Solution of Inverse Thermoelastic Problem of Hollow Cylinder with Internal Heat Generation by Using Integral Transforms

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Abstract

This paper deals with effect of the sine transform on thermoelastic problem of hollow cylinder with internal heat generation. This paper determines the temperature distribution, Thermal displacements and stress functions of hollow cylinder by sine transform and finite Marchi-Zgrablich transform.

Keywords: Inverse thermoelastic problem, Temperature distribution, Stress functions.

1. Introduction

Wankhede P.C. and Deshmukh K.C. [5] investigated an axisymmetric inverse steady-state problem of thermoelastic deformation of finite length hollow cylinder. Sierakowski and Sun [3] have studied an exact solution to the elastic deformation of a finite length hollow cylinder. This paper consist determination of temperature distribution, Thermal displacements and stress function of hollow cylinder with internal heat generation occupying the space$D: a \le r \le b, 0 \le z \le h$. The finite Marchi-Zgrablich transforms and Fourier sine transform technique is used.

2. Statement of the problem

Consider a hollow cylinder of length h occupying the space$D: a \le r \le b, 0 \le z \le h$. The thermoelastic displacement function as in [4] is governed by the Poisson's equation

$$\nabla^2 \emptyset = \frac{(1+v)}{(1-v)} a_t T \tag{1}$$

With $\emptyset = 0$ at $r = a$ and $r = b$

Where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$

$'v'$ And $'a_t'$ are the Poisson's ratio and the linear coefficient of thermal expansion of the material of the cylinder and T is the temperature of the cylinder satisfying the differential equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{g}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{2}$$

Subject to initial condition

$$T[r, z, 0] = 0 \tag{3}$$

The boundary conditions are

$$\left[T + k_1 \frac{\partial T}{\partial r}\right]_{r=a} = F_1(z, t) \tag{4}$$

$$\left[T + k_2 \frac{\partial T}{\partial r}\right]_{r=b} = F_2(z, t) \tag{5}$$

$$T[r, z, t]_{z=\xi} = f(r, t) \text{ (Known)} \tag{6}$$

$$T[r, z, t]_{z=0} = F_3(r, t) \tag{7}$$

The interior condition is

$$T[r, z, t]_{z=h} = \eta(r, t) \text{ (Unknown)} \tag{8}$$

Where α is thermal diffusivity of the material of the cylinder.

The radial and axial displacements U and W satisfying the uncoupled thermoelastic equations as in [3] are

$$\nabla^2 U - \frac{U}{r^2} + (1-2v)^{-1}\frac{\partial e}{\partial r} = 2\frac{(1+v)}{(1-v)} a_t \frac{\partial T}{\partial r} \tag{9}$$

$$\nabla^2 W + (1+2v)^{-1}\frac{\partial e}{\partial z} = 2\frac{(1+v)}{(1-2v)}\frac{\partial T}{\partial z} \tag{10}$$

Where $e = \frac{\partial U}{\partial r} + \frac{U}{r} + \frac{\partial W}{\partial z}$ is the volume dilation and

$$U = \frac{\partial \emptyset}{\partial r} \tag{11}$$

$$W = \frac{\partial \emptyset}{\partial z} \tag{12}$$

The stress functions are given by

$$\tau_{rz}(a,z,t) = 0, \quad \tau_{rz}(b,z,t) = 0, \quad \tau_{rz}(r,z,0) = 0 \tag{13}$$

And

$$\sigma_r(a,z,t) = p_1, \quad \sigma_r(b,z,t) = -p_0, \quad \sigma_r(r,0,t) = 0 \tag{14}$$

Where p_1 and p_0 are the surface pressures assumed to be uniform over the boundaries of the cylinder. The boundary conditions for the stress functions (13) and (14) are expressed in terms of the displacements components by the following relations.

$$\sigma_r = (\lambda + 2G)\frac{\partial U}{\partial r} + \lambda\left[\frac{U}{r} + \frac{\partial W}{\partial z}\right] \tag{15}$$

$$\sigma_z = (\lambda + 2G)\frac{\partial W}{\partial z} + \lambda\left[\frac{\partial U}{\partial r} + \frac{U}{r}\right] \tag{16}$$

$$\sigma_\theta = (\lambda + 2G)\frac{U}{r} + \lambda\left[\frac{\partial U}{\partial r} + \frac{\partial W}{\partial z}\right] \tag{17}$$

$$\tau_{rz} = G\left[\frac{\partial W}{\partial r} + \frac{\partial U}{\partial z}\right] \tag{18}$$

Where $\lambda = \frac{2G\nu}{1-2\nu}$ is the Lame's constant, G is the shear modulus and U and W are the displacement components.

The equation (1) to (18) constitutes the mathematical formulation of the problem.

3. Solution of the problem

The finite Marchi-Zgrablich integral transform of order p is defined as

$$\bar{f}_p(n) = \int_a^b x f(x) S_p(\mathrm{k}_1, \mathrm{k}_2, \mu_n x)\,dx \tag{19}$$

And inverse Marchi-Zgrablich integral transform as

$$f(x) = \sum_{n=1}^{\infty} \frac{\bar{f}_p(n) S_p(\mathrm{k}_1, \mathrm{k}_2, \mu_n x)}{c_n} \tag{20}$$

Where

$$S_p(\mathrm{k}_1, \mathrm{k}_2, \mu_n x) = J_p(\mu_n x)\{G_p(\mathrm{k}_1, \mu_n a) + G_p(\mathrm{k}_2, \mu_n b)\} - G_p(\mu_n x)\{J_p(\mathrm{k}_1, \mu_n a) + J_p(\mathrm{k}_2, \mu_n b)\}$$

$$c_n = \frac{b^2}{2}\{S_p{}^2(\mathrm{k}_1, \mathrm{k}_2, \mu_n b) - S_{p-1}(\mathrm{k}_1, \mathrm{k}_2, \mu_n b).S_{p+1}(\mathrm{k}_1, \mathrm{k}_2, \mu_n b)\} - \{S_p{}^2(\mathrm{k}_1, \mathrm{k}_2, \mu_n a) - S_{p-1}(\mathrm{k}_1, \mathrm{k}_2, \mu_n a).S_{p+1}(\mathrm{k}_1, \mathrm{k}_2, \mu_n a)\}$$

An operational property is given by

$$\int_a^b \left[\frac{\partial^2 f}{\partial x^2} + \frac{1}{x}\frac{\partial f}{\partial x} + \frac{p^2 f}{x^2}\right] S_p{}^2(\mathrm{k}_1, \mathrm{k}_2, \mu_n x) = \frac{b}{\mathrm{k}_2} S_p{}^2(\mathrm{k}_1, \mathrm{k}_2, \mu_n b)\left[f + \mathrm{k}_2 \frac{\partial f}{\partial x}\right]_{x=b} - \frac{a}{\mathrm{k}_1} S_p{}^2(\mathrm{k}_1, \mathrm{k}_2, \mu_n a)\left[f + \mathrm{k}_1 \frac{\partial f}{\partial x}\right]_{x=a} - \mu_n{}^2 \bar{f}_p(n) \tag{21}$$

Also

If $f(x)$ satisfies Dirichelet's conditions in interval $(0, a)$ and if for that range its Fourier sine transform is defined to be

$$f_s^*(n) = \int_0^\xi f(z) \sin\frac{n\pi z}{\xi}\,dz \tag{22}$$

Then at each point $(0, a)$ at which $f(z)$ is continuous,

$$f(z) = \frac{2}{\xi}\sum_{n=1}^{\infty} f_s^*(n)\sin\frac{n\pi z}{\xi} \tag{23}$$

The property of sine transform is

$$\int_0^\xi \frac{\partial^2 f}{\partial z^2}\sin\frac{n\pi z}{\xi}\,dz = \frac{n\pi}{\xi}[(-1)^{n+1} f(\xi) + f(0)] - \frac{m^2\pi^2}{\xi^2} f_s^*(n) \tag{24}$$

By applying the finite Marchi-Zgrablich integral transform stated in (19) to (21) to the equations (2), (3), (6) and (7), using (4), (5) and then applying the Fourier sine transform stated in (22) to (24), by using the boundary conditions and again taking their inverses, it gets

$$T = \frac{2}{\xi}\sum_{m,n=1}^{\infty}\left\{\left[\left\langle e^{-\alpha\left(\mu_m^2 + \frac{n^2\pi^2}{\xi^2}\right)t} \int e^{\alpha\left(\mu_m^2 + \frac{n^2\pi^2}{\xi^2}\right)t} F_q\,dt + T_1\right\rangle \frac{\sin(n\pi z)}{\xi}\right] \frac{s_0(\mathrm{k}_1, \mathrm{k}_2, \mu_m r)}{c_m}\right\} \tag{25}$$

Where

$$F_q = \alpha\left[\frac{b}{k_2} s_0^2(\mathrm{k}_1, \mathrm{k}_2, \mu_m b) F_2^* - \frac{a}{k_1} s_0^2(\mathrm{k}_1, \mathrm{k}_2, \mu_m a) F_1^* + \frac{n\pi}{\xi}[(-1)^{n+1} f + F_3] + \frac{\bar{g}^*}{k}\right]$$

And $T_1 = -\left[\int e^{\alpha\left(\mu_m^2+\frac{n^2\pi^2}{\xi^2}\right)t} F_q\, dt\right]_{t=0}$

Substitute the value from (25) in (8), it gets

$$\eta = \frac{2}{\xi}\sum_{m,n=1}^{\infty}\left\{\left[\langle e^{-\alpha\left(\mu_m^2+\frac{n^2\pi^2}{\xi^2}\right)t}\int e^{\alpha\left(\mu_m^2+\frac{n^2\pi^2}{\xi^2}\right)t}F_q\,dt + T_1\rangle\frac{\sin(n\pi h)}{\xi}\right]\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\right\} \quad (26)$$

4. Determination of Thermoelastic displacement

From (1) and (25), Thermoelastic displacement is given by

$$\emptyset = \frac{(1+\nu)}{(1-\nu)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty}\left\{r^2\left[\langle e^{-\alpha\left(\mu_m^2+\frac{n^2\pi^2}{\xi^2}\right)t}\int e^{\alpha\left(\mu_m^2+\frac{n^2\pi^2}{\xi^2}\right)t}F_q\,dt + T_1\rangle\frac{\sin(n\pi z)}{\xi}\right]\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\right\} \quad (27)$$

From (27), equation (11) and (12) gets the radial and axial displacements as

$$U = \frac{(1+\nu)}{(1-\nu)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty}\left\{r^2\left[\left(F_p'+T_1'\right)\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + \left(F_p+T_1\right)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m}\right] + 2r\left[\left(F_p+T_1\right)\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\right]\right\}\frac{\sin(n\pi z)}{\xi}$$

(28)

$$W = \frac{\partial\emptyset}{\partial z} = \frac{(1+\nu)}{(1-\nu)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty} r^2\left[\left(F_p+T_1\right)\frac{n\pi}{\xi}\frac{\cos n\pi z}{\xi} + \left(F_p'+T_1'\right)\frac{\sin n\pi z}{\xi}\right]\frac{s_0(k_1,k_2,\mu_m r)}{c_m} \quad (29)$$

Where $F_p = e^{-\alpha\left(\mu_m^2+\frac{n^2\pi^2}{\xi^2}\right)t}\int e^{\alpha\left(\mu_m^2+\frac{n^2\pi^2}{\xi^2}\right)t}F_q\,dt$

5. Determination of stress functions

From (27), Equation (15), (16), (17) and (18) gives

$$\sigma_r = (\lambda+2G)\frac{(1+\nu)}{(1-\nu)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty}\left\{r^2\langle\left(F_p''+T_1''\right)\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + \left(F_p'+T_1'\right)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m} + \left(F_p+T_1\right)\frac{s_0''(k_1,k_2,\mu_m r)}{c_m} + \left(F_p'+T_1'\right)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m}\rangle + 2r\langle\left(F_p'+T_1'\right)\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + \left(F_p+T_1\right)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m}\rangle + 2r\langle\left(F_p+T_1\right)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m} + \left(F_p'+T_1'\right)\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\rangle + 2\left(F_p+T_1\right)\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\right\}\frac{\sin n\pi z}{\xi} + \lambda\left\{\frac{1}{r}\frac{(1+\nu)}{(1-\nu)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty}\left[r^2\langle\left(F_p'+T_1'\right)\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + \left(F_p+T_1\right)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m}\rangle + 2r\left(F_p+T_1\right)\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\right]\frac{\sin n\pi z}{\xi}\right\} + \lambda\left\{\frac{(1+\nu)}{(1-\nu)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty} r^2\langle 2\left(F_p'+T_1'\right)\frac{n\pi}{\xi}\frac{\cos n\pi z}{\xi} - \left(F_p+T_1\right)\frac{\sin n\pi z}{\xi}\frac{n^2\pi^2}{\xi^2} + \left(F_p''+T_1''\right)\frac{\sin n\pi z}{\xi}\rangle\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\right\} \quad (30)$$

$$\sigma_z = (\lambda+2G)\left\{\frac{(1+\nu)}{(1-\nu)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty} r^2\langle 2\left(F_p'+T_1'\right)\frac{n\pi}{\xi}\frac{\cos n\pi z}{\xi} - \left(F_p+T_1\right)\frac{\sin n\pi z}{\xi}\frac{n^2\pi^2}{\xi^2} + \left(F_p''+T_1''\right)\frac{\sin n\pi z}{\xi}\rangle\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\right\} + \lambda\frac{(1+\nu)}{(1-\nu)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty}\left\{r^2\langle\left(F_p''+T_1''\right)\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + \left(F_p'+T_1'\right)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m} + \left(F_p+T_1\right)\frac{s_0''(k_1,k_2,\mu_m r)}{c_m} + \left(F_p'+T_1'\right)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m}\rangle + 2r\langle\left(F_p'+T_1'\right)\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + \left(F_p+T_1\right)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m}\rangle + 2r\langle\left(F_p+T_1\right)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m} + \left(F_p'+T_1'\right)\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\rangle + 2\left(F_p+T_1\right)\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\right\}\frac{\sin n\pi z}{\xi} + \lambda\left\{\frac{1}{r}\frac{(1+\nu)}{(1-\nu)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty}\left[r^2\langle\left(F_p'+T_1'\right)\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + \left(F_p+T_1\right)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m}\rangle + 2r\left(F_p+T_1\right)\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\right]\frac{\sin n\pi z}{\xi}\right\} \quad (31)$$

$$\sigma_\theta = (\lambda+2G)\left\{\frac{1}{r}\frac{(1+\nu)}{(1-\nu)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty}\left[r^2\langle\left(F_p'+T_1'\right)\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + \left(F_p+T_1\right)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m}\rangle + 2r\left(F_p+T_1\right)\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\right]\frac{\sin n\pi z}{\xi}\right\} + \lambda\frac{(1+\nu)}{(1-\nu)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty}\left\{r^2\langle\left(F_p''+T_1''\right)\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + \left(F_p'+T_1'\right)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m} + \left(F_p+T_1\right)\frac{s_0''(k_1,k_2,\mu_m r)}{c_m} + \left(F_p'+T_1'\right)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m}\rangle + 2r\langle\left(F_p'+T_1'\right)\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + \left(F_p+T_1\right)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m}\rangle + 2r\langle\left(F_p+T_1\right)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m} + \left(F_p'+T_1'\right)\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\rangle + 2\left(F_p+T_1\right)\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\right\}\frac{\sin n\pi z}{\xi} + \lambda\left\{\frac{(1+\nu)}{(1-\nu)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty} r^2\langle 2\left(F_p'+T_1'\right)\frac{n\pi}{\xi}\frac{\cos n\pi z}{\xi} - \left(F_p+T_1\right)\frac{\sin n\pi z}{\xi}\frac{n^2\pi^2}{\xi^2} + \left(F_p''+T_1''\right)\frac{\sin n\pi z}{\xi}\rangle\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\right\} \quad (32)$$

$$T_{r,z} = G\frac{(1+\nu)}{(1-\nu)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty}\left\{\left[\langle r^2\left(F_p'+T_1'\right) + 2r\left(F_p+T_1\right)\rangle\frac{n\pi}{\xi}\frac{\cos n\pi z}{\xi} + \langle r^2\left(F_p''+T_1''\right) + 2r\left(F_p'+T_1'\right)\rangle\frac{\sin n\pi z}{\xi}\right]\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + \left[r^2\langle\left(F_p + \right.\right.\right.$$

$$T_1)\frac{n\pi}{\xi}\frac{\cos n\pi z}{\xi} + (F_p' + T_1')\frac{\sin n\pi z}{\xi}\rangle\Big]\frac{s_0'(k_1,k_2,\mu_m r)}{c_m} + \Big[r^2\langle(F_p' + T_1')\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + (F_p + T_1)\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\rangle + 2r(F_p + T_1)\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\Big]\frac{n\pi}{\xi}\frac{\cos n\pi z}{\xi} + \Big[r^2\langle(F_p'' + T_1'')\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + (F_p' + T_1')\frac{s_0'(k_1,k_2,\mu_m r)}{c_m}\rangle + 2r(F_p' + T_1')\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\Big]\frac{\sin n\pi z}{\xi}\} \quad (33)$$

6. Special case

Set $f = e^t t(r-a)(\xi-b)$ (34)

And $g = \delta(r-r_0)\delta(t-t_0)$ (35)

Applying Finite Marchi-Zegrablich transform then Fourier sine transform to (35) it gets

$$.\bar{g}^* = A \quad (36)$$

Substitute the value from (34) and (36) in (25) to (33) it gets

$$T = \frac{2}{\xi}\sum_{m,n=1}^{\infty}\{\Big[\langle e^{-\alpha\left(\mu_m^2+\frac{n^2\pi^2}{\xi^2}\right)t}\int e^{\alpha\left(\mu_m^2+\frac{n^2\pi^2}{\xi^2}\right)t}F_q\,dt + T_1\rangle\frac{\sin(n\pi z)}{\xi}\Big]\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\}$$

$$\eta = T = \frac{2}{\xi}\sum_{m,n=1}^{\infty}\{\Big[\langle e^{-\alpha\left(\mu_m^2+\frac{n^2\pi^2}{\xi^2}\right)t}\int e^{\alpha\left(\mu_m^2+\frac{n^2\pi^2}{\xi^2}\right)t}F_q\,dt + T_1\rangle\frac{\sin(n\pi h)}{\xi}\Big]\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\}$$

$$\emptyset = \frac{(1+v)}{(1-v)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty}\{r^2\Big[\langle e^{-\alpha\left(\mu_m^2+\frac{n^2\pi^2}{\xi^2}\right)t}\int e^{\alpha\left(\mu_m^2+\frac{n^2\pi^2}{\xi^2}\right)t}F_q\,dt + T_1\rangle\frac{\sin(n\pi z)}{\xi}\Big]\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\}$$

$$U = \frac{(1+v)}{(1-v)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty}\{r^2\Big[(F_p' + T_1')\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + (F_p + T_1)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m}\Big] + 2r\Big[(F_p + T_1)\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\Big]\}\frac{\sin(n\pi z)}{\xi}$$

$$W = \frac{\partial\emptyset}{\partial z} = \frac{(1+v)}{(1-v)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty}r^2\Big[(F_p + T_1)\frac{n\pi}{\xi}\frac{\cos n\pi z}{\xi} + (F_p' + T_1')\frac{\sin n\pi z}{\xi}\Big]\frac{s_0(k_1,k_2,\mu_m r)}{c_m}$$

$$\sigma_r = (\lambda + 2G)\frac{(1+v)}{(1-v)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty}\{r^2\langle(F_p'' + T_1'')\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + (F_p' + T_1')\frac{s_0'(k_1,k_2,\mu_m r)}{c_m} + (F_p + T_1)\frac{s_0''(k_1,k_2,\mu_m r)}{c_m} + (F_p' + T_1')\frac{s_0'(k_1,k_2,\mu_m r)}{c_m}\rangle + 2r\langle(F_p' + T_1')\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + (F_p + T_1)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m}\rangle + 2r\langle(F_p + T_1)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m} + (F_p' + T_1')\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\rangle + 2(F_p + T_1)\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\}\frac{\sin n\pi z}{\xi} + \lambda\{\frac{1}{r}\frac{(1+v)}{(1-v)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty}\Big[r^2\langle(F_p' + T_1')\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + (F_p + T_1)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m}\rangle + 2r(F_p + T_1)\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\Big]\frac{\sin n\pi z}{\xi}\} + \lambda\{\frac{(1+v)}{(1-v)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty}r^2\langle 2(F_p' + T_1')\frac{n\pi}{\xi}\frac{\cos n\pi z}{\xi} - (F_p + T_1)\frac{\sin n\pi z}{\xi}\frac{n^2\pi^2}{\xi^2} + (F_p'' + T_1'')\frac{\sin n\pi z}{\xi}\rangle\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\}$$

$$\sigma_z = (\lambda + 2G)\{\frac{(1+v)}{(1-v)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty}r^2\langle 2(F_p' + T_1')\frac{n\pi}{\xi}\frac{\cos n\pi z}{\xi} - (F_p + T_1)\frac{\sin n\pi z}{\xi}\frac{n^2\pi^2}{\xi^2} + (F_p'' + T_1'')\frac{\sin n\pi z}{\xi}\rangle\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\} + \lambda\frac{(1+v)}{(1-v)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty}\{r^2\langle(F_p'' + T_1'')\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + (F_p' + T_1')\frac{s_0'(k_1,k_2,\mu_m r)}{c_m} + (F_p + T_1)\frac{s_0''(k_1,k_2,\mu_m r)}{c_m} + (F_p' + T_1')\frac{s_0'(k_1,k_2,\mu_m r)}{c_m}\rangle + 2r\langle(F_p' + T_1')\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + (F_p + T_1)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m}\rangle + 2r\langle(F_p + T_1)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m} + (F_p' + T_1')\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\rangle + 2(F_p + T_1)\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\}\frac{\sin n\pi z}{\xi} + \lambda\{\frac{1}{r}\frac{(1+v)}{(1-v)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty}\Big[r^2\langle(F_p' + T_1')\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + (F_p + T_1)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m}\rangle + 2r(F_p + T_1)\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\Big]\frac{\sin n\pi z}{\xi}\}$$

$$\sigma_\theta = (\lambda + 2G)\{\frac{1}{r}\frac{(1+v)}{(1-v)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty}\Big[r^2\langle(F_p' + T_1')\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + (F_p + T_1)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m}\rangle + 2r(F_p + T_1)\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\Big]\frac{\sin n\pi z}{\xi}\} + \lambda\frac{(1+v)}{(1-v)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty}\{r^2\langle(F_p'' + T_1'')\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + (F_p' + T_1')\frac{s_0'(k_1,k_2,\mu_m r)}{c_m} + (F_p + T_1)\frac{s_0''(k_1,k_2,\mu_m r)}{c_m} + (F_p' + T_1')\frac{s_0'(k_1,k_2,\mu_m r)}{c_m}\rangle + 2r\langle(F_p' + T_1')\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + (F_p + T_1)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m}\rangle + 2r\langle(F_p + T_1)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m} + (F_p' + T_1')\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\rangle + 2(F_p + T_1)\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\}\frac{\sin n\pi z}{\xi} + \lambda\{\frac{(1+v)}{(1-v)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty}r^2\langle 2(F_p' + T_1')\frac{n\pi}{\xi}\frac{\cos n\pi z}{\xi} - (F_p + T_1)\frac{\sin n\pi z}{\xi}\frac{n^2\pi^2}{\xi^2} + (F_p'' + T_1'')\frac{\sin n\pi z}{\xi}\rangle\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\}$$

$T_{r,z} = G\frac{(1+\nu)}{(1-\nu)}\frac{a_t}{2\xi}\sum_{m,n=1}^{\infty}\Big\{\Big[\langle r^2(F_p' + T_1') + 2r(F_p + T_1)\rangle\frac{n\pi}{\xi}\frac{\cos n\pi z}{\xi} + \langle r^2(F_p'' + T_1'') + 2r(F_p' + T_1')\rangle\frac{\sin n\pi z}{\xi}\Big]\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + \Big[r^2\langle(F_p + T_1)\frac{n\pi}{\xi}\frac{\cos n\pi z}{\xi} + (F_p' + T_1')\frac{\sin n\pi z}{\xi}\rangle\Big]\frac{s_0'(k_1,k_2,\mu_m r)}{c_m} + \Big[r^2\langle(F_p' + T_1')\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + (F_p + T_1)\frac{s_0'(k_1,k_2,\mu_m r)}{c_m}\rangle + 2r(F_p + T_1)\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\Big]\frac{n\pi}{\xi}\frac{\cos n\pi z}{\xi} + \Big[r^2\langle(F_p'' + T_1'')\frac{s_0(k_1,k_2,\mu_m r)}{c_m} + (F_p' + T_1')\frac{s_0'(k_1,k_2,\mu_m r)}{c_m}\rangle + 2r(F_p' + T_1')\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\Big]\frac{\sin n\pi z}{\xi}\Big\}$

Where

$F_q = \alpha\Big[\frac{b}{k_2}s_0^2(k_1,k_2,\mu_m b)\frac{te^t(b-a+k_2)}{\xi}\langle\frac{\sin(n\pi\xi)}{n^2\pi^2} - \frac{(\xi-b)\cos(n\pi\xi)}{n\pi} - \frac{b}{n\pi}\rangle - \frac{a}{k_1}s_0^2(k_1,k_2,\mu_m a)\frac{te^t k_1}{\xi}\langle\frac{\sin(n\pi\xi)}{n^2\pi^2} - \frac{(\xi-b)\cos(n\pi\xi)}{n\pi} - \frac{b}{n\pi}\rangle + \frac{n\pi}{\xi}[(-1)^{n+1}e^t t(r-a)(\xi-b) - te^t b(r-a)] + \frac{A}{k}\Big]$

$T_1 = -\Big\{\int e^{\alpha\left(\mu_m^2+\frac{n^2\pi^2}{\xi^2}\right)t}\alpha\Big[\frac{b}{k_2}s_0^2(k_1,k_2,\mu_m b)\frac{te^t(b-a+k_2)}{\xi}\langle\frac{\sin(n\pi\xi)}{n^2\pi^2} - \frac{(\xi-b)\cos(n\pi\xi)}{n\pi} - \frac{b}{n\pi}\rangle - \frac{a}{k_1}s_0^2(k_1,k_2,\mu_m a)\frac{te^t k_1}{\xi}\langle\frac{\sin(n\pi\xi)}{n^2\pi^2} - \frac{(\xi-b)\cos(n\pi\xi)}{n\pi} - \frac{b}{n\pi}\rangle + \frac{n\pi}{\xi}[(-1)^{n+1}e^t t(r-a)(\xi-b) - te^t b(r-a)] + \frac{A}{k}\Big]dt\Big\}_{t=0}$

And

$F_p = e^{-\alpha\left(\mu_m^2+\frac{n^2\pi^2}{\xi^2}\right)t} \times \int e^{\alpha\left(\mu_m^2+\frac{n^2\pi^2}{\xi^2}\right)t}\alpha\Big[\frac{b}{k_2}s_0^2(k_1,k_2,\mu_m b)\frac{te^t(b-a+k_2)}{\xi}\langle\frac{\sin(n\pi\xi)}{n^2\pi^2} - \frac{(\xi-b)\cos(n\pi\xi)}{n\pi} - \frac{b}{n\pi}\rangle - \frac{a}{k_1}s_0^2(k_1,k_2,\mu_m a)\frac{te^t k_1}{\xi}\langle\frac{\sin(n\pi\xi)}{n^2\pi^2} - \frac{(\xi-b)\cos(n\pi\xi)}{n\pi} - \frac{b}{n\pi}\rangle + \frac{n\pi}{\xi}[(-1)^{n+1}e^t t(r-a)(\xi-b) - te^t b(r-a)] + \frac{A}{k}\Big]dt$

7. Numerical results

Take *a=1.5m, b=2m, h=4m,* $\xi = 1m$, *k=0.041*

$\eta = T = 2\sum_{m,n=1}^{\infty}\Big\{\Big[\langle e^{-\alpha(\mu_m^2+9.85n^2)t}\int e^{\alpha(\mu_m^2+9.85n^2)t}F_q\,dt + T_1\rangle\sin(12.56n)\Big]\frac{s_0(k_1,k_2,\mu_m r)}{c_m}\Big\}$

Where

$F_q = \alpha\Big[\frac{2}{k_2}s_0^2(k_1,k_2,\mu_m b)te^t(0.5+k_2)\langle\frac{\sin(3.14n)}{9.85n^2} + \frac{\cos(3.14n)}{3.14n} - \frac{0.63}{n}\rangle - \frac{1.5}{k_1}s_0^2(k_1,k_2,\mu_m 1.5)te^t k_1\langle\frac{\sin(3.14n)}{9.85n^2} + \frac{\cos(3.14n)}{3.14n} - \frac{0.63}{n}\rangle + 3.14n[(-1)^{n+2}e^t t(r-1.5) - te^t b(r-1.5)] + \frac{A}{0.041}\Big]$

Conclusions

In this paper temperature distribution, thermoelastic displacements, thermal stresses have been determined for the hollow cylinder with internal heat generation. The temperature distribution and thermal stresses have been investigated by using finite Marchi-Zgrablich and Fourier sine transform. The results are obtained in the form of Bessel functions and infinite series.

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