii) The level of attraction of firefly is proportional to brightness which reduces with the increase in the distance between two fireflies $I\alpha 1/r^2$ since air absorbs the light. If there is no

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brighter or more attractive firefly than a particular one, it will then move randomly. iii) The brightness or light intensity is determined by the value of the *objective function* of a given problem and it is proportional to the light intensity for a maximization or optimization problem.

Need for Improved FA (IFA)
The reasons behind making FA [16] so popular and successful include: i) The method automatically divides its population into subgroups, because of the fact that local attraction is stronger than long distance (global) attraction. ii) FA does not use historical individual best and explicit global best. This reduces the potential drawbacks of premature convergence. iii) Also FA does not use the velocities hence problems associated with velocities in PSO is automatically eliminated. iv) FA has an inbuilt ability to modify and therefore to control the parameters such as γ, leading to improved results. Hence it can be clearly seen that the FA is more efficient in respects of controlling parameters, local search ability, robustness and elimination of premature convergence.

N.A Amoli et al, 2012 **[7]** used the basic FA in solving static ED with cubic cost functions. However, the method is poor in global searching and optimization, long convergence time, requires more iterations, and low computational speed .These problems can be addressed by using modified (improved) FA [17] and using heuristic and deterministic methods to form hybrid FA [18]. In a hybrid method the weaknesses of the base method are suppressed while its strengths are exalted leading to better realistic results and improved performance of the method. In this paper therefore, a hybrid of Modified FA (MFA) with Levy-Flights (LF) [MFA-LF] coupled with Derived Mutation (DM); [MFA-LF-DM] is proposed.

Modified Firefly Algorithm (MFA) with Levy Flights (LF) and Derived Mutation (DM) [MFA-LF-DM]
The MFA-LF-DM proposed in this paper has six operators. These include brightness, distance, attractiveness, movement, randomness reduction and mutation. These are formulated as follows:

i) Brightness
The brightness I of a firefly at a particular location x can be chosen as

$$I(x) \propto f(x) \qquad (14)$$

ii) Distance
The distance between any two fireflies i and j at x_i and x_j respectively is the Cartesian distance

$$r_{ij} = \|x_i - x_j\| = \sqrt{\sum_{k=1}^{d}(x_{i,k} - x_{j,k})^2} \qquad (15)$$

Where $x_{i,k}$ is the k^{th} component of the spatial component x_i of the i^{th} firefly

The minimum distance between any two fireflies i and j at x_i and x_j is thus given by

$$r = \min r_{ij} = \min\|x_i - x_j\| \qquad (16)$$

iii) Attractiveness
The attractiveness β between two fireflies i and j at a separation distance r_{ij} is given by

$$\beta = \beta_0 e^{-\gamma r^2} \qquad (17)$$

Where β_0 is the attractiveness at $r = 0$.

In actual implementation, the actual implementation $\beta(r)$ is a monotonically decreasing function generalized as

$$\beta(r) = \beta_0 e^{-\gamma r^m} \; m \geq 1 \qquad (18)$$

iv) Movement
The movement of a firefly i is attracted to another more attractive (brighter) firefly j by the relation

$$x'_{i+1} = x_i + \beta_0 e^{-\gamma r_{ij}^2}(x_i - x_j) + \alpha \operatorname{sign}\left[rand - \frac{1}{2}\right] \qquad (19)$$

Where x_i is the current position of a firefly,the second term defines the fireflies attractiveness to light intensity as seen by the adjacent firefly and the third term is for the random movement of a firefly is no brighter firefly is left, α is a randomization parameter, and is a random number generator uniformly distributed over the space [0,1],that is, $rand \in [0,1]$.

In general the solutions can be improved by reducing the randomness by

$$\alpha = \alpha_\infty + (\alpha_0 - \alpha_\infty)e^{-t} \qquad (20)$$

Where $t \in [0, t_{max}]$the pseudo is time for simulation and t_{max} is the maximum number of generations, α_∞ and α_0 are the final and initial values of the randomness parameter

v) Randomness Reduction
Levy flight is a random walk of step lengths having direction of the steps as isotropic and random. The concept propounded by Paul Pierre Levy (1886-1971) is very useful in stochastic measurements and simulations of random and pseudo-random phenomena.

The movement of a firefly i with Levy Flights is defined by the relation

$$x'_{i+1} = x_i + \beta_0 e^{-\gamma r_{ij}^2}(x_i - x_j) + \alpha \operatorname{sign}\left[rand - \frac{1}{2}\right] \oplus \text{Levy} \qquad (21)$$

Where the second term is due to attraction, while the third term is randomization via the Levy Flights with α being the randomization parameter.The product ⊕ means entry wise multiplication
The sign $\left[rand - \frac{1}{2}\right]$ where $rand \in [0,1]$ essentially provides a random sign or direction while the random step length is drawn from a Levy distribution given by

$$Levy \sim u = t^{-\lambda}, (1 < \lambda \leq 3) \qquad (22)$$

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Which has an infinite variance with an infinite mean

vi) Derived Mutation (DM)

To further improve the exploration of or diversity of the candidate solution, the simple mutation corresponding to α from the ant colony optimization (ACO) genetic algorithm (GA), evolutionary programming (EP) and differential evolution (DE)algorithms is adopted in the MFA-LF process. This enhances the accuracy of the optimum results in solving the fully-constrained SODED problem.

3.2 Algorithm for MFA-LF-DM

The proposed MFA-LF-DM algorithm is implemented using the following procedure:

Step 1: Define objective function $f(x)$.

Step 2: Read the system data, cubic cost coefficients, loss coefficients, minimum and maximum power limits of all the generating units and power demand
Step 3: Input the algorithm parameters- randomness (α), attractiveness(β), light absorption coefficient(γ), randomness reduction parameter (λ) , number of fireflies (n), maximum iterations, and stopping criteria.
Step 4: Generate initial population of fireflies $x_i(\ i = 1,2,3 \ldots.n)$ in a random manner

Step 5: Set the iteration counter to 1
Step 6: Evaluate the light intensity I_i or function value at x_i by value of $f\ (x_i)$.
Step 7: while $(t\ < Max\ Generation)$

for$i = 1$*: n all n fireflies*
for $j = 1$ *all n fireflies*
if $I_j > I_i$
Move firefly i towards j in d-dimension via Levy flights
end if

Find the minimum variation distance of all fireflies

$r = min\ (\sum(firefly\ i - firefly\ j))$
Attractiveness varies with distance r via $\exp[-\gamma r]$
Evaluate new solutions and update light intensity
end for j
end for i

Random

Mutation if $random < probability\ of\ mutation$
Rank the fireflies and find the current best
end of while loop

Step 8: Post process results and visualize the same
Step 9: Find the firefly with the highest light Intensity among all fireflies, Gbest
Step 10: Plot the increase of light intensity with time per iteration
Step 11: Plot the objective with respect to time, % best solution with time
Step 12: End of MFA-LF-DM

IV: RESULTS AND ANALYSIS

In this method the initial solution is generated randomly within the feasible range, The FA parameters used in the problem are as shown in table 2.0.The mapping of the parameters to the SODED problem is also given .

The cubic cost coefficient, maximum and minimum power limits, ramp rates and valve points have been taken from [4] and [10].A lossless system is assumed. The results are divided into four parts: SOCED and SODED comparison, ED with cubic cost function under various demands, ED with Cubic and quadratic cost functions and finally a comparison of MFA-LF-DM with other methods in solving the SODED with cubic cost functions.

TABLE 2.0 Parameters for MFA-LF-DM

Parameter	Value
Brightness	$F(x)$
Alpha (α)	0.9
Beta (β)	0.5
Gamma (γ)	1.0
Number of fireflies (n)	50
Maximum no. of iterations	100
Attraction at $r = 0$, (β_0)	2.5
Lambda (λ)	1.5

SOCED and SODED

The optimal generation of the six generating units and the optimal costs are displayed for each of the intervals. The algorithm is first run without any constraints and the optimization does not include the ramp rate constraints, that is, the algorithm is run to optimize a classic economic dispatch problem. The algorithm is then run to solve the classic economic dispatch with minimum generation constraints.

Modifications are then done to include maximum generation constraints. Finally, the algorithm is run to include the inequality, equality and ramp rate constraints. The algorithm optimizes a dynamic economic dispatch problem. The power demand for each interval is taken as [150MW, 300MW, 400MW 500MW]. The key used in interpreting the results in this section include *A: SOCED without constraints, B: SOCED with min generation constraints, C: SOCED with min and MAX constraints, D: SODED with valve points and ramp rate limits. Further, in the tables,t represents computation time(seconds),n ,the number of iterations,L,losses(MW) and C,the optimal cost($)*. From the results tabulated in Table 3.0-5.0, it is clear that the optimal cost increases with the power demand. The cost of operation is directly proportional to the power demand. The cost is highest for the SOCED, then slightly less for the SOCED with minimum generation constraints, lesser when the algorithm is used for SOCED with max and min generation constraints and the cost is least when SODED is used with valve point effects and ramp rate generation constraints. The difference in the optimal cost is more pronounced at higher power demand. This is so because at lower power demand only minimum generating constraints are violated hence the costs tend to be similar. At higher power demands, line constraints and max generation constraints are violated hence the need to keep them in check by not overloading the generators. This is done by defining the highest power demand that a generator can supply.

Table 3.0: Results for a load demand of 150 MW

Unit	A	B	C	D
G1	45.9510	45.9510	45.9510	45.9510
G2	34.4019	34.4019	34.4019	34.4019
G3	19.8484	19.8484	19.8484	19.8484
G4	9.8010	9.8010	9.8010	9.8010
G5	11.0204	11.0204	11.0204	11.0204
G6	15.9781	15.9781	15.9781	15.9781
L	0.3010	0.3025	0.3055	0.3100
t	2.5	2.6	2.8	3.0
n	10	11	12	15
C	489.0303	452.9328	452.9328	454.3845

Table 4.0: Results for a load demand of 300 MW

Unit	A	B	C	D
G1	95.0530	95.0530	95.0530	95.0530
G2	71.1643	71.1643	71.1643	71.1643
G3	41.0587	41.0587	41.0587	41.0587
G4	20.2745	20.2745	20.2745	20.2745
G5	22.7970	22.7970	22.7970	22.7970
G6	33.0525	33.0525	33.0525	33.0525
L	0.6050	0.6085	0.6090	0.7100
t	2.6	2.7	3.0	3.2
n	12	15	17	18
C	927.6174	927.6174	891.7733	886.5009

Table 5.0: Results for a power demand of 400MW

Unit	A	B	C	D
G1	125.5410	125.5410	125.5410	125.5410
G2	93.9901	93.9901	93.9901	93.9901
G3	54.2282	54.2282	54.2282	54.2282
G4	26.7775	26.7775	26.7775	26.7775
G5	30.1091	30.1091	30.1091	30.1091
G6	43.6540	43.6540	43.6540	43.6540
L	1.0500	1.0580	1.1000	1.2000
t	2.6	2.9	3.9	4.1
n	13	16	17	19
C	**1266**	**1266**	**1139**	**1,084.8**

The computation time and the number of iterations increase with system demand. It should be noted that the parameter gamma (γ) which is set to 1.0 in this case characterizes the variation of the attractiveness, beta β ,and it is very crucial in determining the speed of convergence and how the MFA-LF-DM behaves. Theoretically, $\gamma\epsilon[0,\infty$ but in practice $\gamma\epsilon O(1)$ and is determined by the characteristic length of the system to be optimized. By varying γ the computation speed can be improved

Table 6.0: Results for a load demand of 500MW

Unit	A	B	C	D
G1	160.7583	160.7583	160.7583	160.7583
G2	120.35	120.35	120.35	120.35
G3	69.44	69.44	69.44	69.44
G4	34.2829	34.2829	34.2829	34.2829
G5	38.5554	38.5554	38.5554	38.5554
G6	55.900	55.900	55.900	55.900
L	1.8010	1.8080	2.1015	2.3050
t	3.0	4.0	4.5	5.0
n	15	17	18	20
C	**1719.8**	**1719.8**	**1221.6**	**1,209.9**

4.2 ED with Cubic Cost Function under Various Demands

With the demand of [500MW, 600MW, 700MW, 800MW], the results for the cubic cost function under various demands are tabulated in Table 7.0 .In this case the IEEE 6-unit system is used. The optimal cost and the losses increase with power demand. However the Computation time and the number of iterations are not affected by demand in a great extend

Table 7.0: SODED with cubic cost function under various demands

[MW]	500	600	700	800
G1	48.7954	56.9279	65.0605	73.1931
G2	37.8673	44.1786	50.4898	56.8010
G3	21.4006	24.9673	28.5341	32.1009
G4	11.3017	13.1856	15.0689	16.9525
G5	12.7317	14.8537	16.9757	19.0976
G6	17.9033	20.88	23.8711	26.8549
L	2.3065	2.3090	2.5000	2.9950
t	5.5	5.8	6.2	6.0
n	15	18	18	20
C	1,977.1	3,523.3	4,211.0	4,951.29

SODED with Quadratic and Cubic Cost Functions

.The Algorithms were tested with 3 unit, 5 unit and 26 unit test systems and the results compared with the basic methods; FA, MFA, and MFA-LF. The system demands considered are 850MW, 1800MW, 2000MW and 2500MW. The results presented are for the 2500MW demand.

From the results in Table 8.0 it, it is clear that the cubic cost functions provide better and more realistic costs (higher costs) than the quadratic cost functions. The MFA-LF-DM method gave the best optimal results as compared to the FA, MFA and MFA-LF.

SODED with Cubic Cost Functions

Further comparison was done using the 5-unit and the 26-unit systems .The results are as tabulated in table 9.0 -10.0. The results are compared with those in [5] since this is the only work that has considered cubic cost functions in DED. From these tables, it can be observed that optimal cost in industrial power systems increases with the complexity of the system. Further, the system losses also are directly proportional to the system size. The execution time and the number of iterations don't vary to a great extend with the system size and the nature of cost function. It is worth noting that the MFA-LF-DM provide better optimal costs, losses and total output power than all the lower versions of FA, GA and PPSO.

V: CONCLUSION

The objective of this paper was to propose a method for solving SODED with cubic cost functions. Cubic cost functions provided more realistic higher costs which are applicable in an industrial setting in a fully constrained environment. MFA-LF-DM proved effective than FA, MFA, MFA-LF and the basic heuristic methods in the solutions of the industrial cubic SODED, which is a good example of NP hard problems. The pure MFA is also found to be more effective than GA and PSO in cost optimization. This effectiveness is measured in terms of efficiency and success rate. MFA-LF-DM has been found to be very efficient, however a further improvement on the convergence can be achieved by carrying out sensitivity

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studies by varying of parameters such asβ_0,γ,α and more interestinglyλ. Other than mutation, other operators of the biologically inspired heuristic methods can also be considered. For more realistic results, a multi objective dynamic economic dispatch (MODED) problem with thermal cubic cost functions need to be considered. That is, the SODED problem need to be considered simultaneously with Renewable energy, transmission losses and emissions. The security and power wheeling aspects under SODED and MODED with higher order cost functions may form an exciting area for further research

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Table 8.0: Cubic Cost Function on 3-Unit System

	Quadratic				Cubic			
Unit	FA	MFA	MFA-LF	MFA-LF-DM	FA	MFA	MFA-LF	MFA-LF-DM
1	393.170	393.170	393.170	393.169	725.02	724.99	724.99	724.99
2	334.604	334.604	334.603	334.603	910.19	910.19	910.19	910.19
3	122.226	122.226	122.226	122.226	864.88	864.88	864.88	864.88
L	850.00	850.00	850.00	850.00	2,500.00	2,500.00	2,500.00	2,500.00
t	5.0	6.0	6.5	7.0	5.2	6.7	7.0	8.0
n	60	58	52	50	68	62	55	53
C	8,194.35	8,194.35	8,193.30	8,193.20	12,730.14	12,729.35	12,728.15	12,728.05

Table 9.0: Five Unit System with Static Cubic Cost Functions

Unit	GA[5]	PSO[5]	MFFA	MFFA-LF	MFFA-LF-DM
1	320.00	319.90	320.00	320.05	320.10
2	343.74	343.70	343.73	343.70	343.74
3	472.60	472.50	472.40	472.45	472.68
4	320.00	320.08	319.95	320.00	320.00
5	343.74	343.77	343.65	343.74	343.74
L	1800.00	1800.00	1800.00	1800.00	1800.00
t	8.5	9.5	8.5	9.0	10.0
n	72	68	60	55	53
C	18,611.07	18,610.40	18,609.35	18,609.05	18,608.65

Table 10.0: 26-Unit System with Static Cubic Cost Functions

Unit	GA[5]	PSO[5]	MFFA	MFFA-LF	MFFA-LF-DM
1-9	2.40	2.40	2.40	2.40	2.40
10-12	15.20	15.20	15.20	15.20	15.20
13-16	25.00	25.00	25.00	25.00	25.00
17	129.71	124.69	124.69	124.69	124.69
18	124.71	124.69	124.69	124.69	124.69
19	120.42	120.40	120.40	120.40	120.40
20	116.72	116.70	116.70	116.70	116.70
21-23	68.95	68.95	68.95	68.95	68.95
24	337.76	337.85	337.85	337.85	337.85
25-26	400.00	400.00	400.00	400.00	400.00
L	2000.00	2000.00	2000.00	2000.00	2000.00
t	24	26	20	22	25
n	95	92	90	88	85
C	27,671.24441	27,671.2276	27,671.3926	27,672.1113	27,672.3345

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