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Small Scale Axial Turbine Preliminary Design and Modelling

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***Abstract*- Purpose: This paper presents the preliminary design and modelling of an axial turbine suitable for use in a small to medium level, low temperature, solar thermal, organic Rankine cycle (ORC).The work involves thermodynamic and geometrical design and analyses. Empirical loss correlations are used to account for the different kinds of losses. The engineering equation solver (EES) is used to perform the thermodynamic analysis. 2D and 3D computational fluid dynamics (CFD) simulation and the aerofoil design are not done at this stage as they require specialized CFD software. The work is meant to contribute towards the development of optimal cost-effective turbine expanders suitable for small to medium sized operation at low to medium temperatures. There is not sufficient evidence to show that significant research and development work has been done with regard to turbomachinery design and development for small to medium size low temperature organic Rankine cycle (ORC) systems. Most turbine manufacturers and developers put more emphasis on larger scale models in the Megawatts (MW) ranges while most researchers who have shown interest in micro-scale operations in the low temperature applications have concentrated their efforts more on thermodynamic studies regarding the power cycle, and on the proper rules for the selection of the working fluids, with special attention to the power plant efficiency; others have made attempts at adaptation and modification of other equipment, especially positive displacement machines, for use as ORC expanders. A turbine design suitable for small scale and low temperature operation based on the ORC thermodynamic cycle is required because the operating conditions such as speed, flow rate, pressure ratio, etc. are quite different from those of conventional steam and gas turbines; also the properties of the organic fluids used as working fluids are different from those of the conventional steam or fossil-fuel-gas mixtures.**

Keywords: ORC thermodynamic cycle, preliminary design and modelling, thermodynamic and geometrical design, EES

***Paper type*:** Research Paper

Nomenclature

Roman symbols

b	blade height
c_p	isobaric specific heat capacity
d	mean diameter
e	internal energy per unit mass of the medium
$\bar{G}$	gravitational acceleration, electromagnetic acceleration, etc.
h	specific (static) enthalpy (J/kg)
I	rothalpy
$\bar{\bar{I}}$	identity matric equals the kronecker unit tensor
k	kinetic energy per unit mass of the medium
$\dot{m}$	mass flow rate (m_dot in EES code)
P	pressure
PR	pressure ratio
$\bar{Q}$	conductive heat flux
r	mean blade radius (m)
R_r	rotor degree of reaction (also R_s in EES code)
R_n	nozzle degree of reaction
s	entropy
t	time
T	temperature
U	blade velocity (m/s)
$\bar{U}$	fluid velocity vector
V	absolute fluid velocity (m/s)
V_a	axial component of velocity V
V_u	tangential component of velocity V
W	relative velocity of fluid flow to moving blades (m/s)
Ws	specific shaft work
$\dot{w}$	specific shaft work
$\dot{W}_t$	shaft work; turbine work

Greek symbols

α	absolute flow angle
β	relative flow angle
∂	partial differential operator
Φ	flow coefficient
η_{tt}	total-to-total efficiency (eta_tt in EES code)
η_{ts}	total-to-static efficiency (eta_ts in EES code)
θ	tangential (circumferential) component
ρ	specific mass; density
$\bar{\bar{\tau}}$	stress tensor
ω	shaft angular velocity (rad/s)
Ψ	work coefficient (also blade loading)

mathematical operators

∇	gradient operator

numbers

1	inlet to machine; inlet to nozzle blades
2	outlet from machine; inlet to rotor blades
3	outlet from rotor blades
01	stagnation state at station 1
02	stagnation state at station 2
02_{rel}	relative stagnation state at station 2
03_{rel}	relative stagnation state at station 3
2’	static state at station 2 after an isentropic expansion in the nozzle

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3’ static state at station 3 after an isentropic expansion through entire stage
3” static state at station 3 after an isentropic expansion in the rotor blades only
03’ relative stagnation state at station 3 after an isentropic expansion through entire stage

I. INTRODUCTION

In this paper we present work done on the research and development of a turbine suitable for a low temperature solar thermal conversion cycle based on the organic Rankine cycle (ORC). The turbine is the single most critical component in a thermal conversion cycle. The ideal solution should be characterized by maximum efficiency, small footprint, and minimum shaft speed (Cooper *et al*, 2010). Although the research considered all the three possible architectures: single stage radial turbine – cantilever type; single stage radial turbine – Ninety Degrees In-Flow Radial turbine (90° IFR); and single stage axial turbine, this paper only presents findings on the latter.

The turbine design process can be broken down into three stages:

- Preliminary Design (PD);
- Meanline/Streamline (1D/2D) Analysis and Optimization
- Profiling, 3D Blade Design, 3D Modelling and Analysis

Preliminary Design involves finding the optimal flow path, number of stages and distribution of geometrical parameters (heights and angles) based on the given thermodynamic conditions at turbine inlet and outlet. This process can further be subdivided into two tasks:

- initial enthalpy drop distribution: this entails determining the optimal number of stages and appropriately distributing the enthalpy drop between them and finding the first approximation of flow path geometry paths; and
- adjusting design calculations (inverse calculation task): this entails calculation of turbine main performance characteristics as well as exact thermodynamic and kinetic parameters basing on initial enthalpy drop distribution results.

Initial design parameters are the inlet working fluid conditions (pressure, temperature, and enthalpy), outlet pressure, mass flow rate and rotational speed.

To fully develop a final working turbine model, the following factors are of paramount importance:

- manufacturing and material specifications of the rotor and nozzle;
- structural and aerodynamic design of the rotor and nozzle; and
- specifications of the inlet and outlet parameters such as pressures and temperatures.

II. THEORY OF TURBOMACHINERY

Fluid dynamics and hence turbomachinery theory is based on three fundamental principles of conservation of mass (continuity), conservation of momentum and conservation of energy, represented by the following equations [2]:

conservation of mass (continuity):

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\bar{U}) = 0 \quad (1)$$

conservation of momentum:

$$\frac{\partial\rho\bar{U}}{\partial t} + \nabla\cdot\left(\rho\bar{U}\bar{U} + P\bar{\bar{I}} - \bar{\bar{\tau}}\right) - \rho\bar{G} = 0 \quad (2)$$

conservation of energy:

$$\frac{\partial\rho(e+k)}{\partial t} + \nabla\cdot\left(\rho\bar{U}\left(e + \frac{P}{\rho} + k\right) - \bar{\bar{\tau}}\cdot\bar{U} + \bar{Q}\right) - \rho\bar{G}\cdot\bar{U} = 0 \quad (3)$$

The work done by a turbomachine can be represented by the Euler turbine equation which can be written as (Ingram, 2009):

$$\dot{W}_t = \dot{m}\omega(r_2\bar{V}_{\theta 2} - r_1\bar{V}_{\theta 1}) \quad (4)$$

where $\dot{m}$ is the mass flow rate, ω is the shaft angular velocity, r is the mean blade radius, $\bar{V}$ is the working fluid flow velocity, while subscripts 1, 2 and θ represent the inlet and outlet to the machine, and tangential (circumferential) component respectively.

Velocity Triangles and Mollier diagrams are used to aid the analysis of the turbomachinery; typically the velocity triangle is a representation of the equation $\bar{V} = \bar{U} + \bar{W}$ at each station, that is, entry to nozzle, and entry and exit to rotor; where V is absolute fluid velocity, U is blade velocity and W is relative velocity of fluid flow to moving blades; refer to figure 1:

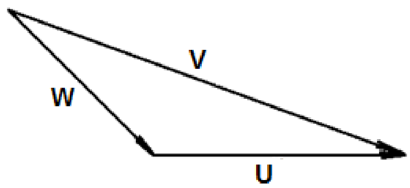

Figure 1: velocity diagram

The Mollier diagram is a plot of enthalpy against entropy for a process in which one property usually pressure or temperature is kept constant [4]; pertaining to turbine expansion process, the mollier diagram aids in visualizing the isentropic and real expansion processes as well as the stagnation and static states of the working fluid; figure 2 shows a typical expansion process on a mollier diagram.

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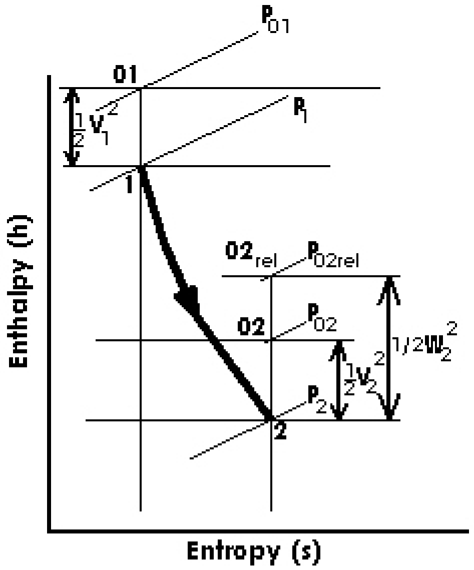

Figure 2: mollier diagram showing a typical expansion process

Stagnation state is represented by state parameters designated as P_0 for stagnation pressure, T_0 for stagnation temperature, and h_0 for stagnation enthalpy; stagnation pressure is a pressure at a state corresponding to zero velocity, a stagnation state, which is representative of an adiabatic throttling process. The throttling process is a representation of flow through inlets, nozzles, stationary turbomachinery blades, and the use of stagnation pressure as a measure of loss is a practice that has widespread application. Stagnation pressure is a key variable in propulsion and power systems.

The stagnation pressure at a given state is defined by the enthalpy equation:

$$h_0 = h + \frac{V^2}{2} \tag{5}$$

where: h_0 is the stagnation enthalpy (J/kg); h is the static enthalpy (J/kg); and V is the fluid speed (m/s)

Rothalpy is a function/property that remains constant throughout a rotating machine, that is, in an adiabatic irreversible process relative to the rotating component [5]. It is defined by the equation:

$$I = h + \frac{W^2}{2} - \frac{U^2}{2} \tag{6}$$

where h is static enthalpy, W is the relative velocity of the fluid, and U is the blade speed.

Thus rothalpy (rotational enthalpy) is conserved between two stations in a rotating reference in any turbomachinery:

$$I_2 = I_3 \tag{7}$$

Stagnation enthalpy is conserved between two points in a fluid flow stream in a non-rotating reference system:

$$h_{01} = h_{02} \tag{8}$$

The degree of reaction is expressed as the relative pressure or enthalpy drop in the nozzle or rotor blades to that of the stage:

Rotor degree of reaction:

$$R_r = \frac{\text{static enthalpy drop in rotor}}{\text{stagnation enthalpy drop in stage}} \tag{9}$$

Nozzle degree of reaction:

$$R_n = \frac{\text{static enthalpy drop in nozzle}}{\text{stagnation enthalpy drop in stage}} \tag{10}$$

III. AXIAL FLOW TURBINE MODEL

Description

The fluid flow in an axial turbine is essentially in a direction parallel to the axis of rotation of the machine. Axial turbines usually have several stages such that each stage only handles a moderate pressure or enthalpy drop. Figure 3 shows a single stage axial turbine rotor.

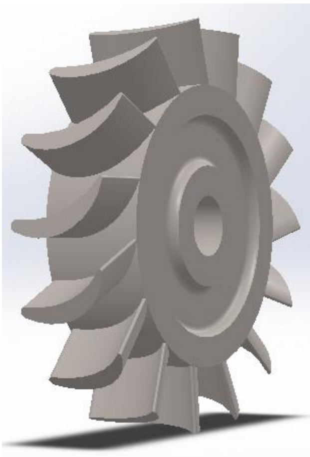

Figure 3: single stage axial turbine rotor

For a single stage the diameter will usually be the same at the turbine inlet and outlet and as such the blade speed remains constant along a flow path; and a combined velocity triangle can be drawn as shown in figure 4.

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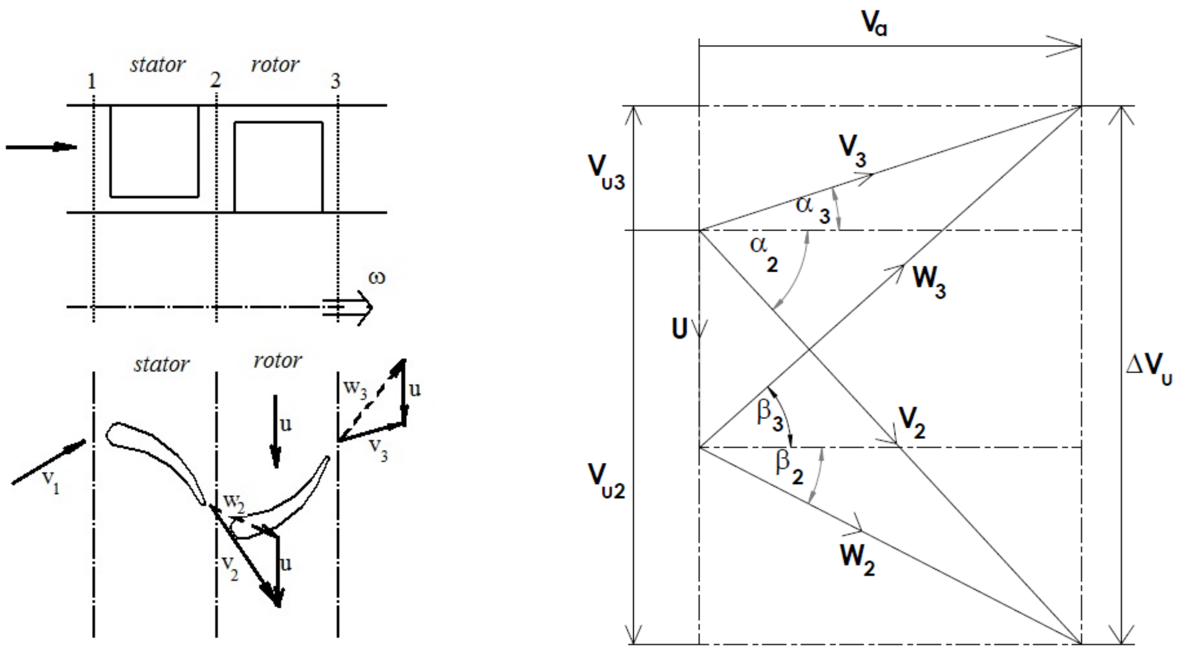

Figure 4: blade arrangement and velocity triangles

Mathematical Model

With reference to figures 4 and 5 the following set of equations can be written for the single stage axial turbine:

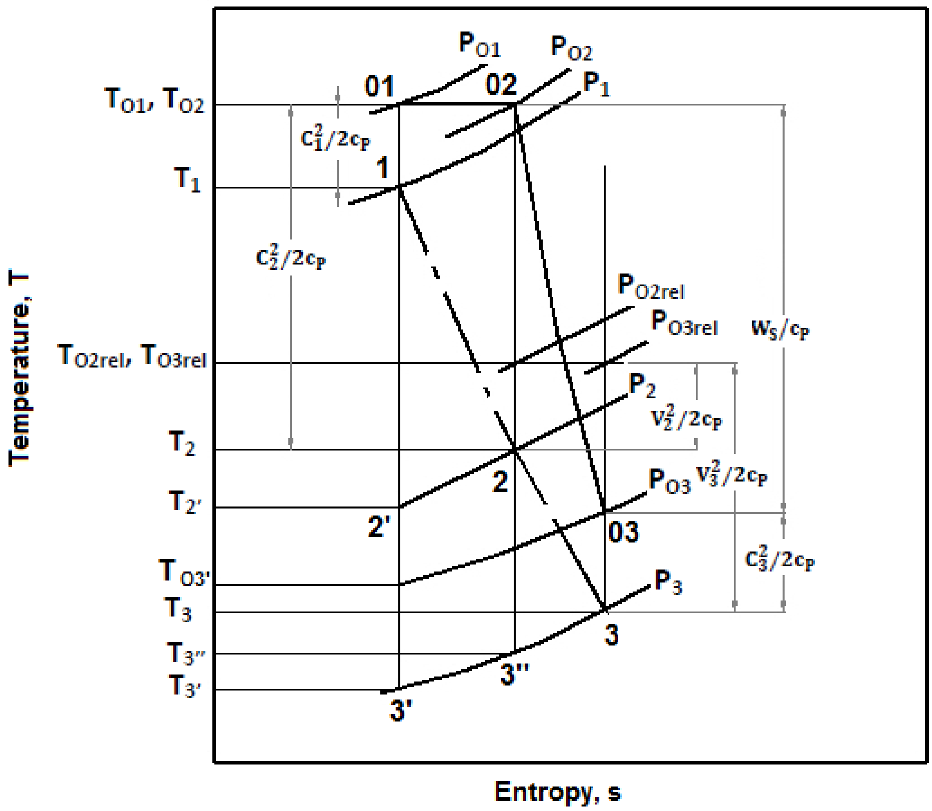

Figure 5: Mollier diagram for an axial turbine stage [6]

The work output per unit mass flow is given by:

$$\dot{w} = U \cdot (V_{u2} + V_{u3}) = U \cdot V_a \cdot (\tan\alpha_2 + \tan\alpha_3) = U \cdot V_a \cdot (\tan\beta_2 + \tan\beta_3) \quad (11)$$

The blade-loading coefficient is used to express work capacity of the stage. It is defined as the ratio of the specific work of the stage to the square of the blade velocity:

$$\Psi = \frac{\dot{w}}{U^2} \quad (12)$$

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The flow coefficient, ϕ, is the ratio of the axial component of the inlet flow velocity to the blade speed:

$$\phi = \frac{V_a}{U} \tag{13}$$

Computer Simulations

Simulations were performed using the engineering equation solver (EES), (Klein, 2014); Soderberg's loss correlations were used (Dixon, 1998). As no convergence could be attained with the given mass flow rates, i.e. from the evaporator model, (Situmbeko and Inambao, 2015), the first simulation was to determine the lowest feasible mass flow rates for all the working fluids by varying the mas flow rates from 0.1 to 2 kg/s; the results showed 0.459 kg/s (instead of 0.241) for isobutene, 0.420 kg/s (instead of 0.207) for n-butane, 0.226 kg/s (instead of 0.396) for R134a and 0.909 kg/s (instead of 0.396) for R245fa. Using these new figures the input conditions are modified and then the simulations progressed; the revised inlet conditions are shown in the following table 1:

Table 1: axial turbine model – revised inlet conditions

Working Fluid	Mass flow rate	Inlet Pressure	Inlet Temperature
	[kg/s]	[Pa]	[C]
R245fa	0.909	810600	80.99
R134a	0.226	810600	80.99
n-butane	0.420	1010000	80.03
isobutane	0.459	1010000	66.82

Three sets of simulations were conducted:

Simulation 1: Rotor exit static pressure was varied within the feasible pressure range and the results are shown in figures 6 and 7; convergence for R245fa could only be attained for pressures 350 to 360 kPa; however, since this range happened to yield higher total-to-total efficiencies, see figure 8, the rotor exit pressure was set constant at 355 kPa for the remainder of the simulations.

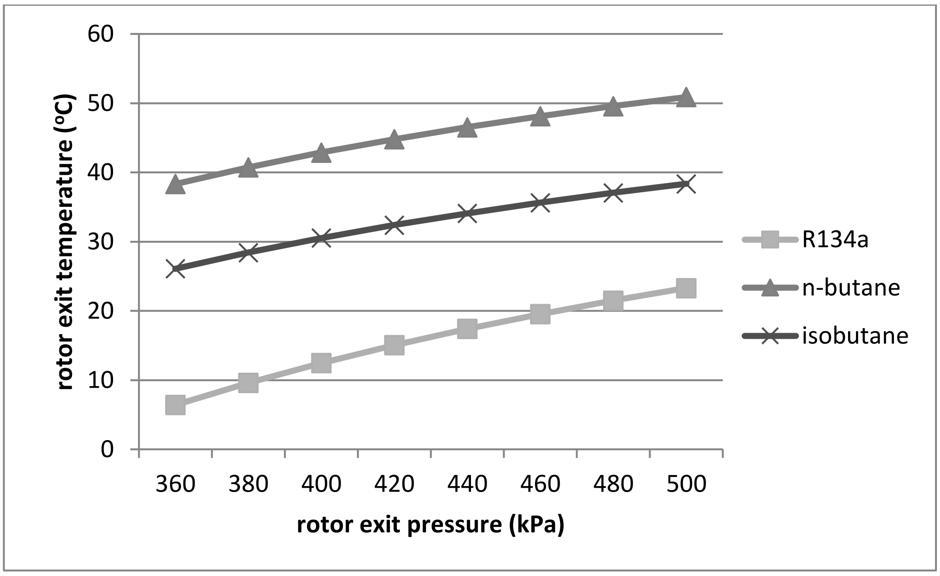

Figure 6: axial turbine model rotor exit - temperature versus pressure

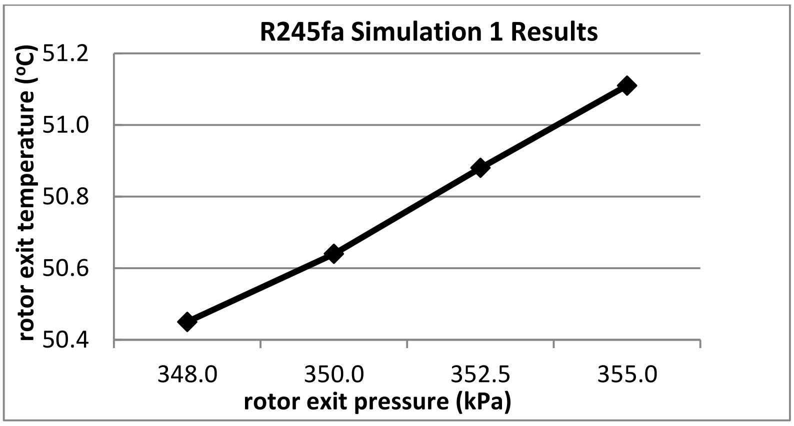

Figure 7: axial turbine model rotor exit - temperature versus pressure for R245fa

Note: The results for n-butane and isobutene appear superimposed in figure 8, although the results for isobutene are slightly superior.

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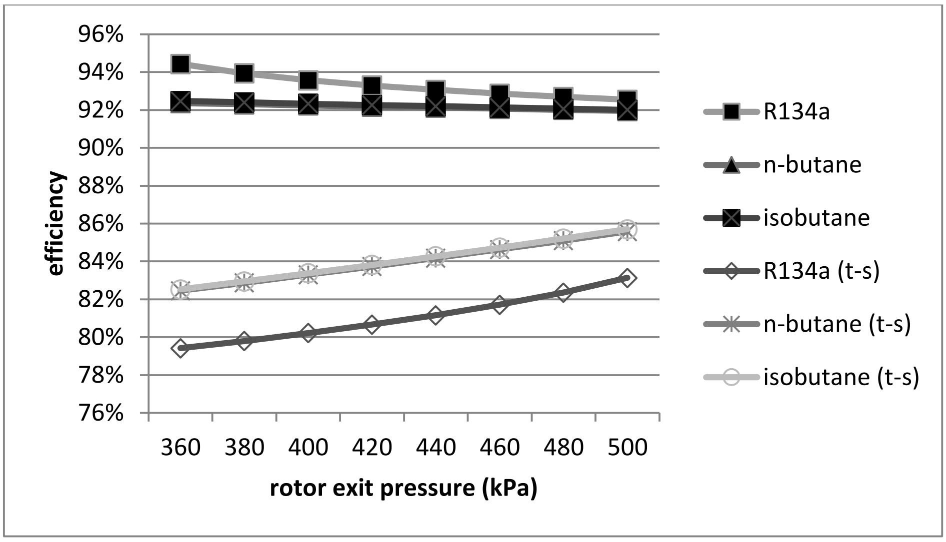

Figure 8: axial turbine model efficiency versus rotor exit pressure
(t-s is for total-to-static; other series are for total-to-total)

With the rotor exit pressure set constant at 355 kPa, the rotor diameter was varied from 26 mm to 160 mm as a way to optimize the machine speed to a lower acceptable level. Results of these simulations are shown in figure 9; from the results it can be seen that any speed between 5000 rpm and 15000 rpm could be considered acceptable; however the speed was set at 20000 rpm as had been done with the radial turbine model (results will be presented in a separate publication). The final optimal results are shown in table 2 and velocity triangles of figures 10 to 13.

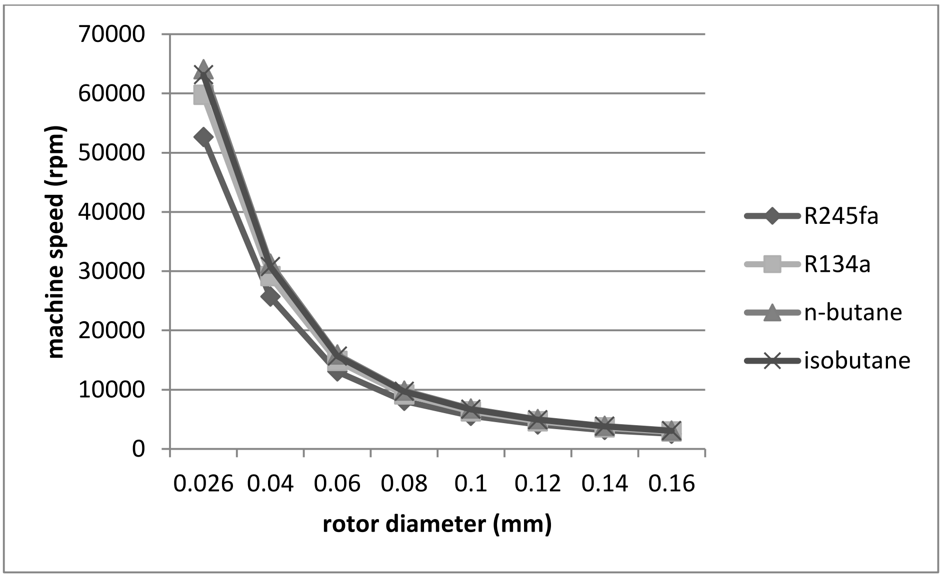

Figure 9: axial turbine model – machine speed versus rotor diameter

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Table 2: axial turbine model simulation results

WF$	m_dot	P_1	P_2	P_3	T_1	T_2	T_3	eta_ts	eta_tt	PR
	[kg/s]	[Pa]	[Pa]	[Pa]	[C]	[C]	[C]	[-]	[-]	
R245fa	2.061	810600	356544	355000	80.99	62.67	61.58	0.8397	0.92	1.777
R134a	1.77	810600	356633	355000	80.99	65.28	63.91	0.8348	0.9196	1.896
n-butane	1.627	1010000	355929	355000	80.03	59.83	59.11	0.863	0.9222	2.226
isobutane	1.657	1010000	355998	355000	66.82	46.23	45.48	0.8632	0.9223	2.211

WF$	C_1	C_2	C_3	U	W_2	W_3	Ma_1	Ma_2	Ma_3	Ma_2rel	Ma_3rel
	[m/s]	[m/s]	[m/s]	[m/s]	[m/s]	[m/s]					
R245fa	44.76	138.2	107.5	55.95	111.9	121.2	0.3478	1.001	0.7809	0.81	0.8803
R134a	48.3	144.6	110.2	60.37	115.1	125.7	0.2932	0.862	0.6586	0.686	0.7509
n-butane	50.37	184.1	156.1	62.97	159.9	168.3	0.2633	0.8666	0.7358	0.753	0.7934
isobutane	49.92	182.6	154.9	62.41	158.6	167	0.2671	0.8772	0.7451	0.762	0.8033

WF$	rpm	d	b	R_s	alpha_1	alpha_2	alpha_3	beta_2	beta_3
	[-/min]	[m]	[m]		[deg]	[deg]	[deg]	[deg]	[deg]
R245fa	20000	0.04646	0.00697	0.1133	0	38.88	0	15.97	27.49
R134a	20000	0.05013	0.00752	0.1204	0	40.33	0	16.76	28.71
n-butane	20000	0.05229	0.00784	0.08103	0	32.01	0	12.51	21.96
isobutane	20000	0.05182	0.00777	0.08094	0	31.99	0	12.5	21.95

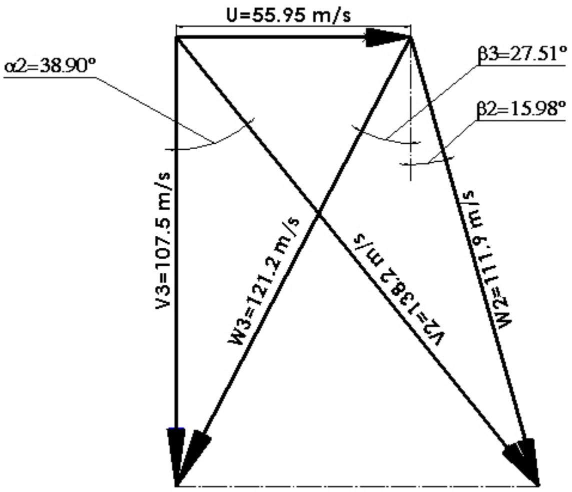

Figure 10: axial turbine velocity triangles for R245fa

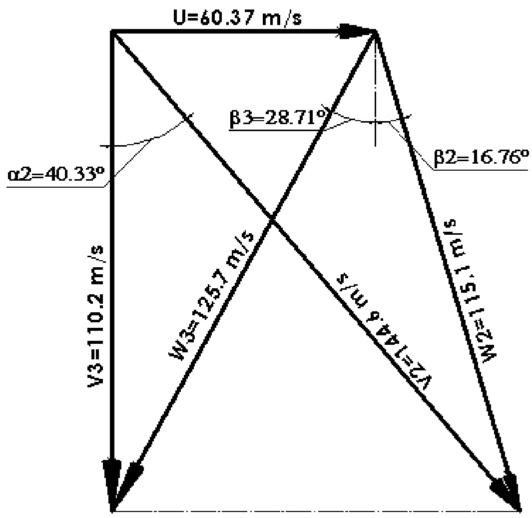

Figure 11: axial turbine velocity triangles for R134a

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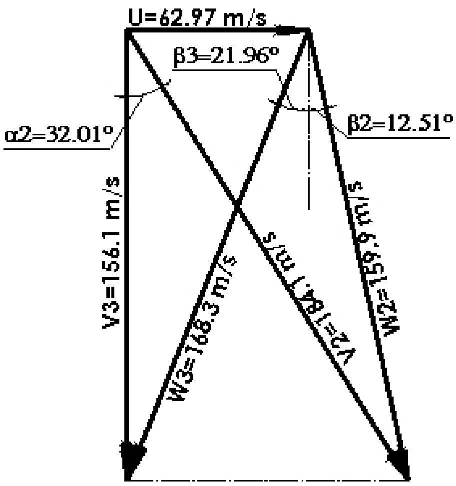

Figure 12: axial turbine velocity triangles for n-butane

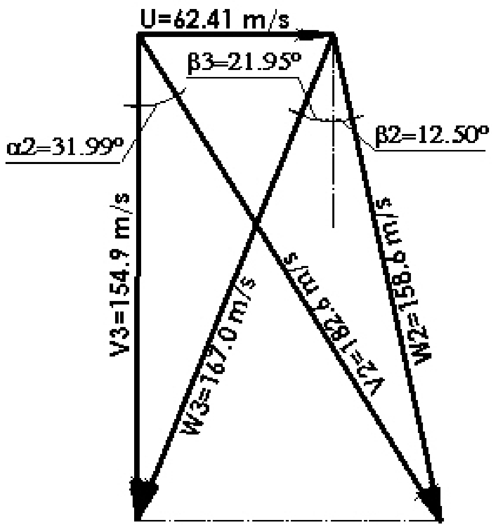

Figure 13: axial turbines velocity triangles for isobutene

IV. DISCUSSIONS AND CONCLUSIONS

The paper has presented the preliminary design models for axial turbines suitable for a 10 kWe low temperature organic Rankine cycle. The preliminary design has been presented in terms of geometric parameters of flow angles, blade diameters and heights; the preliminary design also includes thermodynamic parameters of stagnation and static pressures, temperatures and enthalpy's; the thermodynamic analyses were conducted within the cycle temperature ranges of the evaporator and condenser. Although the presented design models are not complete, this work has shown that small turbines for low temperature cycles are a feasible design option. The turbine preliminary design parameters for the 10 kWe turbine model after parametric optimization are listed in the table 2.

Efficiency: in terms of total-to-total efficiency all the four working fluids performed well of course with varying pressure ratios, mass flow rates and turbines sizes; however when all these factors are taken into account: R245fa requires the least pressure ratio, higher mass flow are and smaller turbine size. R134a performs second best in terms of smaller pressure ratio and smaller turbine size with a remarkable lower (than R245fa) mass flow rate. The performance of the other remaining two working fluids, isobutane and n-butane, is almost a tie, with isobutane having a slighter edge.

In terms of Mach numbers, the flow changes from subsonic to transonic status for all four working fluids but does not extend beyond the sonic stage. This implies that a more detailed study of the blade is required to determine whether the flow passages need to transition from convergent to divergent at any point in the flow passages.

To fully complete the turbine design task it is necessary to employ CFD and FEA analysis and modelling of the detailed blade and nozzle geometry and flow profile design. This would be followed by providing material and manufacturing specifications for prototype construction and testing. AxSTREAM software suite by SoftInWay Inc. is a good package for turbine CFD modelling.

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